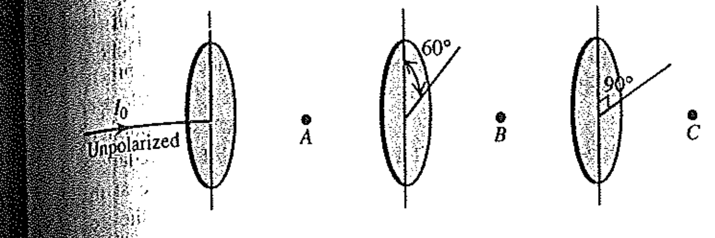
Figure E33.29

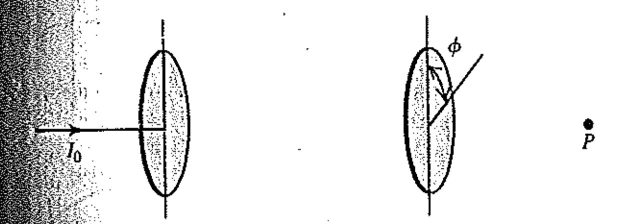


33.30 • Light traveling in water strikes a glass plate at an angle of incidence of 53.0°; part of the beam is reflected and part is refracted. If the reflected and refracted portions make an angle of 90.0° with each other, what is the index of refraction of the glass?

33.31 • A parallel beam of unpolarized light in air is incident at an angle of 54.5° (with respect to the normal) on a plane glass surface. The reflected beam is completely linearly polarized. (a) What is the refractive index of the glass? (b) What is the angle of refraction of the transmitted beam?

33.32 •• Light of original intensity I_0 passes through two ideal polarizing filters having their polarizing axes oriented as shown in Fig. E33.32. You want to adjust the angle ϕ so that the intensity at point P is equal to $I_0/10$. (a) If the original light is unpolarized, what should ϕ be? (b) If the original light is linearly polarized in the same direction as the polarizing axis of the first polarizer the light reaches, what should ϕ be?

Figure E33.32



33.33 •• A beam of polarized light passes through a polarizing filter. When the angle between the polarizing axis of the filter and the direction of polarization of the light is θ, the intensity of the emerging beam is I. If you now want the intensity to be $I/2$, what should be the angle (in terms of θ) between the polarizing angle of the filter and the original direction of polarization of the light?

33.34 • The refractive index of a certain glass is 1.66. For what incident angle is light reflected from the surface of this glass completely polarized if the glass is immersed in (a) air and (b) water?

33.35 •• Unpolarized light of intensity 20.0 W/cm^2 is incident on two polarizing filters. The axis of the first filter is at an angle of 25.0° counterclockwise from the vertical (viewed in the direction the light is traveling), and the axis of the second filter is at 62.0° counterclockwise from the vertical. What is the intensity of the light after it has passed through the second polarizer?

33.36 ••• Three polarizing filters are stacked, with the polarizing axes of the second and third filters at 23.0° and 62.0°, respectively, to that of the first. If unpolarized light is incident on the stack, the light has intensity 75.0 W/cm^2 after it passes through the stack. If the incident intensity is kept constant, what is the intensity of the light after it has passed through the stack if the second polarizer is removed?

33.37 • **Three Polarizing Filters.** Three polarizing filters are stacked with the polarizing axes of the second and third at 45.0° and 90.0°, respectively, with that of the first. (a) If unpolarized light of intensity I_0 is incident on the stack, find the intensity and state of polarization of light emerging from each filter. (b) If the second filter is removed, what is the intensity of the light emerging from each remaining filter?

Section 33.6 Scattering of Light

33.38 • A beam of white light passes through a uniform thickness of air. If the intensity of the scattered light in the middle of the green part of the visible spectrum is I, find the intensity (in terms of I) of scattered light in the middle of (a) the red part of the spectrum and (b) the violet part of the spectrum. Consult Table 32.1.

PROBLEMS

33.39 •• **The Corner Reflector.** An inside corner of a cube is lined with mirrors to make a corner reflector (see Example 33.3 in Section 33.2). A ray of light is reflected successively from each of three mutually perpendicular mirrors; show that its final direction is always exactly opposite to its initial direction.

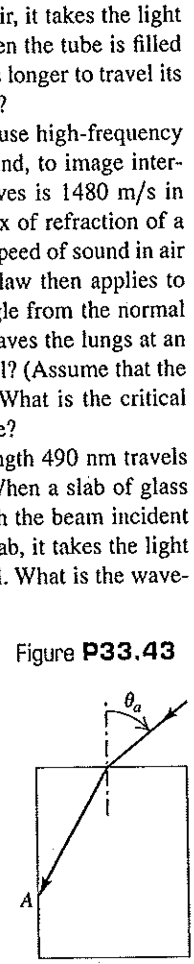
33.40 • A light beam is directed parallel to the axis of a hollow cylindrical tube. When the tube contains only air, it takes the light 8.72 ns to travel the length of the tube, but when the tube is filled with a transparent jelly, it takes the light 2.04 ns longer to travel its length. What is the refractive index of this jelly?

33.41 •• **BIO Heart Sonogram.** Physicians use high-frequency ($f = 1{-}5$ MHz) sound waves, called ultrasound, to image internal organs. The speed of these ultrasound waves is 1480 m/s in muscle and 344 m/s in air. We define the index of refraction of a material for sound waves to be the ratio of the speed of sound in air to the speed of sound in the material. Snell's law then applies to the refraction of sound waves. (a) At what angle from the normal does an ultrasound beam enter the heart if it leaves the lungs at an angle of 9.73° from the normal to the heart wall? (Assume that the speed of sound in the lungs is 344 m/s.) (b) What is the critical angle for sound waves in air incident on muscle?

33.42 ••• In a physics lab, light with wavelength 490 nm travels in air from a laser to a photocell in 17.0 ns. When a slab of glass 0.840 m thick is placed in the light beam, with the beam incident along the normal to the parallel faces of the slab, it takes the light 21.2 ns to travel from the laser to the photocell. What is the wavelength of the light in the glass?

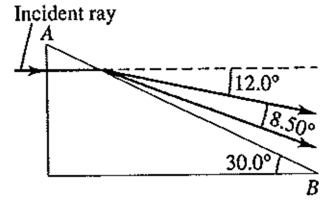
33.43 •• A ray of light is incident in air on a block of a transparent solid whose index of refraction is n. If $n = 1.38$, what is the *largest* angle of incidence θ_a for which total internal reflection will occur at the vertical face (point A shown in Fig. P33.43)?

Figure P33.43



33.44 • A light ray in air strikes the right-angle prism shown in Fig. P33.44. The prism angle at B is 30.0°. This ray consists of two different wavelengths. When it emerges at face AB, it has been split into two different rays that diverge from each other by 8.50°. Find the index of refraction of the prism for each of the two wavelengths.

Figure P33.44



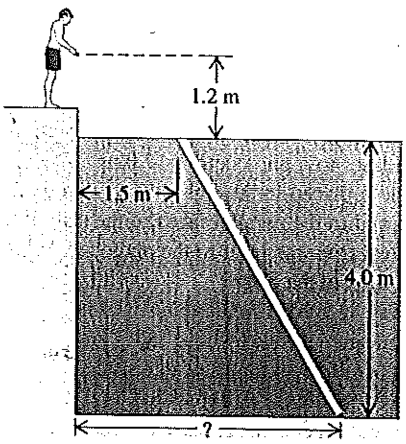
33.45 •• A ray of light traveling *in* a block of glass ($n = 1.52$) is incident on the top surface at an angle of 57.2° with respect to the normal in the glass. If a layer of oil is placed on the top surface of the glass, the ray is totally reflected. What is the maximum possible index of refraction of the oil?

33.46 ••• A glass plate 2.50 mm thick, with an index of refraction of 1.40, is placed between a point source of light with wavelength 540 nm (in vacuum) and a screen. The distance from source to screen is 1.80 cm. How many wavelengths are there between the source and the screen?

33.47 •• Old photographic plates were made of glass with a light-sensitive emulsion on the front surface. This emulsion was somewhat transparent. When a bright point source is focused on the front of the plate, the developed photograph will show a halo around the image of the spot. If the glass plate is 3.10 mm thick and the halos have an inner radius of 5.34 mm, what is the index of refraction of the glass? (*Hint:* Light from the spot on the front surface is scattered in all directions by the emulsion. Some of it is then totally reflected at the back surface of the plate and returns to the front surface.)

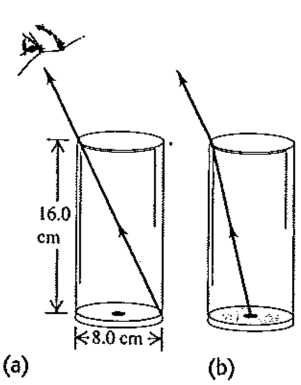
33.48 • After a long day of driving you take a late-night swim in a motel swimming pool. When you go to your room, you realize that you have lost your room key in the pool. You borrow a powerful flashlight and walk around the pool, shining the light into it. The light shines on the key, which is lying on the bottom of the pool, when the flashlight is held 1.2 m above the water surface and is directed at the surface a horizontal distance of 1.5 m from the edge (Fig. P33.48). If the water here is 4.0 m deep, how far is the key from the edge of the pool?

Figure **P33.48**



33.49 • You sight along the rim of a glass with vertical sides so that the top rim is lined up with the opposite edge of the bottom (Fig. P33.49a). The glass is a thin-walled, hollow cylinder 16.0 cm high. The diameter of the top and bottom of the glass is 8.0 cm. While you keep your eye in the same position, a friend fills the glass with a transparent liquid, and you then see a dime that is lying at the center of the bottom of the glass (Fig. P33.49b). What is the index of refraction of the liquid?

Figure **P33.49**

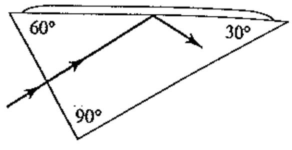


33.50 •• A 45°–45°–90° prism is immersed in water. A ray of light is incident normally on one of its shorter faces. What is the minimum index of refraction that the prism must have if this ray is to be totally reflected within the glass at the long face of the prism?

33.51 • A thin layer of ice ($n = 1.309$) floats on the surface of water ($n = 1.333$) in a bucket. A ray of light from the bottom of the bucket travels upward through the water. (a) What is the largest angle with respect to the normal that the ray can make at the ice–water interface and still pass out into the air above the ice? (b) What is this angle after the ice melts?

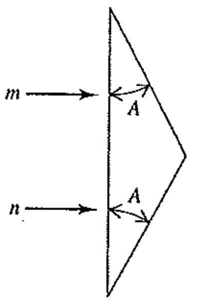
33.52 •• Light is incident normally on the short face of a 30°–60°–90° prism (Fig. P33.52). A drop of liquid is placed on the hypotenuse of the prism. If the index of refraction of the prism is 1.62, find the maximum index that the liquid may have if the light is to be totally reflected.

Figure **P33.52**



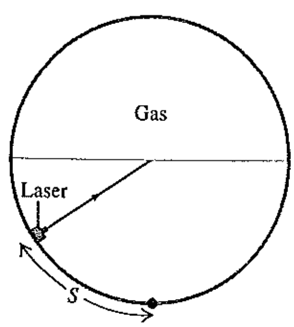
33.53 •• The prism shown in Fig. P33.53 has a refractive index of 1.66, and the angles A are 25.0°. Two light rays m and n are parallel as they enter the prism. What is the angle between them after they emerge?

Figure **P33.53**



33.54 •• A horizontal cylindrical tank 2.20 m in diameter is half full of water. The space above the water is filled with a pressurized gas of unknown refractive index. A small laser can move along the curved bottom of the water and aims a light beam toward the center of the water surface (Fig. P33.54). You observe that when the laser has moved a distance $S = 1.09$ m or more (measured along the curved surface) from the lowest point in the water, no light enters the gas. (a) What is the index of refraction of the gas? (b) What minimum time does it take the light beam to travel from the laser to the rim of the tank when (i) $S > 1.09$ m and (ii) $S < 1.09$ m?

Figure **P33.54**



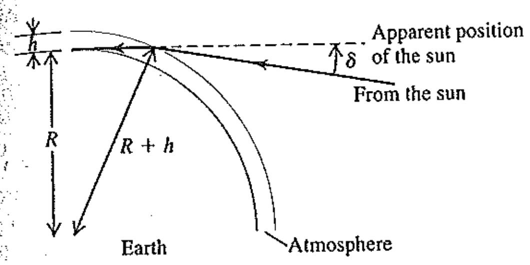
33.55 •• When the sun is either rising or setting and appears to be just on the horizon, it is in fact *below* the horizon. The explanation for this seeming paradox is that light from the sun bends slightly when entering the earth's atmosphere, as shown in Fig. P33.55. Since our perception is based on the idea that light travels in straight lines, we perceive the light to be coming from an apparent position that is an angle δ above the sun's true position. (a) Make the simplifying assumptions that the atmosphere has uniform density, and hence uniform index of refraction n, and extends to a height h above the earth's surface, at which point it abruptly stops. Show that the angle δ is given by

$$\delta = \arcsin\left(\frac{nR}{R+h}\right) - \arcsin\left(\frac{R}{R+h}\right)$$

where $R = 6378$ km is the radius of the earth. (b) Calculate δ using $n = 1.0003$ and $h = 20$ km. How does this compare to the angular radius of the sun, which is about one quarter of a degree?

(In actuality a light ray from the sun bends gradually, not abruptly, since the density and refractive index of the atmosphere change gradually with altitude.)

Figure **P33.55**

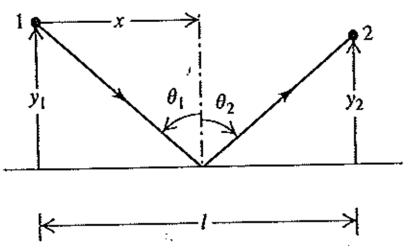


33.56 •• CALC Fermat's Principle of Least Time. A ray of light traveling with speed c leaves point 1 shown in Fig. P33.56 and is reflected to point 2. The ray strikes the reflecting surface a horizontal distance x from point 1. (a) Show that the time t required for the light to travel from 1 to 2 is

$$t = \frac{\sqrt{y_1^2 + x^2} + \sqrt{y_2^2 + (l - x)^2}}{c}$$

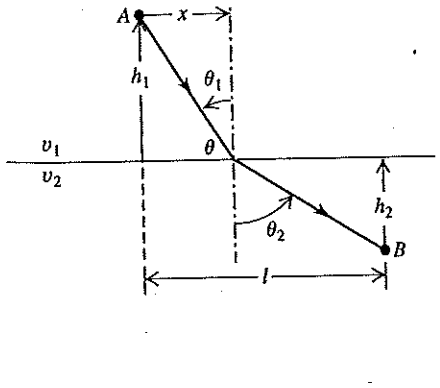
(b) Take the derivative of t with respect to x. Set the derivative equal to zero to show that this time reaches its *minimum* value when $\theta_1 = \theta_2$, which is the law of reflection and corresponds to the actual path taken by the light. This is an example of Fermat's *principle of least time,* which states that among all possible paths between two points, the one actually taken by a ray of light is that for which the time of travel is a *minimum.* (In fact, there are some cases in which the time is a maximum rather than a minimum.)

Figure **P33.56**



33.57 •• CALC A ray of light goes from point A in a medium in which the speed of light is v_1 to point B in a medium in which the speed is v_2 (Fig. P33.57). The ray strikes the interface a horizontal distance x to the right of point A. (a) Show that the time required for the light to go from A to B is

Figure **P33.57**



$$t = \frac{\sqrt{h_1^2 + x^2}}{v_1} + \frac{\sqrt{h_2^2 + (l - x)^2}}{v_2}$$

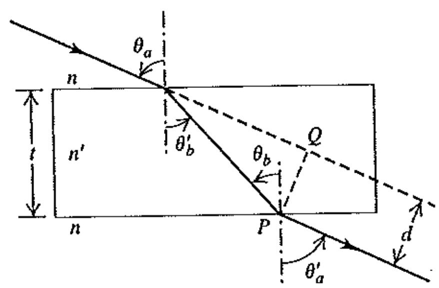
(b) Take the derivative of t with respect to x. Set this derivative equal to zero to show that this time reaches its *minimum* value when $n_1 \sin\theta_1 = n_2 \sin\theta_2$. This is Snell's law and corresponds to the actual path taken by the light. This is another example of Fermat's principle of least time (see Problem 33.56).

33.58 •• Light is incident in air at an angle θ_a (Fig. P33.58) on the upper surface of a transparent plate, the surfaces of the plate being plane and parallel to each other. (a) Prove that $\theta_a = \theta_a'$. (b) Show that this is true for any number of different parallel plates. (c) Prove that the lateral displacement d of the emergent beam is given by the relationship

$$d = t\frac{\sin(\theta_a - \theta_b')}{\cos\theta_b'}$$

where t is the thickness of the plate. (d) A ray of light is incident at an angle of 66.0° on one surface of a glass plate 2.40 cm thick with an index of refraction of 1.80. The medium on either side of the plate is air. Find the lateral displacement between the incident and emergent rays.

Figure **P33.58**

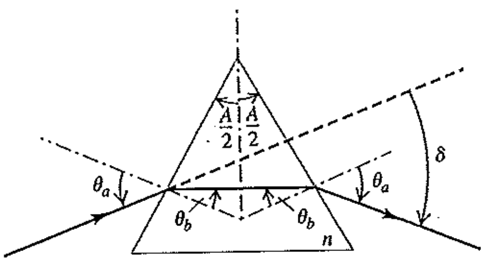


33.59 •• Angle of Deviation. The incident angle θ_a shown in Fig. P33.59 is chosen so that the light passes symmetrically through the prism, which has refractive index n and apex angle A. (a) Show that the angle of deviation δ (the angle between the initial and final directions of the ray) is given by

$$\sin\frac{A + \delta}{2} = n\sin\frac{A}{2}$$

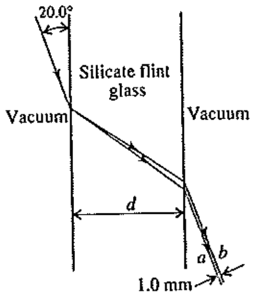
(When the light passes through symmetrically, as shown, the angle of deviation is a minimum.) (b) Use the result of part (a) to find the angle of deviation for a ray of light passing symmetrically through a prism having three equal angles ($A = 60.0°$) and $n = 1.52$. (c) A certain glass has a refractive index of 1.61 for red light (700 nm) and 1.66 for violet light (400 nm). If both colors pass through symmetrically, as described in part (a), and if $A = 60.0°$, find the difference between the angles of deviation for the two colors.

Figure **P33.59**



33.60 • A thin beam of white light is directed at a flat sheet of silicate flint glass at an angle of 20.0° to the surface of the sheet. Due to dispersion in the glass, the beam is spread out in a spectrum as shown in Fig. P33.60. The refractive index of silicate flint glass versus wavelength is graphed in Fig. 33.18. (a) The rays a and b shown in Fig. P33.60 correspond to the extreme wavelengths shown in Fig. 33.18. Which corresponds to red and which to violet? Explain your reasoning. (b) For what thickness d of the glass sheet will the spectrum be 1.0 mm wide, as shown (see Problem 33.58)?

Figure **P33.60**



33.61 • A beam of light traveling horizontally is made of an unpolarized component with intensity I_0 and a polarized component with intensity I_p. The plane of polarization of the polarized component is oriented at an angle of θ with respect to the vertical. The data in the table give the intensity measured through a polarizer with an orientation of ϕ with respect to the vertical. (a) What is the orientation of the polarized component? (That is, what is the angle θ?) (b) What are the values of I_0 and I_p?

ϕ(°)	I_{total}(W/m²)	ϕ(°)	I_{total}(W/m²)
0	18.4	100	8.6
10	21.4	110	6.3
20	23.7	120	5.2
30	24.8	130	5.2
40	24.8	140	6.3
50	23.7	150	8.6
60	21.4	160	11.6
70	18.4	170	15.0
80	15.0	180	18.4
90	11.6		

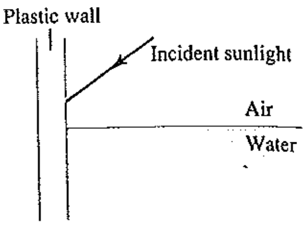
33.62 •• **BIO Optical Activity of Biological Molecules.** Many biologically important molecules are optically active. When linearly polarized light traverses a solution of compounds containing these molecules, its plane of polarization is rotated. Some compounds rotate the polarization clockwise; others rotate the polarization counterclockwise. The amount of rotation depends on the amount of material in the path of the light. The following data give the amount of rotation through two amino acids over a path length of 100 cm:

Rotation (°)		
l-leucine	*d*-glutamic acid	Concentration (g/100 mL)
−0.11	0.124	1.0
−0.22	0.248	2.0
−0.55	0.620	5.0
−1.10	1.24	10.0
−2.20	2.48	20.0
−5.50	6.20	50.0
−11.0	12.4	100.0

From these data, find the relationship between the concentration C (in grams per 100 mL) and the rotation of the polarization (in degrees) of each amino acid. (*Hint:* Graph the concentration as a function of the rotation angle for each amino acid.)

33.63 •• A beam of unpolarized sunlight strikes the vertical plastic wall of a water tank at an unknown angle. Some of the light reflects from the wall and enters the water (Fig. P33.63). The refractive index of the plastic wall is 1.61. If the light that has been reflected from the wall into the water is observed to be completely polarized, what angle does this beam make with the normal inside the water?

Figure **P33.63**



33.64 •• A certain birefringent material has indexes of refraction n_1 and n_2 for the two perpendicular components of linearly polarized light passing through it. The corresponding wavelengths are $\lambda_1 = \lambda_0/n_1$ and λ_0/n_2, where λ_0 is the wavelength in vacuum. (a) If the crystal is to function as a quarter-wave plate, the number of wavelengths of each component within the material must differ by $\frac{1}{4}$. Show that the minimum thickness for a quarter-wave plate is

$$d = \frac{\lambda_0}{4(n_1 - n_2)}$$

(b) Find the minimum thickness of a quarter-wave plate made of siderite ($FeO \cdot CO_2$) if the indexes of refraction are $n_1 = 1.875$ and $n_2 = 1.635$ and the wavelength in vacuum is $\lambda_0 = 589$ nm.

CHALLENGE PROBLEMS

33.65 ••• Consider two vibrations of equal amplitude and frequency but differing in phase, one along the x-axis,

$$x = a\sin(\omega t - \alpha)$$

and the other along the y-axis,

$$y = a\sin(\omega t - \beta)$$

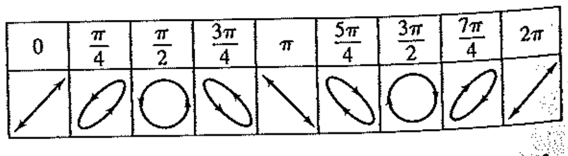
These can be written as follows:

$$\frac{x}{a} = \sin\omega t\cos\alpha - \cos\omega t\sin\alpha \qquad (1)$$

$$\frac{y}{a} = \sin\omega t\cos\beta - \cos\omega t\sin\beta \qquad (2)$$

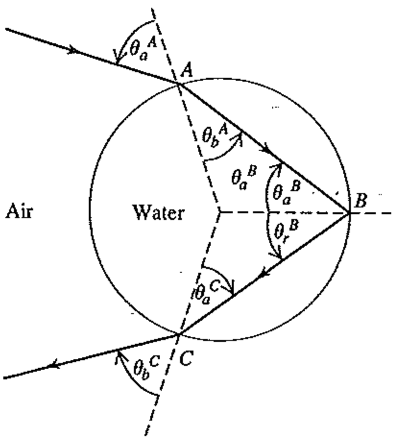
(a) Multiply Eq. (1) by $\sin\beta$ and Eq. (2) by $\sin\alpha$, and then subtract the resulting equations. (b) Multiply Eq. (1) by $\cos\beta$ and Eq. (2) by $\cos\alpha$, and then subtract the resulting equations. (c) Square and add the results of parts (a) and (b). (d) Derive the equation $x^2 + y^2 - 2xy\cos\delta = a^2\sin^2\delta$, where $\delta = \alpha - \beta$. (e) Use the above result to justify each of the diagrams in Fig. P33.65. In the figure, the angle given is the phase difference between two simple harmonic motions of the same frequency and amplitude, one horizontal (along the x-axis) and the other vertical (along the y-axis). The figure thus shows the resultant motion from the superposition of the two perpendicular harmonic motions.

Figure **P33.65**



33.66 ••• **CALC** A rainbow is produced by the reflection of sunlight by spherical drops of water in the air. Figure P33.66 shows a ray that refracts into a drop at point A, is reflected from the back surface of the drop at point B, and refracts back into the air at point C. The angles of incidence and refraction, θ_a and θ_b, are shown at points A and C, and the angles of incidence and reflection, θ_a and θ_r, are shown at point B. (a) Show that $\theta_a{}^B = \theta_b{}^A$, $\theta_a{}^C = \theta_b{}^A$, and $\theta_b{}^C = \theta_a{}^A$. (b) Show that the angle in radians between the ray before it enters the drop at A and after it exits at C (the total angular deflection of the ray) is $\Delta = 2\theta_a{}^A - 4\theta_b{}^A + \pi$. (*Hint:* Find the angular deflections that occur at A, B, and C, and add them to get Δ.) (c) Use Snell's law to write Δ in terms of $\theta_a{}^A$ and n, the refractive index of the water in the drop. (d) A rainbow will form when the angular deflection Δ is *stationary* in the incident angle $\theta_a{}^A$—that is, when $d\Delta/d\theta_a{}^A = 0$. If this condition is satisfied, all the rays with incident angles close to $\theta_a{}^A$ will be sent back in the same direction, producing a bright zone in the sky. Let θ_1 be the value of $\theta_a{}^A$ for which this occurs. Show that $\cos^2\theta_1 = \frac{1}{3}(n^2 - 1)$. (*Hint:* You may find the derivative formula $d(\arcsin u(x))/dx = (1 - u^2)^{-1/2}(du/dx)$ helpful.) (e) The index of refraction in water is 1.342 for violet light and 1.330 for red light. Use the results of parts (c) and (d) to find θ_1 and Δ for violet and red light. Do your results agree with the angles shown in Fig. 33.20d? When you view the rainbow, which color, red or violet, is higher above the horizon?

Figure **P33.66**



33.67 ••• **CALC** A *secondary rainbow* is formed when the incident light undergoes two internal reflections in a spherical drop of water as shown in Fig. 33.20e. (See Challenge Problem 33.66.) (a) In terms of the incident angle θ_a^A and the refractive index n of the drop, what is the angular deflection Δ of the ray? That is, what is the angle between the ray before it enters the drop and after it exits? (b) What is the incident angle θ_2 for which the derivative of Δ with respect to the incident angle θ_a^A is zero? (c) The indexes of refraction for red and violet light in water are given in part (e) of Challenge Problem 33.66. Use the results of parts (a) and (b) to find θ_2 and Δ for violet and red light. Do your results agree with the angles shown in Fig. 33.20e? When you view a secondary rainbow, is red or violet higher above the horizon? Explain.

Answers

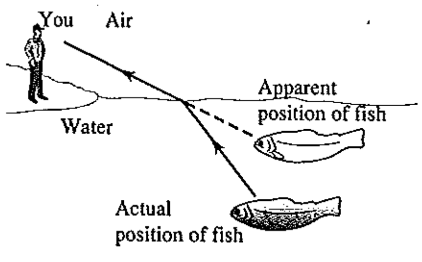
Chapter Opening Question ?

This is the same effect as shown in Fig. 33.31. The drafting tools are placed between two polarizing filters whose polarizing axes are perpendicular. In places where the clear plastic is under stress, the plastic becomes birefringent; that is, light travels through it at a speed that depends on its polarization. The result is that the light that emerges from the plastic has a different polarization than the light that enters. A spot on the plastic appears bright if the emerging light has the same polarization as the second polarizing filter. The amount of birefringence depends on the wavelength of the light as well as the amount of stress on the plastic, so different colors are seen at different locations on the plastic.

Test Your Understanding Questions

33.1 Answer: (iii) The waves go farther in the y-direction in a given amount of time than in the other directions, so the wave fronts are elongated in the y-direction.

33.2 Answers: (a) (ii), (b) (iii) As shown in the figure, light rays coming from the fish bend away from the normal when they pass from the water ($n = 1.33$) into the air ($n = 1.00$). As a result, the fish appears to be higher in the water than it actually is. Hence you should aim a spear *below* the apparent position of the fish. If you use a laser beam, you should aim *at* the apparent position of the fish: The beam of laser light takes the same path from you to the fish as ordinary light takes from the fish to you (though in the opposite direction).



33.3 Answers: (i), (ii) Total internal reflection can occur only if two conditions are met: n_b must be less than n_a, and the critical angle θ_{crit} (where $\sin\theta_{crit} = n_b/n_a$) must be smaller than the angle of incidence θ_a. In the first two cases both conditions are met: The critical angles are (i) $\theta_{crit} = \sin^{-1}(1/1.33) = 48.8°$ and (ii) $\theta_{crit} = \sin^{-1}(1.33/1.52) = 61.0°$, both of which are smaller than $\theta_a = 70°$. In the third case $n_b = 1.52$ is greater than $n_a = 1.33$, so total internal reflection cannot occur for any incident angle.

33.5 Answer: (ii) The sunlight reflected from the windows of the high-rise building is partially polarized in the vertical direction, since each window lies in a vertical plane. The Polaroid filter in front of the lens is oriented with its polarizing axis perpendicular to the dominant direction of polarization of the reflected light.

33.7 Answer: (ii) Huygens's principle applies to waves of all kinds, including sound waves. Hence this situation is exactly like that shown in Fig. 33.36, with material a representing the warm air, material b representing the cold air in which the waves travel more slowly, and the interface between the materials representing the weather front. North is toward the top of the figure and east is toward the right, so Fig. 33.36 shows that the rays (which indicate the direction of propagation) deflect toward the east.

Bridging Problem

Answer: 1.93×10^8 m/s