Combinatorial problems from synchronization theory

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A finite automaton is *synchronizing* if there is a sequence of inputs (a *reset word*) which brings it to a known state from any initial state. The big open problem is the *Černý conjecture*: if an n-state automaton is synchronizing, then it has a reset word of length at most $(n-1)^2$.

Synchronization can be formulated in the language of transformation monoids: an automaton is a transformation monoid with a distinguished generating set; it is synchronizing if it contains a transformation of rank 1.

In the last eight years or so, we have developed the theory of *synchronizing groups*, permutation groups G with the property that the monoid $\langle a, G \rangle$ is synchronizing for all non-permutations a.

It turns out that a monoid is synchronizing if and only if it is not contained in the endomorphism monoid of a graph with clique number equal to chromatic number. Synchronizing groups are necessarily primitive, and form an interesting class between primitive and 2-homogeneous.

If you try to decide which groups in some class are synchronizing, you get very quickly into difficult problems in combinatorics, finite geometry, design theory, and such areas. The point of the talk is to outline this process, and to bring to the attention of experts some of the problems that arise.

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