Resolutions of $t$–designs were studied as early as 1847 by Reverend T. P. Kirkman [2, 3] who proposed the famous 15 schoolgirls problem (see also [1]). Kirkman’s problem is equivalent to finding a resolvable 2-(15, 3, 1) design with $r = 7$, and $b = 35$. We define and discuss $\tau$-resolutions of $t$-designs, large sets of $t$-designs, orthogonal resolutions of $t$-designs, Room rectangles [4, 5], and Steiner tableaux. We briefly discuss recursive constructions of large sets, but spend more time on techniques for constructing starter large sets which can then be used in the recursive techniques to obtain infinite families of large sets. We pay particular attention to coherence techniques, i.e. construction methods which assume particular automorphism groups under which the above combinatorial objects are invariant. We give examples of large sets and super-large sets constructed by means of groups. We discuss infinite families of semiregular large sets arising from 3-homogeneous actions of the groups $PSL_2(q)$ on the projective line, and time permitting, we discuss the construction of certain sporadic, as well as infinite families of Room rectangles arising from block-transitive but imprimitive group actions. Finally, we present some tantalizing open problems.

References