

Reminders from Lecture 5

LU Factorization

$$A = A^{(1)} \sim A^{(2)} \sim \dots \sim A^{(n-1)} \sim A^{(n)} = U$$

$$A^{(j)} \sim \{\mu_{(j+1)j}, \dots, \mu_{nj}\} A^{(j+1)}$$

LU Factorization

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$$A = \begin{bmatrix} 1 & 0 & & & 0 \\ -\mu_{21} & 1 & & & \\ -\mu_{31} & -\mu_{32} & \ddots & & \\ & & \ddots & 1 & 0 \\ -\mu_{n1} & & & -\mu_{n(n-1)} & 1 \end{bmatrix} U$$

LU Factorization

$$\begin{bmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \\ -3 & -2 & 1 \end{bmatrix} \sim \{\mu_{21} = 2, \mu_{31} = 3\} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 8 \\ 0 & 4 & 10 \end{bmatrix}$$
$$\sim \{\mu_{32} = -4/5\} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 8 \\ 0 & 0 & 18/5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \\ -3 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 4/5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 8 \\ 0 & 0 & 18/5 \end{bmatrix}$$

LU Factorization with Partial Pivoting

- ▶ Avoid large multipliers, so at stage j swap rows j and k s.t.

$$\left| a_{kj}^{(j)} \right| = \max_{i=j, \dots, n} \left| a_{ij}^{(j)} \right|$$

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$$P^{(kj)} A^{(j)} \sim \{ \mu_{(j+1)j}, \dots, \mu_{nj} \} A^{(j+1)}$$

$P^{(kj)}$ - row interchange matrix;
obtained from I by swapping its j th, k th rows.

LU Factorization with Partial Pivoting

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$$|\mu_{\ell j}| \leq 1, \quad j = 1, \dots, n-1, \ell = j+1, \dots, n$$

LU Factorization with Partial Pivoting

$$\begin{bmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \\ -3 & -2 & 1 \end{bmatrix} \sim 1 \leftrightarrow 3$$

$$\sim \mu_{21} = -2/3, \mu_{31} = 1/3$$

$$\sim \mu_{32} = -4/7$$

$$\begin{bmatrix} -3 & -2 & 1 \\ -2 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} -3 & -2 & 1 \\ 0 & 7/3 & 4/3 \\ 0 & 4/3 & 10/3 \end{bmatrix}$$

$$\begin{bmatrix} -3 & -2 & 1 \\ 0 & 7/3 & 4/3 \\ 0 & 0 & 18/7 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \\ -3 & -2 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2/3 & 1 & 0 \\ -1/3 & 4/7 & 1 \end{bmatrix} \begin{bmatrix} -3 & -2 & 1 \\ 0 & 7/3 & 4/3 \\ 0 & 0 & 18/7 \end{bmatrix}$$