BRIDGING PROBLEM Observing the Expanding Universe

An astronomer who is studying the light from a galaxy has identified the spectrum of hydrogen but finds that the wavelengths are somewhat shifted from those found in the laboratory. In the lab, the H_α line in the hydrogen spectrum has a wavelength of 656.3 nm. The astronomer is using a transmission diffraction grating having 5758 lines/cm in the first order and finds that the first bright fringe for the H_α line occurs at $\pm 23.41°$ from the central spot. How fast is the galaxy moving? Express your answer in m/s and as a percentage of the speed of light. Is the galaxy moving toward us or away from us?

SOLUTION GUIDE

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IDENTIFY and SET UP

1. You can use the information about the grating to find the wavelength of the H_α line in the galaxy's spectrum.
2. In Section 16.8 we learned about the Doppler effect for electromagnetic radiation: The frequency that we receive from a moving source, such as the galaxy, is different from the frequency that is emitted. Equation (16.30) relates the emitted frequency, the received frequency, and the velocity of the source (the target variable). The equation $c = f\lambda$ relates the frequency f and wavelength λ through the speed of light c.

EXECUTE

3. Find the wavelength of the H_α spectral line in the received light.
4. Rewrite Eq. (16.30) as a formula for the velocity v of the galaxy in terms of the received wavelength and the wavelength emitted by the source.
5. Solve for v. Express it in m/s and as a percentage of c, and decide whether the galaxy is moving toward us or moving away.

EVALUATE

6. Is your answer consistent with the relative sizes of the received wavelength and the emitted wavelength?

Problems

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•, ••, •••: Problems of increasing difficulty. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q36.1 Why can we readily observe diffraction effects for sound waves and water waves, but not for light? Is this because light travels so much faster than these other waves? Explain.

Q36.2 What is the difference between Fresnel and Fraunhofer diffraction? Are they different *physical* processes? Explain.

Q36.3 You use a lens of diameter D and light of wavelength λ and frequency f to form an image of two closely spaced and distant objects. Which of the following will increase the resolving power? (a) Use a lens with a smaller diameter; (b) use light of higher frequency; (c) use light of longer wavelength. In each case justify your answer.

Q36.4 Light of wavelength λ and frequency f passes through a single slit of width a. The diffraction pattern is observed on a screen a distance x from the slit. Which of the following will *decrease* the width of the central maximum? (a) Decrease the slit width; (b) decrease the frequency f of the light; (c) decrease the wavelength λ of the light; (d) decrease the distance x of the screen from the slit. In each case justify your answer.

Q36.5 In a diffraction experiment with waves of wavelength λ, there will be *no* intensity minima (that is, no dark fringes) if the slit width is small enough. What is the maximum slit width for which this occurs? Explain your answer.

Q36.6 The predominant sound waves used in human speech have wavelengths in the range from 1.0 to 3.0 meters. Using the ideas of diffraction, explain how it is possible to hear a person's voice even when he is facing away from you.

Q36.7 In single-slit diffraction, what is $\sin(\beta/2)$ when $\theta = 0$? In view of your answer, why is the single-slit intensity *not* equal to zero at the center?

Q36.8 A rainbow ordinarily shows a range of colors (see Section 33.4). But if the water droplets that form the rainbow are small enough, the rainbow will appear white. Explain why, using diffraction ideas. How small do you think the raindrops would have to be for this to occur?

Q36.9 Some loudspeaker horns for outdoor concerts (at which the entire audience is seated on the ground) are wider vertically than horizontally. Use diffraction ideas to explain why this is more efficient at spreading the sound uniformly over the audience than either a square speaker horn or a horn that is wider horizontally than vertically. Would this still be the case if the audience were seated at different elevations, as in an amphitheater? Why or why not?

Q36.10 Figure 31.12 (Section 31.2) shows a loudspeaker system. Low-frequency sounds are produced by the *woofer,* which is a speaker with large diameter; the *tweeter,* a speaker with smaller diameter, produces high-frequency sounds. Use diffraction ideas to explain why the tweeter is more effective for distributing high-frequency sounds uniformly over a room than is the woofer.

Q36.11 Information is stored on an audio compact disc, CD-ROM, or DVD disc in a series of pits on the disc. These pits are scanned by a laser beam. An important limitation on the amount of information that can be stored on such a disc is the width of the laser beam. Explain why this should be, and explain how using a shorter-wavelength laser allows more information to be stored on a disc of the same size.

Q36.12 With which color of light can the Hubble Space Telescope see finer detail in a distant astronomical object: red, blue, or ultraviolet? Explain your answer.

Q36.13 At the end of Section 36.4, the following statements were made about an array of N slits. Explain, using phasor diagrams, why each statement is true. (a) A minimum occurs whenever ϕ is an integral multiple of $2\pi/N$, except when ϕ is an integral multiple of 2π (which gives a principal maximum). (b) There are $(N-1)$ minima between each pair of principal maxima.

Q36.14 Could x-ray diffraction effects with crystals be observed by using visible light instead of x rays? Why or why not?

Q36.15 Why is a diffraction grating better than a two-slit setup for measuring wavelengths of light?

Q36.16 One sometimes sees rows of evenly spaced radio antenna towers. A student remarked that these act like diffraction gratings. What did she mean? Why would one want them to act like a diffraction grating?

Q36.17 If a hologram is made using 600-nm light and then viewed with 500-nm light, how will the images look compared to those observed when viewed with 600-nm light? Explain.

Q36.18 A hologram is made using 600-nm light and then viewed by using white light from an incandescent bulb. What will be seen? Explain.

Q36.19 Ordinary photographic film reverses black and white, in the sense that the most brightly illuminated areas become blackest upon development (hence the term *negative*). Suppose a hologram negative is viewed directly, without making a positive transparency. How will the resulting images differ from those obtained with the positive? Explain.

EXERCISES

Section 36.2 Diffraction from a Single Slit

36.1 •• Monochromatic light from a distant source is incident on a slit 0.750 mm wide. On a screen 2.00 m away, the distance from the central maximum of the diffraction pattern to the first minimum is measured to be 1.35 mm. Calculate the wavelength of the light.

36.2 • Parallel rays of green mercury light with a wavelength of 546 nm pass through a slit covering a lens with a focal length of 60.0 cm. In the focal plane of the lens the distance from the central maximum to the first minimum is 10.2 mm. What is the width of the slit?

36.3 •• Light of wavelength 585 nm falls on a slit 0.0666 mm wide. (a) On a very large and distant screen, how many *totally* dark fringes (indicating complete cancellation) will there be, including both sides of the central bright spot? Solve this problem *without* calculating all the angles! (*Hint:* What is the largest that $\sin\theta$ can be? What does this tell you is the largest that m can be?) (b) At what angle will the dark fringe that is most distant from the central bright fringe occur?

36.4 • Light of wavelength 633 nm from a distant source is incident on a slit 0.750 mm wide, and the resulting diffraction pattern is observed on a screen 3.50 m away. What is the distance between the two dark fringes on either side of the central bright fringe?

36.5 •• Diffraction occurs for all types of waves, including sound waves. High-frequency sound from a distant source with wavelength 9.00 cm passes through a slit 12.0 cm wide. A microphone is placed 8.00 m directly in front of the center of the slit, corresponding to point O in Fig. 36.5a. The microphone is then moved in a direction perpendicular to the line from the center of the slit to point O. At what distances from O will the intensity detected by the microphone be zero?

36.6 • **CP Tsunami!** On December 26, 2004, a violent earthquake of magnitude 9.1 occurred off the coast of Sumatra. This quake triggered a huge tsunami (similar to a tidal wave) that killed more than 150,000 people. Scientists observing the wave on the open ocean measured the time between crests to be 1.0 h and the speed of the wave to be 800 km/h. Computer models of the evolution of this enormous wave showed that it bent around the continents and spread to all the oceans of the earth. When the wave reached the gaps between continents, it diffracted between them as through a slit. (a) What was the wavelength of this tsunami? (b) The distance between the southern tip of Africa and northern Antarctica is about 4500 km, while the distance between the southern end of Australia and Antarctica is about 3700 km. As an approximation, we can model this wave's behavior by using Fraunhofer diffraction. Find the smallest angle away from the central maximum for which the waves would cancel after going through each of these continental gaps.

36.7 •• **CP** A series of parallel linear water wave fronts are traveling directly toward the shore at 15.0 cm/s on an otherwise placid lake. A long concrete barrier that runs parallel to the shore at a distance of 3.20 m away has a hole in it. You count the wave crests and observe that 75.0 of them pass by each minute, and you also observe that no waves reach the shore at ±61.3 cm from the point directly opposite the hole, but waves do reach the shore everywhere within this distance. (a) How wide is the hole in the barrier? (b) At what other angles do you find no waves hitting the shore?

36.8 • Monochromatic electromagnetic radiation with wavelength λ from a distant source passes through a slit. The diffraction pattern is observed on a screen 2.50 m from the slit. If the width of the central maximum is 6.00 mm, what is the slit width a if the wavelength is (a) 500 nm (visible light); (b) 50.0 μm (infrared radiation); (c) 0.500 nm (x rays)?

36.9 •• **Doorway Diffraction.** Sound of frequency 1250 Hz leaves a room through a 1.00-m-wide doorway (see Exercise 36.5). At which angles relative to the centerline perpendicular to the doorway will someone outside the room hear no sound? Use 344 m/s for the speed of sound in air and assume that the source and listener are both far enough from the doorway for Fraunhofer diffraction to apply. You can ignore effects of reflections.

36.10 • **CP** Light waves, for which the electric field is given by $E_y(x, t) = E_{max}\sin[(1.20 \times 10^7\ \text{m}^{-1})x - \omega t]$, pass through a slit and produce the first dark bands at ±28.6° from the center of the diffraction pattern. (a) What is the frequency of this light? (b) How wide is the slit? (c) At which angles will other dark bands occur?

36.11 •• Parallel rays of light with wavelength 620 nm pass through a slit covering a lens with a focal length of 40.0 cm. The diffraction pattern is observed in the focal plane of the lens, and the distance from the center of the central maximum to the first minimum is 36.5 cm. What is the width of the slit? (*Note:* The angle that locates the first minimum is *not* small.)

36.12 •• Red light of wavelength 633 nm from a helium–neon laser passes through a slit 0.350 mm wide. The diffraction pattern is observed on a screen 3.00 m away. Define the width of a bright fringe as the distance between the minima on either side. (a) What is the width of the central bright fringe? (b) What is the width of the first bright fringe on either side of the central one?

Section 36.3 Intensity in the Single-Slit Pattern

36.13 •• Monochromatic light of wavelength 580 nm passes through a single slit and the diffraction pattern is observed on a screen. Both the source and screen are far enough from the slit for Fraunhofer diffraction to apply. (a) If the first diffraction minima are at ±90.0°, so the central maximum completely fills the screen, what is the width of the slit? (b) For the width of the slit as calculated in part (a), what is the ratio of the intensity at $\theta = 45.0°$ to the intensity at $\theta = 0$?

36.14 •• Monochromatic light of wavelength $\lambda = 620$ nm from a distant source passes through a slit 0.450 mm wide. The diffraction pattern is observed on a screen 3.00 m from the slit. In terms of the intensity I_0 at the peak of the central maximum, what is the intensity of the light at the screen the following distances from the center of the central maximum: (a) 1.00 mm; (b) 3.00 mm; (c) 5.00 mm?

36.15 •• A slit 0.240 mm wide is illuminated by parallel light rays of wavelength 540 nm. The diffraction pattern is observed on a screen that is 3.00 m from the slit. The intensity at the center of the central maximum ($\theta = 0°$) is 6.00×10^{-6} W/m^2. (a) What is the distance on the screen from the center of the central maximum to the first minimum? (b) What is the intensity at a point on the screen midway between the center of the central maximum and the first minimum?

36.16 • Monochromatic light of wavelength 486 nm from a distant source passes through a slit that is 0.0290 mm wide. In the resulting diffraction pattern, the intensity at the center of the central maximum ($\theta = 0°$) is 4.00×10^{-5} W/m^2. What is the intensity at a point on the screen that corresponds to $\theta = 1.20°$.

36.17 •• A single-slit diffraction pattern is formed by monochromatic electromagnetic radiation from a distant source passing through a slit 0.105 mm wide. At the point in the pattern 3.25° from the center of the central maximum, the total phase difference between wavelets from the top and bottom of the slit is 56.0 rad. (a) What is the wavelength of the radiation? (b) What is the intensity at this point, if the intensity at the center of the central maximum is I_0?

36.18 • Consider a single-slit diffraction experiment in which the amplitude of the wave at point O in Fig. 36.5a is E_0. For each of the following cases, draw a phasor diagram like that in Fig. 36.8c and determine *graphically* the amplitude of the wave at the point in question. (*Hint:* Use Eq. (36.6) to determine the value of β for each case.) Compute the intensity and compare to Eq. (36.5). (a) $\sin\theta = \lambda/2a$; (b) $\sin\theta = \lambda/a$; (c) $\sin\theta = 3\lambda/2a$.

36.19 •• Public Radio station KXPR-FM in Sacramento broadcasts at 88.9 MHz. The radio waves pass between two tall skyscrapers that are 15.0 m apart along their closest walls. (a) At what horizontal angles, relative to the original direction of the waves, will a distant antenna not receive any signal from this station? (b) If the maximum intensity is 3.50 W/m^2 at the antenna, what is the intensity at $\pm 5.00°$ from the center of the central maximum at the distant antenna?

Section 36.4 Multiple Slits

36.20 •• **Diffraction and Interference Combined.** Consider the interference pattern produced by two parallel slits of width a and separation d, in which $d = 3a$. The slits are illuminated by normally incident light of wavelength λ. (a) First we ignore diffraction effects due to the slit width. At what angles θ from the central maximum will the next four maxima in the two-slit interference pattern occur? Your answer will be in terms of d and λ. (b) Now we include the effects of diffraction. If the intensity at $\theta = 0$ is I_0, what is the intensity at each of the angles in part (a)? (c) Which double-slit interference maxima are missing in the pattern? (d) Compare your results to those illustrated in Fig. 36.12c. In what ways is your result different?

36.21 • **Number of Fringes in a Diffraction Maximum.** In Fig. 36.12c the central diffraction maximum contains exactly seven interference fringes, and in this case $d/a = 4$. (a) What must the ratio d/a be if the central maximum contains exactly five fringes? (b) In the case considered in part (a), how many fringes are contained within the first diffraction maximum on one side of the central maximum?

36.22 • An interference pattern is produced by eight parallel and equally spaced, narrow slits. There is an interference minimum when the phase difference ϕ between light from adjacent slits is $\pi/4$. The phasor diagram is given in Fig. 36.14b. For which pairs of slits is there totally destructive interference?

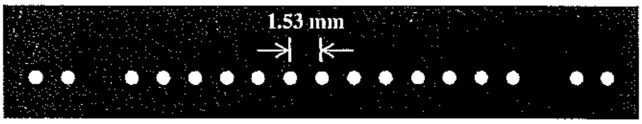
36.23 •• An interference pattern is produced by light of wavelength 580 nm from a distant source incident on two identical parallel slits separated by a distance (between centers) of 0.530 mm. (a) If the slits are very narrow, what would be the angular positions of the first-order and second-order, two-slit, interference maxima? (b) Let the slits have width 0.320 mm. In terms of the intensity I_0 at the center of the central maximum, what is the intensity at each of the angular positions in part (a)?

36.24 • Parallel rays of monochromatic light with wavelength 568 nm illuminate two identical slits and produce an interference pattern on a screen that is 75.0 cm from the slits. The centers of the slits are 0.640 mm apart and the width of each slit is 0.434 mm. If the intensity at the center of the central maximum is 5.00×10^{-4} W/m^2, what is the intensity at a point on the screen that is 0.900 mm from the center of the central maximum?

36.25 •• An interference pattern is produced by four parallel and equally spaced, narrow slits. By drawing appropriate phasor diagrams, show that there is an interference minimum when the phase difference ϕ from adjacent slits is (a) $\pi/2$; (b) π; (c) $3\pi/2$. In each case, for which pairs of slits is there totally destructive interference?

36.26 •• A diffraction experiment involving two thin parallel slits yields the pattern of closely spaced bright and dark fringes shown in Fig. E36.26. Only the central portion of the pattern is shown in the figure. The bright spots are equally spaced at 1.53 mm center to center (except for the missing spots) on a screen 2.50 m from the slits. The light source was a He-Ne laser producing a wavelength of 632.8 nm. (a) How far apart are the two slits? (b) How wide is each one?

Figure **E36.26**



36.27 •• Laser light of wavelength 500.0 nm illuminates two identical slits, producing an interference pattern on a screen 90.0 cm from the slits. The bright bands are 1.00 cm apart, and the third bright bands on either side of the central maximum are missing in the pattern. Find the width and the separation of the two slits.

Section 36.5 The Diffraction Grating

36.28 •• Monochromatic light is at normal incidence on a plane transmission grating. The first-order maximum in the interference pattern is at an angle of 8.94°. What is the angular position of the fourth-order maximum?

36.29 • If a diffraction grating produces its third-order bright band at an angle of 78.4° for light of wavelength 681 nm, find (a) the number of slits per centimeter for the grating and (b) the angular location of the first-order and second-order bright bands. (c) Will there be a fourth-order bright band? Explain.

36.30 • If a diffraction grating produces a third-order bright spot for red light (of wavelength 700 nm) at 65.0° from the central maximum, at what angle will the second-order bright spot be for violet light (of wavelength 400 nm)?

36.31 • Visible light passes through a diffraction grating that has 900 slits/cm, and the interference pattern is observed on a screen that is 2.50 m from the grating. (a) Is the angular position of the first-order spectrum small enough for $\sin\theta \approx \theta$ to be a good approximation? (b) In the first-order spectrum, the maxima for two different wavelengths are separated on the screen by 3.00 mm. What is the difference in these wavelengths?

36.32 • The wavelength range of the visible spectrum is approximately 380–750 nm. White light falls at normal incidence on a diffraction grating that has 350 slits/mm. Find the angular width of the visible spectrum in (a) the first order and (b) the third order. (*Note:* An advantage of working in higher orders is the greater angular spread and better resolution. A disadvantage is the overlapping of different orders, as shown in Example 36.4.)

36.33 • When laser light of wavelength 632.8 nm passes through a diffraction grating, the first bright spots occur at ±17.8° from the central maximum. (a) What is the line density (in lines/cm) of this grating? (b) How many additional bright spots are there beyond the first bright spots, and at what angles do they occur?

36.34 • (a) What is the wavelength of light that is deviated in the first order through an angle of 13.5° by a transmission grating having 5000 slits/cm? (b) What is the second-order deviation of this wavelength? Assume normal incidence.

36.35 • Plane monochromatic waves with wavelength 520 nm are incident normally on a plane transmission grating having 350 slits/mm. Find the angles of deviation in the first, second, and third orders.

36.36 • **Identifying Isotopes by Spectra.** Different isotopes of the same element emit light at slightly different wavelengths. A wavelength in the emission spectrum of a hydrogen atom is 656.45 nm; for deuterium, the corresponding wavelength is 656.27 nm. (a) What minimum number of slits is required to resolve these two wavelengths in second order? (b) If the grating has 500.00 slits/mm, find the angles and angular separation of these two wavelengths in the second order.

36.37 • A typical laboratory diffraction grating has 5.00×10^3 lines/cm, and these lines are contained in a 3.50-cm width of grating. (a) What is the chromatic resolving power of such a grating in the first order? (b) Could this grating resolve the lines of the sodium doublet (see Section 36.5) in the first order? (c) While doing spectral analysis of a star, you are using this grating in the *second* order to resolve spectral lines that are very close to the 587.8002-nm spectral line of iron. (i) For wavelengths longer than the iron line, what is the shortest wavelength you could distinguish from the iron line? (ii) For wavelengths shorter than the iron line, what is the longest wavelength you could distinguish from the iron line? (iii) What is the range of wavelengths you could *not* distinguish from the iron line?

36.38 • The light from an iron arc includes many different wavelengths. Two of these are at $\lambda = 587.9782$ nm and $\lambda = 587.8002$ nm. You wish to resolve these spectral lines in first order using a grating 1.20 cm in length. What minimum number of slits per centimeter must the grating have?

Section 36.6 X-Ray Diffraction

36.39 • X rays of wavelength 0.0850 nm are scattered from the atoms of a crystal. The second-order maximum in the Bragg reflection occurs when the angle θ in Fig. 36.22 is 21.5°. What is the spacing between adjacent atomic planes in the crystal?

36.40 • If the planes of a crystal are 3.50 Å (1 Å = 10^{-10} m = 1 Ångstrom unit) apart, (a) what wavelength of electromagnetic waves is needed so that the first strong interference maximum in the Bragg reflection occurs when the waves strike the planes at an angle of 15.0°, and in what part of the electromagnetic spectrum do these waves lie? (See Fig. 32.4.) (b) At what other angles will strong interference maxima occur?

36.41 • Monochromatic x rays are incident on a crystal for which the spacing of the atomic planes is 0.440 nm. The first-order maximum in the Bragg reflection occurs when the incident and reflected x rays make an angle of 39.4° with the crystal planes. What is the wavelength of the x rays?

Section 36.7 Circular Apertures and Resolving Power

36.42 • BIO If you can read the bottom row of your doctor's eye chart, your eye has a resolving power of 1 arcminute, equal to $\frac{1}{60}$ degree. If this resolving power is diffraction limited, to what effective diameter of your eye's optical system does this correspond? Use Rayleigh's criterion and assume $\lambda = 550$ nm.

36.43 • Two satellites at an altitude of 1200 km are separated by 28 km. If they broadcast 3.6-cm microwaves, what minimum receiving-dish diameter is needed to resolve (by Rayleigh's criterion) the two transmissions?

36.44 •• The VLBA (Very Long Baseline Array) uses a number of individual radio telescopes to make one unit having an equivalent diameter of about 8000 km. When this radio telescope is focusing radio waves of wavelength 2.0 cm, what would have to be the diameter of the mirror of a visible-light telescope focusing light of wavelength 550 nm so that the visible-light telescope has the same resolution as the radio telescope?

36.45 •• Monochromatic light with wavelength 620 nm passes through a circular aperture with diameter 7.4 μm. The resulting diffraction pattern is observed on a screen that is 4.5 m from the aperture. What is the diameter of the Airy disk on the screen?

36.46 • **Photography.** A wildlife photographer uses a moderate telephoto lens of focal length 135 mm and maximum aperture $f/4.00$ to photograph a bear that is 11.5 m away. Assume the wavelength is 550 nm. (a) What is the width of the smallest feature on the bear that this lens can resolve if it is opened to its maximum aperture? (b) If, to gain depth of field, the photographer stops the lens down to $f/22.0$, what would be the width of the smallest resolvable feature on the bear?

36.47 • **Observing Jupiter.** You are asked to design a space telescope for earth orbit. When Jupiter is 5.93×10^8 km away (its closest approach to the earth), the telescope is to resolve, by Rayleigh's criterion, features on Jupiter that are 250 km apart. What minimum-diameter mirror is required? Assume a wavelength of 500 nm.

36.48 • A converging lens 7.20 cm in diameter has a focal length of 300 mm. If the resolution is diffraction limited, how far away can an object be if points on it 4.00 mm apart are to be resolved (according to Rayleigh's criterion)? Use $\lambda = 550$ nm.

36.49 •• **Hubble Versus Arecibo.** The Hubble Space Telescope has an aperture of 2.4 m and focuses visible light (380–750 nm). The Arecibo radio telescope in Puerto Rico is 305 m (1000 ft) in diameter (it is built in a mountain valley) and focuses radio waves of wavelength 75 cm. (a) Under optimal viewing conditions, what is the smallest crater that each of these telescopes could resolve on our moon? (b) If the Hubble Space Telescope were to be converted to surveillance use, what is the highest orbit above the surface of the earth it could have and still be able to resolve the license plate (not the letters, just the plate) of a car on the ground? Assume optimal viewing conditions, so that the resolution is diffraction limited.

36.50 • **Searching for Starspots.** The Hale Telescope on Palomar Mountain in California has a mirror 200 in. (5.08 m) in diameter and it focuses visible light. Given that a large sunspot is about 10,000 mi in diameter, what is the most distant star on which this telescope could resolve a sunspot to see whether other stars have them? (Assume optimal viewing conditions, so that the resolution is diffraction limited.) Are there any stars this close to us, besides our sun?

PROBLEMS

36.51 •• **BIO Thickness of Human Hair.** Although we have discussed single-slit diffraction only for a slit, a similar result holds when light bends around a straight, thin object, such as a strand of hair. In that case, a is the width of the strand. From actual laboratory measurements on a human hair, it was found that when a beam of light of wavelength 632.8 nm was shone on a single strand of hair, and the diffracted light was viewed on a screen 1.25 m away, the first dark fringes on either side of the central bright spot were 5.22 cm apart. How thick was this strand of hair?

36.52 •• Suppose the entire apparatus (slit, screen, and space in between) in Exercise 36.4 is immersed in water ($n = 1.333$). Then what is the distance between the two dark fringes?

36.53 ••• Laser light of wavelength 632.8 nm falls normally on a slit that is 0.0250 mm wide. The transmitted light is viewed on a distant screen where the intensity at the center of the central bright fringe is 8.50 W/m^2. (a) Find the maximum number of totally dark fringes on the screen, assuming the screen is large enough to show them all. (b) At what angle does the dark fringe that is most distant from the center occur? (c) What is the maximum intensity of the bright fringe that occurs immediately before the dark fringe in part (b)? Approximate the angle at which this fringe occurs by assuming it is midway between the angles to the dark fringes on either side of it.

36.54 •• **CP** A loudspeaker having a diaphragm that vibrates at 1250 Hz is traveling at 80.0 m/s directly toward a pair of holes in a very large wall in a region for which the speed of sound is 344 m/s. You observe that the sound coming through the openings first cancels at $\pm 11.4°$ with respect to the original direction of the speaker when observed far from the wall. (a) How far apart are the two openings? (b) At what angles would the sound first cancel if the source stopped moving?

36.55 • **Measuring Refractive Index.** A thin slit illuminated by light of frequency f produces its first dark band at $\pm 38.2°$ in air. When the entire apparatus (slit, screen, and space in between) is immersed in an unknown transparent liquid, the slit's first dark bands occur instead at $\pm 21.6°$. Find the refractive index of the liquid.

36.56 • **Grating Design.** Your boss asks you to design a diffraction grating that will disperse the first-order visible spectrum through an angular range of 21.0° (see Example 36.4 in Section 36.5). (a) What must the number of slits per centimeter be for this grating? (b) At what angles will the first-order visible spectrum begin and end?

36.57 •• A slit 0.360 mm wide is illuminated by parallel rays of light that have a wavelength of 540 nm. The diffraction pattern is observed on a screen that is 1.20 m from the slit. The intensity at the center of the central maximum ($\theta = 0°$) is I_0. (a) What is the distance on the screen from the center of the central maximum to the first minimum? (b) What is the distance on the screen from the center of the central maximum to the point where the intensity has fallen to $I_0/2$?

36.58 ••• **CALC** The intensity of light in the Fraunhofer diffraction pattern of a single slit is

$$I = I_0\left(\frac{\sin\gamma}{\gamma}\right)^2$$

where

$$\gamma = \frac{\pi a \sin\theta}{\lambda}$$

(a) Show that the equation for the values of γ at which I is a maximum is $\tan\gamma = \gamma$. (b) Determine the three smallest positive values of γ that are solutions of this equation. (*Hint:* You can use a trial-and-error procedure. Guess a value of γ and adjust your guess to bring $\tan\gamma$ closer to γ. A graphical solution of the equation is very helpful in locating the solutions approximately, to get good initial guesses.)

36.59 •• **Angular Width of a Principal Maximum.** Consider N evenly spaced, narrow slits. Use the small-angle approximation $\sin\theta = \theta$ (for θ in radians) to prove the following: For an intensity maximum that occurs at an angle θ, the intensity minima immediately adjacent to this maximum are at angles $\theta + \lambda/Nd$ and $\theta - \lambda/Nd$, so that the angular width of the principal maximum is $2\lambda/Nd$. This is proportional to $1/N$, as we concluded in Section 36.4 on the basis of energy conservation.

36.60 •• **CP CALC** In a large vacuum chamber, monochromatic laser light passes through a narrow slit in a thin aluminum plate and forms a diffraction pattern on a screen that is 0.620 m from the slit. When the aluminum plate has a temperature of 20.0°C, the width of the central maximum in the diffraction pattern is 2.75 mm. What is the change in the width of the central maximum when the temperature of the plate is raised to 520.0°C? Does the width of the central diffraction maximum increase or decrease when the temperature is increased?

36.61 • **Phasor Diagram for Eight Slits.** An interference pattern is produced by eight equally spaced, narrow slits. Figure 36.14 shows phasor diagrams for the cases in which the phase difference ϕ between light from adjacent slits is $\phi = \pi$, $\phi = \pi/4$, and $\phi = \pi/2$. Each of these cases gives an intensity minimum. The caption for Fig. 36.14 also claims that minima occur for $\phi = 3\pi/4$, $\phi = 5\pi/4$, $\phi = 3\pi/2$, and $\phi = 7\pi/4$. (a) Draw the phasor diagram for each of these four cases, and explain why each diagram proves that there is in fact a minimum. (*Note:* You may find it helpful to use a different colored pencil for each slit!) (b) For each of the four cases $\phi = 3\pi/4$, $\phi = 5\pi/4$, $\phi = 3\pi/2$, and $\phi = 7\pi/4$, for which pairs of slits is there totally destructive interference?

36.62 •• **CP** In a laboratory, light from a particular spectrum line of helium passes through a diffraction grating and the second-order maximum is at 18.9° from the center of the central bright fringe. The same grating is then used for light from a distant galaxy that is moving away from the earth with a speed of 2.65×10^7 m/s. For the light from the galaxy, what is the angular location of the second-order maximum for the same spectral line as was observed in the lab? (See Section 16.8.)

36.63 • What is the longest wavelength that can be observed in the third order for a transmission grating having 9200 slits/cm? Assume normal incidence.

36.64 •• (a) Figure 36.16 shows plane waves of light incident *normally* on a diffraction grating. If instead the light strikes the grating at an angle of incidence θ' (measured from the normal), show that the condition for an intensity maximum is *not* Eq. (36.13), but rather

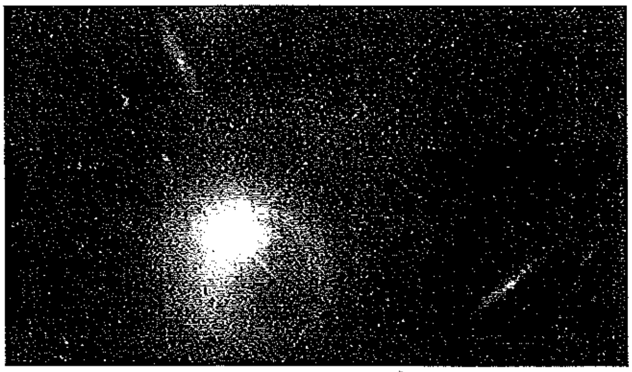
$$d(\sin\theta + \sin\theta') = m\lambda \quad (m = 0, \pm 1, \pm 2, \pm 3, \ldots)$$

(b) For the grating described in Example 36.4 (Section 36.5), with 600 slits/mm, find the angles of the maxima corresponding to $m = 0$, 1, and -1 with red light ($\lambda = 650$ nm) for the cases $\theta' = 0$ (normal incidence) and $\theta' = 20.0°$.

36.65 • A diffraction grating has 650 slits/mm. What is the highest order that contains the entire visible spectrum? (The wavelength range of the visible spectrum is approximately 380–750 nm.)

36.66 •• *Quasars,* an abbreviation for *quasi-stellar radio sources,* are distant objects that look like stars through a telescope but that emit far more electromagnetic radiation than an entire normal galaxy of stars. An example is the bright object below and to the left of center in Fig. P36.66; the other elongated objects in this image are normal galaxies. The leading model for the structure of a quasar is a galaxy with a supermassive black hole at its center. In this model, the radiation is emitted by interstellar gas and dust within the galaxy as this material falls toward the black hole. The radiation is thought to emanate from a region just a few light-years in diameter. (The diffuse glow surrounding the bright quasar shown in Fig. P36.66 is thought to be this quasar's host galaxy.) To investigate this model of quasars and to study other exotic astronomical objects, the Russian Space Agency plans to place a radio telescope in an orbit that extends to 77,000 km from the earth. When the signals from this telescope are combined with signals from the ground-based telescopes of the VLBA, the resolution will be that of a single radio telescope 77,000 km in diameter. What is the size of the smallest detail that this arrangement could resolve in quasar 3C 405, which is 7.2×10^8 light-years from earth, using radio waves at a frequency of 1665 MHz? (*Hint:* Use Rayleigh's criterion.) Give your answer in light-years and in kilometers.

Figure **P36.66**



36.67 ••• **Phased-Array Radar.** In one common type of radar installation, a rotating antenna sweeps a radio beam around the sky. But in a *phased-array* radar system, the antennas remain stationary and the beam is swept electronically. To see how this is done, consider an array of N antennas that are arranged along the horizontal x-axis at $x = 0, \pm d, \pm 2d, \ldots, \pm(N-1)d/2$. (The number N is odd.) Each antenna emits radiation uniformly in all directions in the horizontal xy-plane. The antennas all emit radiation coherently, with the same amplitude E_0 and the same wavelength λ. The relative phase δ of the emission from adjacent antennas can be varied, however. If the antenna at $x = 0$ emits a signal that is given by $E_0 \cos \omega t$, as measured at a point next to the antenna, the antenna at $x = d$ emits a signal given by $E_0 \cos(\omega t + \delta)$, as measured at a point next to that antenna. The corresponding quantity for the antenna at $x = -d$ is $E_0 \cos(\omega t - \delta)$; for the antennas at $x = \pm 2d$, it is $E_0 \cos(\omega t \pm 2\delta)$; and so on. (a) If $\delta = 0$, the interference pattern at a distance from the antennas is large compared to d and has a principal maximum at $\theta = 0$ (that is, in the $+y$-direction, perpendicular to the line of the antennas). Show that if $d < \lambda$, this is the *only* principal interference maximum in the angular range $-90° < \theta < 90°$. Hence this principal maximum describes a beam emitted in the direction $\theta = 0$. As described in Section 36.4, if N is large, the beam will have a large intensity and be quite narrow. (b) If $\delta \neq 0$, show that the principal intensity maximum described in part (a) is located at

$$\theta = \arcsin\left(\frac{\delta \lambda}{2\pi d}\right)$$

where δ is measured in radians. Thus, by varying δ from positive to negative values and back again, which can easily be done electronically, the beam can be made to sweep back and forth around $\theta = 0$. (c) A weather radar unit to be installed on an airplane emits radio waves at 8800 MHz. The unit uses 15 antennas in an array 28.0 cm long (from the antenna at one end of the array to the antenna at the other end). What must the maximum and minimum values of δ be (that is, the most positive and most negative values) if the radar beam is to sweep 45° to the left or right of the airplane's direction of flight? Give your answer in radians.

36.68 •• **Underwater Photography.** An underwater camera has a lens of focal length 35.0 mm and a maximum aperture of $f/2.80$. The film it uses has an emulsion that is sensitive to light of frequency 6.00×10^{14} Hz. If the photographer takes a picture of an object 2.75 m in front of the camera with the lens wide open, what is the width of the smallest resolvable detail on the subject if the object is (a) a fish underwater with the camera in the water and (b) a person on the beach with the camera out of the water?

36.69 •• An astronaut in the space shuttle can just resolve two point sources on earth that are 65.0 m apart. Assume that the resolution is diffraction limited and use Rayleigh's criterion. What is the astronaut's altitude above the earth? Treat his eye as a circular aperture with a diameter of 4.00 mm (the diameter of his pupil), and take the wavelength of the light to be 550 nm. Ignore the effect of fluid in the eye.

36.70 •• BIO **Resolution of the Eye.** The maximum resolution of the eye depends on the diameter of the opening of the pupil (a diffraction effect) and the size of the retinal cells. The size of the retinal cells (about 5.0 μm in diameter) limits the size of an object at the near point (25 cm) of the eye to a height of about 50 μm. (To get a reasonable estimate without having to go through complicated calculations, we shall ignore the effect of the fluid in the eye.) (a) Given that the diameter of the human pupil is about 2.0 mm, does the Rayleigh criterion allow us to resolve a 50-μm-tall object at 25 cm from the eye with light of wavelength 550 nm? (b) According to the Rayleigh criterion, what is the shortest object we could resolve at the 25-cm near point with light of wavelength 550 nm? (c) What angle would the object in part (b) subtend at the eye? Express your answer in minutes (60 min = 1°), and compare it with the experimental value of about 1 min. (d) Which effect is more important in limiting the resolution of our eyes: diffraction or the size of the retinal cells?

36.71 •• A glass sheet is covered by a very thin opaque coating. In the middle of this sheet there is a thin scratch 0.00125 mm thick. The sheet is totally immersed beneath the surface of a liquid. Parallel rays of monochromatic coherent light with wavelength 612 nm in air strike the sheet perpendicular to its surface and pass through the scratch. A screen is placed in the liquid a distance of 30.0 cm away from the sheet and parallel to it. You observe that the first dark fringes on either side of the central bright fringe on the screen are 22.4 cm apart. What is the refractive index of the liquid?

36.72 •• Observing Planets Beyond Our Solar System. NASA is considering a project called *Planet Imager* that would give astronomers the ability to see details on planets orbiting other stars. Using the same principle as the Very Large Array (see Section 36.7), *Planet Imager* will use an array of infrared telescopes spread over thousands of kilometers of space. (Visible light would give even better resolution. Unfortunately, at visible wavelengths, stars are so bright that a planet would be lost in the glare. This is less of a problem at infrared wavelengths.) (a) If *Planet Imager* has an effective diameter of 6000 km and observes infrared radiation at a wavelength of 10 μm, what is the greatest distance at which it would be able to observe details as small as 250 km across (about the size of the greater Los Angeles area) on a planet? Give your answer in light-years (see Appendix E). (*Hint:* Use Rayleigh's criterion.) (b) For comparison, consider the resolution of a single infrared telescope in space that has a diameter of 1.0 m and that observes 10-μm radiation. What is the size of the smallest details that such a telescope could resolve at the distance of the nearest star to the sun, Proxima Centauri, which is 4.22 light-years distant? How does this compare to the diameter of the earth (1.27×10^4 km)? To the average distance from the earth to the sun (1.50×10^8 km)? Would a single telescope of this kind be able to detect the presence of a planet like the earth, in an orbit the size of the earth's orbit, around *any* other star? Explain. (c) Suppose *Planet Imager* is used to observe a planet orbiting the star 70 Virginis, which is 59 light-years from our solar system. A planet (though not an earthlike one) has in fact been detected orbiting this star, not by imaging it directly but by observing the slight "wobble" of the star as both it and the planet orbit their common center of mass. What is the size of the smallest details that *Planet Imager* could hope to resolve on the planet of 70 Virginis? How does this compare to the diameter of the planet, assumed to be comparable to that of Jupiter (1.38×10^5 km)? (Although the planet of 70 Virginis is thought to be at least 6.6 times more massive than Jupiter, its radius is probably not too different from that of Jupiter. The reason is that such large planets are thought to be composed primarily of gases, not rocky material, and hence can be greatly compressed by the mutual gravitational attraction of different parts of the planet.)

CHALLENGE PROBLEMS

36.73 ••• CALC It is possible to calculate the intensity in the single-slit Fraunhofer diffraction pattern *without* using the phasor method of Section 36.3. Let y' represent the position of a point within the slit of width a in Fig. 36.5a, with $y' = 0$ at the center of the slit so that the slit extends from $y' = -a/2$ to $y' = a/2$. We imagine dividing the slit up into infinitesimal strips of width dy', each of which acts as a source of secondary wavelets. (a) The amplitude of the total wave at the point O on the distant screen in Fig. 36.5a is E_0. Explain why the amplitude of the wavelet from each infinitesimal strip within the slit is $E_0(dy'/a)$, so that the electric field of the wavelet a distance x from the infinitesimal strip is $dE = E_0(dy'/a)\sin(kx - \omega t)$. (b) Explain why the wavelet from each strip as detected at point P in Fig. 36.5a can be expressed as

$$dE = E_0\frac{dy'}{a}\sin[k(D - y'\sin\theta) - \omega t]$$

where D is the distance from the center of the slit to point P and $k = 2\pi/\lambda$. (c) By integrating the contributions dE from all parts of the slit, show that the total wave detected at point P is

$$E = E_0\sin(kD - \omega t)\frac{\sin[ka(\sin\theta)/2]}{ka(\sin\theta)/2}$$
$$= E_0\sin(kD - \omega t)\frac{\sin[\pi a(\sin\theta)/\lambda]}{\pi a(\sin\theta)/\lambda}$$

(The trigonometric identities in Appendix B will be useful.) Show that at $\theta = 0$, corresponding to point O in Fig. 36.5a, the wave is $E = E_0\sin(kD - \omega t)$ and has amplitude E_0, as stated in part (a). (d) Use the result of part (c) to show that if the intensity at point O is I_0, then the intensity at a point P is given by Eq. (36.7).

36.74 ••• Intensity Pattern of N Slits. (a) Consider an arrangement of N slits with a distance d between adjacent slits. The slits emit coherently and in phase at wavelength λ. Show that at a time t, the electric field at a distant point P is

$$E_P(t) = E_0\cos(kR - \omega t) + E_0\cos(kR - \omega t + \phi) + E_0\cos(kR - \omega t + 2\phi) + \cdots + E_0\cos(kR - \omega t + (N-1)\phi)$$

where E_0 is the amplitude at P of the electric field due to an individual slit, $\phi = (2\pi d\sin\theta)/\lambda$, θ is the angle of the rays reaching P (as measured from the perpendicular bisector of the slit arrangement), and R is the distance from P to the most distant slit. In this problem, assume that R is much larger than d. (b) To carry out the sum in part (a), it is convenient to use the complex-number relationship

$$e^{iz} = \cos z + i\sin z$$

where $i = \sqrt{-1}$. In this expression, $\cos z$ is the *real part* of the complex number e^{iz}, and $\sin z$ is its *imaginary part.* Show that the electric field $E_P(t)$ is equal to the real part of the complex quantity

$$\sum_{n=0}^{N-1} E_0 e^{i(kR - \omega t + n\phi)}$$

(c) Using the properties of the exponential function that $e^A e^B = e^{(A+B)}$ and $(e^A)^n = e^{nA}$, show that the sum in part (b) can be written as

$$E_0\left(\frac{e^{iN\phi} - 1}{e^{i\phi} - 1}\right)e^{i(kR-\omega t)}$$
$$= E_0\left(\frac{e^{iN\phi/2} - e^{-iN\phi/2}}{e^{i\phi/2} - e^{-i\phi/2}}\right)e^{i[kR - \omega t + (N-1)\phi/2]}$$

Then, using the relationship $e^{iz} = \cos z + i\sin z$, show that the (real) electric field at point P is

$$E_P(t) = \left[E_0\frac{\sin(N\phi/2)}{\sin(\phi/2)}\right]\cos[kR - \omega t + (N-1)\phi/2]$$

The quantity in the first square brackets in this expression is the amplitude of the electric field at P. (d) Use the result for the electric-field amplitude in part (c) to show that the intensity at an angle θ is

$$I = I_0\left[\frac{\sin(N\phi/2)}{\sin(\phi/2)}\right]^2$$

where I_0 is the maximum intensity for an individual slit. (e) Check the result in part (d) for the case $N = 2$. It will help to recall that $\sin 2A = 2\sin A\cos A$. Explain why your result differs from Eq. (35.10), the expression for the intensity in two-source interference, by a factor of 4. (*Hint:* Is I_0 defined in the same way in both expressions?)

36.75 ••• CALC Intensity Pattern of N Slits, Continued. Part (d) of Challenge Problem 36.74 gives an expression for the intensity in the interference pattern of N identical slits. Use this result to verify the following statements. (a) The maximum intensity in the pattern is $N^2 I_0$. (b) The principal maximum at the center of the pattern extends from $\phi = -2\pi/N$ to $\phi = 2\pi/N$, so its width is inversely proportional to $1/N$. (c) A minimum occurs whenever ϕ is an integral multiple of $2\pi/N$, except when ϕ is an integral multiple of 2π (which gives a principal maximum). (d) There are $(N-1)$ minima between each pair of principal maxima. (e) Halfway between two principal maxima, the intensity can be no greater than I_0; that is, it can be no greater than $1/N^2$ times the intensity at a principal maximum.

Answers

Chapter Opening Question ?

The shorter wavelength of a Blu-ray scanning laser gives it superior resolving power, so information can be more tightly packed onto a Blu-ray disc than a DVD. See Section 36.7 for details.

Test Your Understanding Questions

36.1 Answer: yes When you hear the voice of someone standing around a corner, you are hearing sound waves that underwent diffraction. If there were no diffraction of sound, you could hear sounds only from objects that were in plain view.

36.2 Answer: (ii), (i) and (iv) (tie), (iii) The angle θ of the first dark fringe is given by Eq. (36.2) with $m = 1$, or $\sin\theta = \lambda/a$. The larger the value of the ratio λ/a, the larger the value of $\sin\theta$ and hence the value of θ. The ratio λ/a in each case is (i) $(400\text{ nm})/(0.20\text{ mm}) = (4.0\times10^{-7}\text{ m})/(2.0\times10^{-4}\text{ m}) = 2.0\times10^{-3}$; (ii) $(600\text{ nm})/(0.20\text{ mm}) = (6.0\times10^{-7}\text{ m})/(2.0\times10^{-4}\text{ m}) = 3.0\times10^{-3}$; (iii) $(400\text{ nm})/(0.30\text{ mm}) = (4.0\times10^{-7}\text{ m})/(3.0\times10^{-4}\text{ m}) = 1.3\times10^{-3}$; (iv) $(600\text{ nm})/(0.30\text{ mm}) = (6.0\times10^{-7}\text{ m})/(3.0\times10^{-4}\text{ m}) = 2.0\times10^{-3}$.

36.3 Answers: (ii) and (iii) If the slit width a is less than the wavelength λ, there are no points in the diffraction pattern at which the intensity is zero (see Fig. 36.10a). The slit width is $0.0100\text{ mm} = 1.00\times10^{-5}\text{ m}$, so this condition is satisfied for (ii) ($\lambda = 10.6\ \mu\text{m} = 1.06\times10^{-5}\text{ m}$) and (iii) ($\lambda = 1.00\text{ mm} = 1.00\times10^{-3}\text{ m}$) but not for (i) ($\lambda = 500\text{ nm} = 5.00\times10^{-7}\text{ m}$) or (iv) ($\lambda = 50.0\text{ nm} = 5.00\times10^{-8}\text{ m}$).

36.4 Answers: yes; $m_i = \pm5, \pm10, \ldots$ A "missing maximum" satisfies both $d\sin\theta = m_i\lambda$ (the condition for an interference maximum) and $a\sin\theta = m_d\lambda$ (the condition for a diffraction minimum). Substituting $d = 2.5a$, we can combine these two conditions into the relationship $m_i = 2.5m_d$. This is satisfied for $m_i = \pm5$ and $m_d = \pm2$ (the fifth interference maximum is missing because it coincides with the second diffraction minimum), $m_i = \pm10$ and $m_d = \pm4$ (the tenth interference maximum is missing because it coincides with the fourth diffraction minimum), and so on.

36.5 Answer: (i) As described in the text, the resolving power needed is $R = Nm = 1000$. In the first order ($m = 1$) we need $N = 1000$ slits, but in the fourth order ($m = 4$) we need only $N = R/m = 1000/4 = 250$ slits. (These numbers are only approximate because of the arbitrary nature of our criterion for resolution and because real gratings always have slight imperfections in the shapes and spacings of the slits.)

36.6 Answer: no The angular position of the mth maximum is given by Eq. (36.16), $2d\sin\theta = m\lambda$. With $d = 0.200\text{ nm}$, $\lambda = 0.100\text{ nm}$, and $m = 5$, this gives $\sin\theta = m\lambda/2d = (5)(0.100\text{ nm})/(2)(0.200\text{ nm}) = 1.25$. Since the sine function can never be greater than 1, this means that there is no solution to this equation and the $m = 5$ maximum does not appear.

36.7 Answer: (iii), (ii), (iv), (i) Rayleigh's criterion combined with Eq. (36.17) shows that the smaller the value of the ratio λ/D, the better the resolving power of a telescope of diameter D. For the four telescopes, this ratio is equal to (i) $(21\text{ cm})/(100\text{ m}) = (0.21\text{ m})/(100\text{ m}) = 2.1\times10^{-3}$; (ii) $(500\text{ nm})/(2.0\text{ m}) = (5.0\times10^{-7}\text{ m})/(2.0\text{ m}) = 2.5\times10^{-7}$; (iii) $(100\text{ nm})/(1.0\text{ m}) = (1.0\times10^{-7}\text{ m})/(1.0\text{ m}) = 1.0\times10^{-7}$; (iv) $(10\ \mu\text{m})/(2.0\text{ m}) = (1.0\times10^{-5}\text{ m})/(2.0\text{ m}) = 5.0\times10^{-6}$.

Bridging Problem

Answers: 1.501×10^{7} m/s or 5.00% of c; away from us