Quaternary Hadamard Codes and Their Intersections

Mercè Villanueva
Universitat Autònoma de Barcelona
merce.villanueva@uab.cat

(joint work with Josep Rifà and Faina Solov'eva)

A *Hadamard matrix* H of order n is an $n \times n$ matrix of $+1$'s and -1's such that $HH^T = nI$, where I is the $n \times n$ identity matrix. We can change the first row and column of H into $+1$'s and we obtain an equivalent Hadamard matrix H', which is called *normalized*. If $+1$'s are replaced by 0's and -1's by 1's, H' is changed into a *(binary) Hadamard matrix* $c(H')$. The binary code H consisting of the rows of $c(H')$ and their complements is called a *(binary) Hadamard code*. A Hadamard code of length n has $2n$ codewords and minimum distance $n/2$.

Let $\mathbb{Z}_4^\beta$ be the set of all quaternary vectors of length β. A subgroup $\mathcal{C}$ of $\mathbb{Z}_4^\beta$ is called a *quaternary linear code*. Since $\mathcal{C}$ is a subgroup of $\mathbb{Z}_4^\beta$, it is isomorphic to $\mathbb{Z}_2^\gamma \times \mathbb{Z}_4^\delta$ and $|\mathcal{C}| = 2^\gamma 4^\delta$. We call such code $\mathcal{C}$ a *quaternary linear code of type* $2^\gamma 4^\delta$, and $C = \Phi(\mathcal{C})$ the corresponding $\mathbb{Z}_4$*-linear code*, where $\Phi : \mathbb{Z}_4^\beta \rightarrow \mathbb{Z}_2^{2\beta}$ is the usual Gray map, that is, $\Phi(y_1, \ldots, y_\beta) = (\varphi(y_1), \ldots, \varphi(y_\beta))$, and $\varphi(0) = (0,0), \varphi(1) = (0,1), \varphi(2) = (1,1)$, $\varphi(3) = (1,0)$.

A *quaternary extended perfect code* is a quaternary linear code such that, after the Gray map, give a code with the parameters of an extended perfect $\mathbb{Z}_4$-linear code. For each $\delta \in \{1, \ldots, \lfloor (t+1)/2 \rfloor\}$ there exists a unique (up to isomorphism) extended perfect $\mathbb{Z}_4$-linear code C of binary length $2^t \geq 16$, such that the $\mathbb{Z}_4$-dual code of C is of type $2^\gamma 4^\delta$, where $\gamma = t + 1 - 2\delta$. A *quaternary Hadamard code* is the dual code of a quaternary extended perfect code. The intersection problem for binary perfect codes was proposed by Etzion and Vardy in 1998. For Hadamard codes of length 2^t and of length $2^t s$ (s odd and $t \geq 6$), the intersection problem was solved in 2006, as long as there exists a Hadamard matrix of length $4s$. In this work, we will study the intersection problem for quaternary Hadamard codes.

The possibilities for the number of words in the intersection of two quaternary Hadamard codes $\mathcal{C}_1$ and $\mathcal{C}_2$ of the same length are computed. *For any* $t \geq 3$ *there exist two quaternary linear Hadamard codes* $\mathcal{C}_1$ *and* $\mathcal{C}_2$ *of length* $\beta = 2^{t-1}$, *such that* $\eta(\mathcal{C}_1, \mathcal{C}_2) = 2^l$, *where* l *is any value from* 2 *to* $t+1$. Moreover, codes which have any of this given intersection values are constructed. For any two quaternary Hadamard codes $\mathcal{C}_1$ and $\mathcal{C}_2$, the abelian group structure of the intersection $\mathcal{C}_1 \cap \mathcal{C}_2$ is characterized. The parameters of this abelian group structure corresponding to the intersection codes are computed, lower and upper bounds for these parameters are established and the existence of quaternary linear codes with these parameters is given.

MSC2000: 94B05, 94B25, 05B20.

Keywords: intersection, quaternary Hadamard codes.

.