0.0800 mm thick placed under one end. The space between the plates is filled with air. The glass is illuminated from above with light having a wavelength in air of 656 nm. How many interference fringes are observed per centimeter in the reflected light?

35.29 •• A uniform film of TiO_2, 1036 nm thick and having index of refraction 2.62, is spread uniformly over the surface of crown glass of refractive index 1.52. Light of wavelength 520.0 nm falls at normal incidence onto the film from air. You want to increase the thickness of this film so that the reflected light cancels. (a) What is the *minimum* thickness of TiO_2 that you must *add* so the reflected light cancels as desired? (b) After you make the adjustment in part (a), what is the path difference between the light reflected off the top of the film and the light that cancels it after traveling through the film? Express your answer in (i) nanometers and (ii) wavelengths of the light in the TiO_2 film.

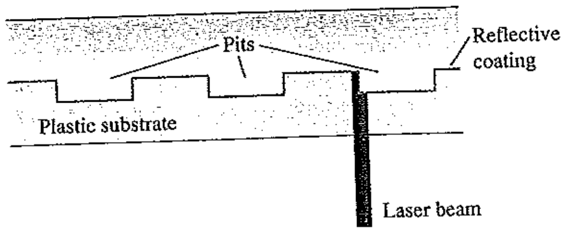
35.30 • A plastic film with index of refraction 1.85 is put on the surface of a car window to increase the reflectivity and thus to keep the interior of the car cooler. The window glass has index of refraction 1.52. (a) What minimum thickness is required if light with wavelength 550 nm in air reflected from the two sides of the film is to interfere constructively? (b) It is found to be difficult to manufacture and install coatings as thin as calculated in part (a). What is the next greatest thickness for which there will also be constructive interference?

35.31 • The walls of a soap bubble have about the same index of refraction as that of plain water, $n = 1.33$. There is air both inside and outside the bubble. (a) What wavelength (in air) of visible light is most strongly reflected from a point on a soap bubble where its wall is 290 nm thick? To what color does this correspond (see Fig. 32.4 and Table 32.1)? (b) Repeat part (a) for a wall thickness of 340 nm.

35.32 •• Light with wavelength 648 nm in air is incident perpendicularly from air on a film 8.76 μm thick and with refractive index 1.35. Part of the light is reflected from the first surface of the film, and part enters the film and is reflected back at the second surface, where the film is again in contact with air. (a) How many waves are contained along the path of this second part of the light in its round trip through the film? (b) What is the phase difference between these two parts of the light as they leave the film?

35.33 •• **Compact Disc Player.** A compact disc (CD) is read from the bottom by a semiconductor laser with wavelength 790 nm passing through a plastic substrate of refractive index 1.8. When the beam encounters a pit, part of the beam is reflected from the pit and part from the flat region between the pits, so these two beams interfere with each other (Fig. E35.33). What must the minimum pit depth be so that the part of the beam reflected from a pit cancels the part of the beam reflected from the flat region? (It is this cancellation that allows the player to recognize the beginning and end of a pit.)

Figure **E35.33**



35.34 • What is the thinnest soap film (excluding the case of zero thickness) that appears black when illuminated with light with wavelength 480 nm? The index of refraction of the film is 1.33, and there is air on both sides of the film.

Section 35.5 The Michelson Interferometer

35.35 • How far must the mirror M_2 (see Fig. 35.19) of the Michelson interferometer be moved so that 1800 fringes of He-Ne laser light ($\lambda = 633$ nm) move across a line in the field of view?

35.36 • Jan first uses a Michelson interferometer with the 606-nm light from a krypton-86 lamp. He displaces the movable mirror away from him, counting 818 fringes moving across a line in his field of view. Then Linda replaces the krypton lamp with filtered 502-nm light from a helium lamp and displaces the movable mirror toward her. She also counts 818 fringes, but they move across the line in her field of view opposite to the direction they moved for Jan. Assume that both Jan and Linda counted to 818 correctly. (a) What distance did each person move the mirror? (b) What is the resultant displacement of the mirror?

PROBLEMS

35.37 ••• The radius of curvature of the convex surface of a planoconvex lens is 68.4 cm. The lens is placed convex side down on a perfectly flat glass plate that is illuminated from above with red light having a wavelength of 580 nm. Find the diameter of the second bright ring in the interference pattern.

35.38 •• Newton's rings can be seen when a planoconvex lens is placed on a flat glass surface. For a particular lens with an index of refraction of $n = 1.50$ and a glass plate with an index of $n = 1.80$, the diameter of the third bright ring is 0.720 mm. If water ($n = 1.33$) now fills the space between the lens and the plate, what is the new diameter of this ring?

35.39 • **BIO Coating Eyeglass Lenses.** Eyeglass lenses can be coated on the *inner* surfaces to reduce the reflection of stray light to the eye. If the lenses are medium flint glass of refractive index 1.62 and the coating is fluorite of refractive index 1.432, (a) what minimum thickness of film is needed on the lenses to cancel light of wavelength 550 nm reflected toward the eye at normal incidence? (b) Will any other wavelengths of visible light be cancelled or enhanced in the reflected light?

35.40 •• **BIO Sensitive Eyes.** After an eye examination, you put some eyedrops on your sensitive eyes. The cornea (the front part of the eye) has an index of refraction of 1.38, while the eyedrops have a refractive index of 1.45. After you put in the drops, your friends notice that your eyes look red, because red light of wavelength 600 nm has been reinforced in the reflected light. (a) What is the minimum thickness of the film of eyedrops on your cornea? (b) Will any other wavelengths of visible light be reinforced in the reflected light? Will any be cancelled? (c) Suppose you had contact lenses, so that the eyedrops went on them instead of on your corneas. If the refractive index of the lens material is 1.50 and the layer of eyedrops has the same thickness as in part (a), what wavelengths of visible light will be reinforced? What wavelengths will be cancelled?

35.41 •• Two flat plates of glass with parallel faces are on a table, one plate on the other. Each plate is 11.0 cm long and has a refractive index of 1.55. A very thin sheet of metal foil is inserted under the end of the upper plate to raise it slightly at that end, in a manner similar to that discussed in Example 35.4. When you view the glass plates from above with reflected white light, you observe that, at 1.15 mm from the line where the sheets are in contact, the violet light of wavelength 400.0 nm is enhanced in this reflected light, but no visible light is enhanced closer to the line of contact.

(a) How far from the line of contact will green light (of wavelength 550 nm) and orange light (of wavelength 600.0 nm) first be enhanced? (b) How far from the line of contact will the violet, green, and orange light again be enhanced in the reflected light? (c) How thick is the metal foil holding the ends of the plates apart?

35.42 •• In a setup similar to that of Problem 35.41, the glass has an index of refraction of 1.53, the plates are each 8.00 cm long, and the metal foil is 0.015 mm thick. The space between the plates is filled with a jelly whose refractive index is not known precisely, but is known to be greater than that of the glass. When you illuminate these plates from above with light of wavelength 525 nm, you observe a series of equally spaced dark fringes in the reflected light. You measure the spacing of these fringes and find that there are 10 of them every 6.33 mm. What is the index of refraction of the jelly?

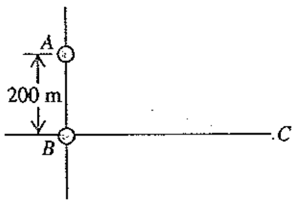
35.43 ••• Suppose you illuminate two thin slits by monochromatic coherent light in air and find that they produce their first interference *minima* at ±35.20° on either side of the central bright spot. You then immerse these slits in a transparent liquid and illuminate them with the same light. Now you find that the first minima occur at ±19.46° instead. What is the index of refraction of this liquid?

35.44 •• **CP CALC** A very thin sheet of brass contains two thin parallel slits. When a laser beam shines on these slits at normal incidence and room temperature (20.0°C), the first interference dark fringes occur at ±32.5° from the original direction of the laser beam when viewed from some distance. If this sheet is now slowly heated up to 135°C, by how many degrees do these dark fringes change position? Do they move closer together or get farther apart? See Table 17.1 for pertinent information, and ignore any effects that might occur due to change in the thickness of the slits. (*Hint:* Since thermal expansion normally produces very small changes in length, you can use differentials to find the change in the angle.)

35.45 •• Two speakers, 2.50 m apart, are driven by the same audio oscillator so that each one produces a sound consisting of *two* distinct frequencies, 0.900 kHz and 1.20 kHz. The speed of sound in the room is 344 m/s. Find all the angles relative to the usual centerline in front of (and far from) the speakers at which *both* frequencies interfere constructively.

35.46 •• Two radio antennas radiating in phase are located at points A and B, 200 m apart (Fig. P35.46). The radio waves have a frequency of 5.80 MHz. A radio receiver is moved out from point B along a line perpendicular to the line connecting A and B (line BC shown in Fig. P35.46). At what distances from B will there be *destructive* interference? (*Note:* The distance of the receiver from the sources is not large in comparison to the separation of the sources, so Eq. (35.5) does not apply.)

Figure **P35.46**

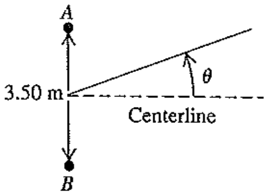


35.47 •• One round face of a 3.25-m, solid, cylindrical plastic pipe is covered with a thin black coating that completely blocks light. The opposite face is covered with a fluorescent coating that glows when it is struck by light. Two straight, thin, parallel scratches, 0.225 mm apart, are made in the center of the black face. When laser light of wavelength 632.8 nm shines through the slits perpendicular to the black face, you find that the central bright fringe on the opposite face is 5.82 mm wide, measured between the dark fringes that border it on either side. What is the index of refraction of the plastic?

35.48 • A uniform thin film of material of refractive index 1.40 coats a glass plate of refractive index 1.55. This film has the proper thickness to cancel normally incident light of wavelength 525 nm that strikes the film surface from air, but it is somewhat greater than the minimum thickness to achieve this cancellation. As time goes by, the film wears away at a steady rate of 4.20 nm per year. What is the minimum number of years before the reflected light of this wavelength is now enhanced instead of cancelled?

35.49 •• Two speakers A and B are 3.50 m apart, and each one is emitting a frequency of 444 Hz. However, because of signal delays in the cables, speaker A is one-fourth of a period *ahead* of speaker B. For points far from the speakers, find all the angles relative to the centerline (Fig. P35.49) at which the sound from these speakers cancels. Include angles on *both* sides of the centerline. The speed of sound is 340 m/s.

Figure **P35.49**



35.50 ••• **CP** The electric fields received at point P from two identical, coherent wave sources are $E_1(t) = E\cos(\omega t + \phi)$ and $E_2(t) = E\cos\omega t$. (a) Use the trigonometric identities in Appendix B to show that the resultant wave is $E_P(t) = 2E\cos(\phi/2)\cos(\omega t + \phi/2)$. (b) Show that the amplitude of this resultant wave is given by Eq. (35.7). (c) Use the result of part (a) to show that at an interference maximum, the amplitude of the resultant wave is in phase with the original waves $E_1(t)$ and $E_2(t)$. (d) Use the result of part (a) to show that near an interference minimum, the resultant wave is approximately $\frac{1}{4}$ cycle out of phase with either of the original waves. (e) Show that the *instantaneous* Poynting vector at point P has magnitude $S = 4\epsilon_0 cE^2\cos^2(\phi/2)\cos^2(\omega t + \phi/2)$ and that the *time-averaged* Poynting vector is given by Eq. (35.9).

35.51 •• **CP** A thin uniform film of refractive index 1.750 is placed on a sheet of glass of refractive index 1.50. At room temperature (20.0°C), this film is just thick enough for light with wavelength 582.4 nm reflected off the top of the film to be cancelled by light reflected from the top of the glass. After the glass is placed in an oven and slowly heated to 170°C, you find that the film cancels reflected light with wavelength 588.5 nm. What is the coefficient of linear expansion of the film? (Ignore any changes in the refractive index of the film due to the temperature change.)

35.52 ••• **GPS Transmission.** The GPS (Global Positioning System) satellites are approximately 5.18 m across and transmit two low-power signals, one of which is at 1575.42 MHz (in the UHF band). In a series of laboratory tests on the satellite, you put two 1575.42-MHz UHF transmitters at opposite ends of the satellite. These broadcast in phase uniformly in all directions. You measure the intensity at points on a circle that is several hundred meters in radius and centered on the satellite. You measure angles on this circle relative to a point that lies along the centerline of the satellite (that is, the perpendicular bisector of a line that extends from one transmitter to the other). At this point on the circle, the measured intensity is 2.00 W/m^2. (a) At how many other angles in the range $0° < \theta < 90°$ is the intensity also 2.00 W/m^2? (b) Find the four smallest angles in the range $0° < \theta < 90°$ for which the intensity is 2.00 W/m^2. (c) What is the intensity at a point on the circle at an angle of 4.65° from the centerline?

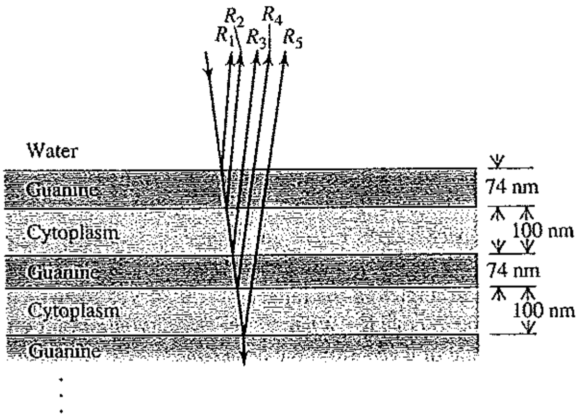
35.53 •• Consider a two-slit interference pattern, for which the intensity distribution is given by Eq. (35.14). Let θ_m be the angular position of the mth bright fringe, where the intensity is I_0. Assume that θ_m is small, so that $\sin\theta_m \cong \theta_m$. Let θ_m^{+} and θ_m^{-} be the two angles on either side of θ_m for which $I = \frac{1}{2}I_0$. The quantity $\Delta\theta_m = |\theta_m^{+} - \theta_m^{-}|$ is the half-width of the mth fringe. Calculate $\Delta\theta_m$. How does $\Delta\theta_m$ depend on m?

35.54 •• White light reflects at normal incidence from the top and bottom surfaces of a glass plate ($n = 1.52$). There is air above and below the plate. Constructive interference is observed for light whose wavelength in air is 477.0 nm. What is the thickness of the plate if the next longer wavelength for which there is constructive interference is 540.6 nm?

35.55 ••• A source S of monochromatic light and a detector D are both located in air a distance h above a horizontal plane sheet of glass and are separated by a horizontal distance x. Waves reaching D directly from S interfere with waves that reflect off the glass. The distance x is small compared to h so that the reflection is at close to normal incidence. (a) Show that the condition for constructive interference is $\sqrt{x^2 + 4h^2} - x = \left(m + \frac{1}{2}\right)\lambda$, and the condition for destructive interference is $\sqrt{x^2 + 4h^2} - x = m\lambda$. (*Hint:* Take into account the phase change on reflection.) (b) Let $h = 24$ cm and $x = 14$ cm. What is the longest wavelength for which there will be constructive interference?

35.56 •• **BIO Reflective Coatings and Herring.** Herring and related fish have a brilliant silvery appearance that camouflages them while they are swimming in a sunlit ocean. The silveriness is due to *platelets* attached to the surfaces of these fish. Each platelet is made up of several alternating layers of crystalline guanine ($n = 1.80$) and of cytoplasm ($n = 1.333$, the same as water), with a guanine layer on the outside in contact with the surrounding water (Fig. P35.56). In one typical platelet, the guanine layers are 74 nm thick and the cytoplasm layers are 100 nm thick. (a) For light striking the platelet surface at normal incidence, for which vacuum wavelengths of visible light will all of the reflections R_1, R_2, R_3, R_4, and R_5, shown in Fig. P35.56, be approximately in phase? If white light is shone on this platelet, what color will be most strongly reflected (see Fig. 32.4)? The surface of a herring has very many platelets side by side with layers of different thickness, so that *all* visible wavelengths are reflected. (b) Explain why such a "stack" of layers is more reflective than a single layer of guanine with cytoplasm underneath. (A stack of five guanine layers separated by cytoplasm layers reflects more than 80% of incident light at the wavelength for which it is "tuned.") (c) The color that is most strongly reflected from a platelet depends on the angle at which it is viewed. Explain why this should be so. (You can see these changes in color by examining a herring from different angles. Most of the platelets on these fish are oriented in the same way, so that they are vertical when the fish is swimming.)

Figure **P35.56**



35.57 • Two thin parallel slits are made in an opaque sheet of film. When a monochromatic beam of light is shone through them at normal incidence,.the first bright fringes in the transmitted light occur in air at ±18.0° with the original direction of the light beam on a distant screen when the apparatus is in air. When the apparatus is immersed in a liquid, the same bright fringes now occur at ±12.6°. Find the index of refraction of the liquid.

35.58 •• Red light with wavelength 700 nm is passed through a two-slit apparatus. At the same time, monochromatic visible light with another wavelength passes through the same apparatus. As a result, most of the pattern that appears on the screen is a mixture of two colors; however, the center of the third bright fringe ($m = 3$) of the red light appears pure red, with none of the other color. What are the possible wavelengths of the second type of visible light? Do you need to know the slit spacing to answer this question? Why or why not?

35.59 ••• In a Young's two-slit experiment a piece of glass with an index of refraction n and a thickness L is placed in front of the upper slit. (a) Describe qualitatively what happens to the interference pattern. (b) Derive an expression for the intensity I of the light at points on a screen as a function of n, L, and θ. Here θ is the usual angle measured from the center of the two slits. That is, determine the equation analogous to Eq. (35.14) but that also involves L and n for the glass plate. (c) From your result in part (b) derive an expression for the values of θ that locate the maxima in the interference pattern [that is, derive an equation analogous to Eq. (35.4)].

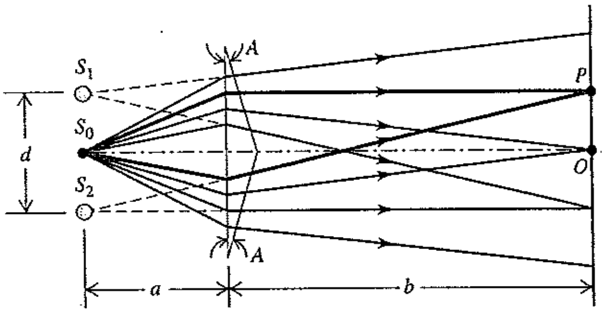
35.60 •• After a laser beam passes through two thin parallel slits, the first completely dark fringes occur at ±19.0° with the original direction of the beam, as viewed on a screen far from the slits. (a) What is the ratio of the distance between the slits to the wavelength of the light illuminating the slits? (b) What is the smallest angle, relative to the original direction of the laser beam, at which the intensity of the light is $\frac{1}{10}$ the maximum intensity on the screen?

CHALLENGE PROBLEMS

35.61 ••• **CP** The index of refraction of a glass rod is 1.48 at $T = 20.0°\text{C}$ and varies linearly with temperature, with a coefficient of $2.50 \times 10^{-5}/\text{C}°$. The coefficient of linear expansion of the glass is $5.00 \times 10^{-6}/\text{C}°$. At 20.0°C the length of the rod is 3.00 cm. A Michelson interferometer has this glass rod in one arm, and the rod is being heated so that its temperature increases at a rate of 5.00 C°/min. The light source has wavelength $\lambda = 589$ nm, and the rod initially is at $T = 20.0°\text{C}$. How many fringes cross the field of view each minute?

35.62 ••• **CP** Figure P35.62 shows an interferometer known as *Fresnel's biprism.* The magnitude of the prism angle A is

Figure **P35.62**



extremely small. (a) If S_0 is a very narrow source slit, show that the separation of the two virtual coherent sources S_1 and S_2 is given by $d = 2aA(n - 1)$, where n is the index of refraction of the material of the prism. (b) Calculate the spacing of the fringes of green light with wavelength 500 nm on a screen 2.00 m from the biprism. Take $a = 0.200$ m, $A = 3.50$ mrad, and $n = 1.50$.

Answers

Chapter Opening Question

The colors appear due to constructive interference between light waves reflected from the outer and inner surfaces of the soap bubble. The thickness of the bubble walls at each point determines the wavelength of light for which the most constructive interference occurs and hence the color that appears the brightest at that point (see Example 35.4 in Section 35.4).

Test Your Understanding Questions

35.1 Answer: (i) At any point P on the positive y-axis above S_1, the distance r_2 from S_2 to P is greater than the distance r_1 from S_1 to P by 4λ. This corresponds to $m = 4$ in Eq. (35.1), the equation for constructive interference. Hence all such points make up an antinodal curve.

35.2 Answer: (ii) Blue light has a shorter wavelength than red light (see Section 32.1). Equation (35.6) tells us that the distance y_m from the center of the pattern to the mth bright fringe is proportional to the wavelength λ. Hence all of the fringes will move toward the center of the pattern as the wavelength decreases, and the spacing between fringes will decrease.

35.3 Answer: (i), (iv), (ii), (iii) In cases (i) and (iii) we are given the wavelength λ and path difference $d\sin\theta$. Hence we use Eq. (35.14), $I = I_0\cos^2[(\pi d\sin\theta)/\lambda]$. In parts (ii) and (iii) we are given the phase difference ϕ and we use Eq. (35.10), $I = I_0\cos^2(\phi/2)$. We find:

(i) $I = I_0\cos^2[\pi(4.00 \times 10^{-7}\text{ m})/(5.00 \times 10^{-7}\text{ m})] = I_0\cos^2(0.800\pi\text{ rad}) = 0.655I_0$;
(ii) $I = I_0\cos^2[(4.00\text{ rad})/2] = I_0\cos^2(2.00\text{ rad}) = 0.173I_0$;
(iii) $I = I_0\cos^2[\pi(7.50 \times 10^{-7}\text{ m})/(5.00 \times 10^{-7}\text{ m})] = I_0\cos^2(1.50\pi\text{ rad}) = 0$;
(iv) $I = I_0\cos^2[(2.00\text{ rad})/2] = I_0\cos^2(1.00\text{ rad}) = 0.292I_0$.

35.4 Answers: (i) and (iii) Benzene has a larger index of refraction than air, so light that reflects off the upper surface of the benzene undergoes a half-cycle phase shift. Fluorite has a *smaller* index of refraction than benzene, so light that reflects off the benzene–fluorite interface does not undergo a phase shift. Hence the equation for constructive reflection is Eq. (35.18a), $2t = \left(m + \frac{1}{2}\right)\lambda$, which we can rewrite as $t = \left(m + \frac{1}{2}\right)\lambda/2 = \left(m + \frac{1}{2}\right)(400\text{ mm})/2 = 100$ nm, 300 nm, 500 nm, ...

35.5 Answer: yes Changing the index of refraction changes the wavelength of the light inside the compensator plate, and so changes the number of wavelengths within the thickness of the plate. Hence this has the same effect as changing the distance L_1 from the beam splitter to mirror M_1, which would change the interference pattern.

Bridging Problem

Answers: (a) 441 nm (b) 551 nm