Strong Monochromatic Connectivity of Digraphs

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(joint work with Diego González-Moreno and Juan José Montellano)

An interesting generalization of the concept of connectivity in graphs, due to Chartrand, Johns, McKeon and Zhang [2], is the rainbow connecting colorings. An edge-colored graph G is *rainbow connected* if there exists a path, with no two edges colored the same, between any two vertices of G. For more information on rainbow connectivity see the book of Li and Sun [3]. Caro and Yuster [1], as a naturally oposite question, introduced the concept of monochromatic-connecting coloring of a graph. An edge-coloring of a graph G is a *monochromatic-connecting coloring* if there exists a monochromatic path between any two vertices of G. The above definition can be naturally extended for digraphs. An arc-coloring of a digraph D is a *strongly monochromatic-connecting coloring* (SMC-coloring, for short) if for every pair u, v of vertices in D there exists an (u, v)-monochromatic path and a (v, u)-monochromatic path. Since every strongly connected digraph has an SMC-coloring, a natural question is: which is the maximum number of colors that can have an SMC-coloring? The *strong monochromatic connection number* of a strong digraph D, denoted by $smc(D)$, is defined as the maximum number of colors used in an SMC-coloring of D. In this talk we show that if D is a strongly connected digraph with size m, then $smc(D) = m - \Omega(D) + 1$, where $\Omega(D)$ is the minimum size of a spanning strongly connected subdigraph of D.

References

[1] Y. Caro, R. Yuster, *Colorful monochromatic connectivity*, Disc. Math., **311** (2011), 1786–1792.

[2] G. Chartrand, G.L. Johns, K.A. McKeon, P. Zhang, *Rainbow connection in graphs*, Mathematica Bohemica, **133** (2008), 85–98.

[3] X. Li, Y. Sun, *Rainbow Connections of Graphs*, Springer, London, 2013.

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