Decompositions of Complete Graphs into Circulants

Mariusz Meszka
AGH University of Science and Technology, Kraków, Poland
meszka@agh.edu.pl

(joint work with Roman Nedela, Alexander Rosa, Martin Škoviera)

For a positive integer n and a set $S \subseteq \{1, 2, \ldots, \lfloor \frac{n}{2} \rfloor\}$, a *circulant* $C(n; S)$ of *order* n and *connection* set S is a graph $G = (V, E)$ such that $V = \mathbb{Z}_n$ and $E = \{\{u, v\} : \delta(u, v) \in S\}$ where $\delta(u, v) = \min\{\pm|u - v| \pmod n\}$. Circulants are Cayley graphs of the cyclic group, and are recognized as an important class of vertex-transitive graphs.

Let $C(n; S)$ be a fixed circulant. The main problem is to determine the spectrum for values of v such that the complete graph K_v admits an edge-disjoint decomposition into subgraphs each of which is isomorphic to $C(n; S)$. A complete solution in not to be expected since some instances correspond to famous existence problems (for instance, the existence of BIBD's with $\lambda = 1$). On the other hand, the well-known result of Richard Wilson guarantees the asymptotic existence of a decomposition.

Results with respect to circulants of small degree will be discussed. In particular, the existence spectrum for Moebius ladders M_4, M_5, M_6 and the prism Pr_5 will be presented.

MSC2000: 05C70.

Keywords: graph decomposition; circulant.