On Edge Orbits and Cyclic and r-Pyramidal Hypergraph Designs

Saad I. El-Zanati

Department of Mathematics, Illinois State University,
Normal, IL 61790-4520, USA

Let $\mathbb{Z}_n$ denote the group of integers modulo n and let $\mathcal{E}_n^{(k)}$ be the set of all k-element subsets of $\mathbb{Z}_n$ where $1 \leq k < n$. If $E \in \mathcal{E}_n^{(k)}$, let $[E] = \{E + r : r \in \mathbb{Z}_n\}$. Then $[E]$ is the orbit of E where $\mathbb{Z}_n$ acts on $\mathcal{E}_n^{(k)}$ via $(r, E) \mapsto E + r$. Furthermore, $\{[E] : E \in \mathcal{E}_n^{(k)}\}$ is a partition of $\mathcal{E}_n^{(k)}$ into $\mathbb{Z}_n$-orbits. We show how to count the total number of $\mathbb{Z}_n$-orbits of $\mathcal{E}_n^{(k)}$, count the number of orbits of each size, and determine the corresponding results when fixed points are introduced. We also give an application to cyclic and r-pyramidal decompositions of certain classes of uniform hypergraphs into isomorphic subgraphs.