BRIDGING PROBLEM Saturation of ^{128}I Production

In an experiment, the iodine isotope $^{128}\mathrm{I}$ is created by irradiating a sample of $^{127}\mathrm{I}$ with a beam of neutrons, yielding 1.50×10^6 $^{128}\mathrm{I}$ nuclei per second. Initially no $^{128}\mathrm{I}$ nuclei are present. A $^{128}\mathrm{I}$ nucleus decays by β^- emission with a half-life of 25.0 min. (a) To what nuclide does $^{128}\mathrm{I}$ decay? (b) Could that nuclide decay back to $^{128}\mathrm{I}$ by β^+ emission? Why or why not? (c) After the sample has been irradiated for a long time, what is the maximum number of $^{128}\mathrm{I}$ atoms that can be present in the sample? What is the maximum activity that can be produced? (This steady-state situation is called *saturation*.) (d) Find an expression for the number of $^{128}\mathrm{I}$ atoms present in the sample as a function of time.

SOLUTION GUIDE

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IDENTIFY and SET UP

1. What happens to the values of Z, N, and A in β^- decay? What must be true for β^- decay to be possible? For β^+ decay to be possible?
2. You'll need to write an equation for the rate of change dN/dt of the number N of $^{128}\mathrm{I}$ atoms in the sample, taking account of both the creation of $^{128}\mathrm{I}$ by the neutron irradiation and the decay of any $^{128}\mathrm{I}$ present. In the steady state, how do the rates of these two processes compare?
3. List the unknown quantities for each part of the problem and identify your target variables.

EXECUTE

4. Find the values of Z and N of the nuclide produced by the decay of $^{128}\mathrm{I}$. What element is this?
5. Decide whether this nuclide can decay back to $^{128}\mathrm{I}$.
6. Inspect your equation for dN/dt. What is the value of dN/dt in the steady state? Use this to solve for the steady-state values of N and the activity.
7. Solve your dN/dt equation for the function $N(t)$. (*Hint:* See Section 26.4.)

EVALUATE

8. Your result from step 6 tells you the value of N after a long time (that is, for large values of t). Is this consistent with your result from step 7? What would constitute a "long time" under these conditions?

Problems

For instructor-assigned homework, go to www.masteringphysics.com

•, ••, •••: Problems of increasing difficulty. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q43.1 BIO Neutrons have a magnetic dipole moment and can undergo spin flips by absorbing electromagnetic radiation. Why, then, are protons rather than neutrons used in MRI of body tissues? (See Fig. 43.1.)

Q43.2 In Eq. (43.11), as the total number of nucleons becomes larger, the importance of the second term in the equation decreases relative to that of the first term. Does this make physical sense? Explain.

Q43.3 Why aren't the masses of all nuclei integer multiples of the mass of a single nucleon?

Q43.4 Can you tell from the value of the mass number A whether to use a plus value, a minus value, or zero for the fifth term of Eq. (43.11)? Explain.

Q43.5 What are the six known elements for which Z is a magic number? Discuss what properties these elements have as a consequence of their special values of Z.

Q43.6 The binding energy per nucleon for most nuclides doesn't vary much (see Fig. 43.2). Is there similar consistency in the *atomic* energy of atoms, on an "energy per electron" basis? If so, why? If not, why not?

Q43.7 Heavy, unstable nuclei usually decay by emitting an α or β particle. Why don't they usually emit a single proton or neutron?

Q43.8 The only two stable nuclides with more protons than neutrons are $^{1}_{1}\mathrm{H}$ and $^{3}_{2}\mathrm{He}$. Why is $Z > N$ so uncommon?

Q43.9 Since lead is a stable element, why doesn't the $^{238}\mathrm{U}$ decay series shown in Fig. 43.7 stop at lead, $^{214}\mathrm{Pb}$?

Q43.10 In the $^{238}\mathrm{U}$ decay series shown in Fig. 43.7, some nuclides in the series are found much more abundantly in nature than others, even though every $^{238}\mathrm{U}$ nucleus goes through every step in the series before finally becoming $^{206}\mathrm{Pb}$. Why don't the intermediate nuclides all have the same abundance?

Q43.11 Compared to α particles with the same energy, β particles can much more easily penetrate through matter. Why is this?

Q43.12 If ${}^{A}_{Z}\mathrm{El}_i$ represents the initial nuclide, what is the decay process or processes if the final nuclide is (a) ${}^{A}_{Z+1}\mathrm{El}_f$; (b) ${}^{A-4}_{Z-2}\mathrm{El}_f$; (c) ${}^{A}_{Z-1}\mathrm{El}_f$?

Q43.13 In a nuclear decay equation, why can we represent an electron as ${}^{\;0}_{-1}\beta^-$? What are the equivalent representations for a positron, a neutrino, and an antineutrino?

Q43.14 Why is the alpha, beta, or gamma decay of an unstable nucleus unaffected by the *chemical* situation of the atom, such as the nature of the molecule or solid in which it is bound? The chemical situation of the atom can, however, have an effect on the half-life in electron capture. Why is this?

Q43.15 In the process of *internal conversion,* a nucleus decays from an excited state to a ground state by giving the excitation energy directly to an atomic electron rather than emitting a gamma-ray photon. Why can this process also produce x-ray photons?

Q43.16 In Example 43.9 (Section 43.4), the activity of atmospheric carbon *before* 1900 was given. Discuss why this activity may have changed since 1900.

Q43.17 BIO One problem in radiocarbon dating of biological samples, especially very old ones, is that they can easily be contaminated with modern biological material during the measurement process. What effect would such contamination have on the estimated age? Why is such contamination a more serious problem for samples of older material than for samples of younger material?

Q43.18 The most common radium isotope found on earth, ${}^{226}\text{Ra}$, has a half-life of about 1600 years. If the earth was formed well over 10^9 years ago, why is there any radium left now?

Q43.19 Fission reactions occur only for nuclei with large nucleon numbers, while exoergic fusion reactions occur only for nuclei with small nucleon numbers. Why is this?

Q43.20 When a large nucleus splits during nuclear fission, the daughter nuclei of the fission fly apart with enormous kinetic energy. Why does this happen?

Q43.21 As stars age, they use up their supply of hydrogen and eventually begin producing energy by a reaction that involves the fusion of three helium nuclei to form a carbon nucleus. Would you expect the interiors of these old stars to be hotter or cooler than the interiors of younger stars? Explain.

EXERCISES

Section 43.1 Properties of Nuclei

43.1 • How many protons and how many neutrons are there in a nucleus of the most common isotope of (a) silicon, ${}^{28}_{14}\text{Si}$; (b) rubidium, ${}^{85}_{37}\text{Rb}$; (c) thallium, ${}^{205}_{81}\text{Tl}$?

43.2 •• **CP** Hydrogen atoms are placed in an external 1.65-T magnetic field. (a) The *protons* can make transitions between states where the nuclear spin component is parallel and antiparallel to the field by absorbing or emitting a photon. Which state has lower energy: the state with the nuclear spin component parallel or antiparallel to the field? What are the frequency and wavelength of the photon? In which region of the electromagnetic spectrum does it lie? (b) The *electrons* can make transitions between states where the electron spin component is parallel and antiparallel to the field by absorbing or emitting a photon. Which state has lower energy: the state with the electron spin component parallel or antiparallel to the field? What are the frequency and wavelength of the photon? In which region of the electromagnetic spectrum does it lie?

43.3 • Hydrogen atoms are placed in an external magnetic field. The protons can make transitions between states in which the nuclear spin component is parallel and antiparallel to the field by absorbing or emitting a photon. What magnetic-field magnitude is required for this transition to be induced by photons with frequency 22.7 MHz?

43.4 •• Neutrons are placed in a magnetic field with magnitude 2.30 T. (a) What is the energy difference between the states with the nuclear spin angular momentum components parallel and antiparallel to the field? Which state is lower in energy: the one with its spin component parallel to the field or the one with its spin component antiparallel to the field? How do your results compare with the energy states for a proton in the same field (see Example 43.2)? (b) The neutrons can make transitions from one of these states to the other by emitting or absorbing a photon with energy equal to the energy difference of the two states. Find the frequency and wavelength of such a photon.

Section 43.2 Nuclear Binding and Nuclear Structure

43.5 • The most common isotope of boron is ${}^{11}_{5}\text{B}$. (a) Determine the total binding energy of ${}^{11}_{5}\text{B}$ from Table 43.2 in Section 43.1. (b) Calculate this binding energy from Eq. (43.11). (Why is the fifth term zero?) Compare to the result you obtained in part (a). What is the percent difference? Compare the accuracy of Eq. (43.11) for ${}^{11}_{5}\text{B}$ to its accuracy for ${}^{62}_{28}\text{Ni}$ (see Example 43.4).

43.6 • The most common isotope of uranium, ${}^{238}_{92}\text{U}$, has atomic mass 238.050783 u. Calculate (a) the mass defect; (b) the binding energy (in MeV); (c) the binding energy per nucleon.

43.7 • **CP** What is the maximum wavelength of a γ ray that could break a deuteron into a proton and a neutron? (This process is called photodisintegration.)

43.8 • Calculate (a) the total binding energy and (b) the binding energy per nucleon of ${}^{12}\text{C}$. (c) What percent of the rest mass of this nucleus is its total binding energy?

43.9 • **CP** A photon with a wavelength of 3.50×10^{-13} m strikes a deuteron, splitting it into a proton and a neutron. (a) Calculate the kinetic energy released in this interaction. (b) Assuming the two particles share the energy equally, and taking their masses to be 1.00 u, calculate their speeds after the photodisintegration.

43.10 • Calculate the mass defect, the binding energy (in MeV), and the binding energy per nucleon of (a) the nitrogen nucleus, ${}^{14}_{7}\text{N}$, and (b) the helium nucleus, ${}^{4}_{2}\text{He}$. (c) How does the binding energy per nucleon compare for these two nuclei?

43.11 • Use Eq. (43.11) to calculate the binding energy per nucleon for the nuclei ${}^{86}_{36}\text{Kr}$ and ${}^{180}_{73}\text{Ta}$. Do your results confirm what is shown in Fig. 43.2—that for A greater than 62 the binding energy per nucleon deceases as A increases?

Section 43.3 Nuclear Stability and Radioactivity

43.12 • (a) Is the decay $\text{n} \rightarrow \text{p} + \beta^- + \bar{\nu}_e$ energetically possible? If not, explain why not. If so, calculate the total energy released. (b) Is the decay $\text{p} \rightarrow \text{n} + \beta^+ + \nu_e$ energetically possible? If not, explain why not. If so, calculate the total energy released.

43.13 • What nuclide is produced in the following radioactive decays? (a) α decay of ${}^{239}_{94}\text{Pu}$; (b) β^- decay of ${}^{24}_{11}\text{Na}$; (c) β^+ decay of ${}^{15}_{8}\text{O}$.

43.14 •• **CP** ${}^{238}\text{U}$ decays spontaneously by α emission to ${}^{234}\text{Th}$. Calculate (a) the total energy released by this process and (b) the recoil velocity of the ${}^{234}\text{Th}$ nucleus. The atomic masses are 238.050788 u for ${}^{238}\text{U}$ and 234.043601 u for ${}^{234}\text{Th}$.

43.15 •• The atomic mass of ${}^{14}\text{C}$ is 14.003242 u. Show that the β^- decay of ${}^{14}\text{C}$ is energetically possible, and calculate the energy released in the decay.

43.16 • What particle (α particle, electron, or positron) is emitted in the following radioactive decays? (a) ${}^{27}_{14}\text{Si} \rightarrow {}^{27}_{13}\text{Al}$; (b) ${}^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th}$; (c) ${}^{74}_{33}\text{As} \rightarrow {}^{74}_{34}\text{Se}$.

43.17 •• (a) Calculate the energy released by the electron-capture decay of ${}^{57}_{27}\text{Co}$ (see Example 43.7). (b) A negligible amount of this energy goes to the resulting ${}^{57}_{26}\text{Fe}$ atom as kinetic energy. About 90% of the time, the ${}^{57}_{26}\text{Fe}$ nucleus emits two successive gamma-ray photons after the electron-capture process, of energies 0.122 MeV and 0.014 MeV, respectively, in decaying to its ground state. What is the energy of the neutrino emitted in this case?

43.18 • Tritium (${}^{3}_{1}\text{H}$) is an unstable isotope of hydrogen; its mass, including one electron, is 3.016049 u. (a) Show that tritium must be unstable with respect to beta decay because the decay products (${}^{3}_{2}\text{He}$ plus an emitted electron) have less total mass than the tritium. (b) Determine the total kinetic energy (in MeV) of the decay products, taking care to account for the electron masses correctly.

Section 43.4 Activities and Half-Lives

43.19 • If a 6.13-g sample of an isotope having a mass number of 124 decays at a rate of 0.350 Ci, what is its half-life?

43.20 • BIO Radioactive isotopes used in cancer therapy have a "shelf-life," like pharmaceuticals used in chemotherapy. Just after it has been manufactured in a nuclear reactor, the activity of a sample of ^{60}Co is 5000 Ci. When its activity falls below 3500 Ci, it is considered too weak a source to use in treatment. You work in the radiology department of a large hospital. One of these ^{60}Co sources in your inventory was manufactured on October 6, 2004. It is now April 6, 2007. Is the source still usable? The half-life of ^{60}Co is 5.271 years.

43.21 •• The common isotope of uranium, ^{238}U, has a half-life of 4.47×10^9 years, decaying to ^{234}Th by alpha emission. (a) What is the decay constant? (b) What mass of uranium is required for an activity of 1.00 curie? (c) How many alpha particles are emitted per second by 10.0 g of uranium?

43.22 •• BIO Radiation Treatment of Prostate Cancer. In many cases, prostate cancer is treated by implanting 60 to 100 small seeds of radioactive material into the tumor. The energy released from the decays kills the tumor. One isotope that is used (there are others) is palladium (^{103}Pd), with a half-life of 17 days. If a typical grain contains 0.250 g of ^{103}Pd, (a) what is its initial activity rate in Bq, and (b) what is the rate 68 days later?

43.23 •• A 12.0-g sample of carbon from living matter decays at the rate of 180.0 decays/min due to the radioactive ^{14}C in it. What will be the decay rate of this sample in (a) 1000 years and (b) 50,000 years?

43.24 •• BIO Radioactive Tracers. Radioactive isotopes are often introduced into the body through the bloodstream. Their spread through the body can then be monitored by detecting the appearance of radiation in different organs. ^{131}I, a β^- emitter with a half-life of 8.0 d, is one such tracer. Suppose a scientist introduces a sample with an activity of 375 Bq and watches it spread to the organs. (a) Assuming that the sample all went to the thyroid gland, what will be the decay rate in that gland 24 d (about $3\frac{1}{2}$ weeks) later? (b) If the decay rate in the thyroid 24 d later is actually measured to be 17.0 Bq, what percentage of the tracer went to that gland? (c) What isotope remains after the I-131 decays?

43.25 •• The unstable isotope ^{40}K is used for dating rock samples. Its half-life is 1.28×10^9 y. (a) How many decays occur per second in a sample containing 1.63×10^{-6} g of ^{40}K? (b) What is the activity of the sample in curies?

43.26 • As a health physicist, you are being consulted about a spill in a radiochemistry lab. The isotope spilled was 500 μCi of ^{131}Ba, which has a half-life of 12 days. (a) What mass of ^{131}Ba was spilled? (b) Your recommendation is to clear the lab until the radiation level has fallen 1.00 μCi. How long will the lab have to be closed?

43.27 • Measurements on a certain isotope tell you that the decay rate decreases from 8318 decays/min to 3091 decays/min in 4.00 days. What is the half-life of this isotope?

43.28 • The isotope ^{226}Ra undergoes α decay with a half-life of 1620 years. What is the activity of 1.00 g of ^{226}Ra? Express your answer in Bq and in Ci.

43.29 • The radioactive nuclide ^{199}Pt has a half-life of 30.8 minutes. A sample is prepared that has an initial activity of 7.56×10^{11} Bq. (a) How many ^{199}Pt nuclei are initially present in the sample? (b) How many are present after 30.8 minutes? What is the activity at this time? (c) Repeat part (b) for a time 92.4 minutes after the sample is first prepared.

43.30 •• Radiocarbon Dating. A sample from timbers at an archeological site containing 500 g of carbon provides 3070 decays/min. What is the age of the sample?

Section 43.5 Biological Effects of Radiation

43.31 •• BIO (a) If a chest x ray delivers 0.25 mSv to 5.0 kg of tissue, how many *total* joules of energy does this tissue receive? (b) Natural radiation and cosmic rays deliver about 0.10 mSv per year at sea level. Assuming an RBE of 1, how many rem and rads is this dose, and how many joules of energy does a 75-kg person receive in a year? (c) How many chest x rays like the one in part (a) would it take to deliver the same *total* amount of energy to a 75-kg person as she receives from natural radiation in a year at sea level, as described in part (b)?

43.32 •• BIO A person exposed to fast neutrons receives a radiation dose of 200 rem on part of his hand, affecting 25 g of tissue. The RBE of these neutrons is 10. (a) How many rad did he receive? (b) How many joules of energy did this person receive? (c) Suppose the person received the same rad dosage, but from beta rays with an RBE of 1.0 instead of neutrons. How many rem would he have received?

43.33 •• BIO A nuclear chemist receives an accidental radiation dose of 5.0 Gy from slow neutrons (RBE = 4.0). What does she receive in rad, rem, and J/kg?

43.34 • BIO To Scan or Not to Scan? It has become popular for some people to have yearly whole-body scans (CT scans, formerly called CAT scans) using x rays, just to see if they detect anything suspicious. A number of medical people have recently questioned the advisability of such scans, due in part to the radiation they impart. Typically, one such scan gives a dose of 12 mSv, applied to the *whole body*. By contrast, a chest x ray typically administers 0.20 mSv to only 5.0 kg of tissue. How many chest x rays would deliver the same *total* amount of energy to the body of a 75-kg person as one whole-body scan?

43.35 • BIO Food Irradiation. Food is often irradiated with either x rays or electron beams to help prevent spoilage. A low dose of 5–75 kilorads (krad) helps to reduce and kill inactive parasites, a medium dose of 100–400 krad kills microorganisms and pathogens such as salmonella, and a high dose of 2300–5700 krad sterilizes food so that it can be stored without refrigeration. (a) A dose of 175 krad kills spoilage microorganisms in fish. If x rays are used, what would be the dose in Gy, Sv, and rem, and how much energy would a 220-g portion of fish absorb? (See Table 43.3.) (b) Repeat part (a) if electrons of RBE 1.50 are used instead of x rays.

43.36 • BIO In an industrial accident a 65-kg person receives a lethal whole-body equivalent dose of 5.4 Sv from x rays. (a) What is the equivalent dose in rem? (b) What is the absorbed dose in rad? (c) What is the total energy absorbed by the person's body? How does this amount of energy compare to the amount of energy required to raise the temperature of 65 kg of water 0.010 C°?

43.37 •• BIO A 67-kg person accidentally ingests 0.35 Ci of tritium. (a) Assume that the tritium spreads uniformly throughout the body and that each decay leads on the average to the absorption of 5.0 keV of energy from the electrons emitted in the decay. The half-life of tritium is 12.3 y, and the RBE of the electrons is 1.0. Calculate the absorbed dose in rad and the equivalent dose in rem during one week. (b) The β^- decay of tritium releases more than 5.0 keV of energy. Why is the average energy absorbed less than the total energy released in the decay?

43.38 •• CP BIO In a diagnostic x-ray procedure, 5.00×10^{10} photons are absorbed by tissue with a mass of 0.600 kg. The x-ray wavelength is 0.0200 nm. (a) What is the total energy absorbed by the tissue? (b) What is the equivalent dose in rem?

Section 43.6 Nuclear Reactions, Section 43.7 Nuclear Fission, and Section 43.8 Nuclear Fusion

43.39 • Consider the nuclear reaction

$$ {}^{2}_{1}\mathrm{H} + {}^{14}_{7}\mathrm{N} \rightarrow \mathrm{X} + {}^{10}_{5}\mathrm{B} $$

where X is a nuclide. (a) What are Z and A for the nuclide X? (b) Calculate the reaction energy Q (in MeV). (c) If the ${}^{2}_{1}\mathrm{H}$ nucleus is incident on a stationary ${}^{14}_{7}\mathrm{N}$ nucleus, what minimum kinetic energy must it have for the reaction to occur?

43.40 • **Energy from Nuclear Fusion.** Calculate the energy released in the fusion reaction

$$ {}^{3}_{2}\mathrm{He} + {}^{2}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{1}_{1}\mathrm{H} $$

43.41 • Consider the nuclear reaction

$$ {}^{2}_{1}\mathrm{H} + {}^{9}_{4}\mathrm{Be} \rightarrow \mathrm{X} + {}^{4}_{2}\mathrm{He} $$

where X is a nuclide. (a) What are the values of Z and A for the nuclide X? (b) How much energy is liberated? (c) Estimate the threshold energy for this reaction.

43.42 • The United States uses 1.0×10^{20} J of electrical energy per year. If all this energy came from the fission of ${}^{235}\mathrm{U}$, which releases 200 MeV per fission event, (a) how many kilograms of ${}^{235}\mathrm{U}$ would be used per year and (b) how many kilograms of uranium would have to be mined per year to provide that much ${}^{235}\mathrm{U}$? (Recall that only 0.70% of naturally occurring uranium is ${}^{235}\mathrm{U}$.)

43.43 •• At the beginning of Section 43.7 the equation of a fission process is given in which ${}^{235}\mathrm{U}$ is struck by a neutron and undergoes fission to produce ${}^{144}\mathrm{Ba}$, ${}^{89}\mathrm{Kr}$, and three neutrons. The measured masses of these isotopes are 235.043930 u (${}^{235}\mathrm{U}$), 143.922953 u (${}^{144}\mathrm{Ba}$), 88.917630 u (${}^{89}\mathrm{Kr}$), and 1.0086649 u (neutron). (a) Calculate the energy (in MeV) released by each fission reaction. (b) Calculate the energy released per gram of ${}^{235}\mathrm{U}$, in MeV/g.

43.44 •• Consider the nuclear reaction

$$ {}^{28}_{14}\mathrm{Si} + \gamma \rightarrow {}^{24}_{12}\mathrm{Mg} + \mathrm{X} $$

where X is a nuclide. (a) What are Z and A for the nuclide X? (b) Ignoring the effects of recoil, what minimum energy must the photon have for this reaction to occur? The mass of a ${}^{28}_{14}\mathrm{Si}$ atom is 27.976927 u, and the mass of a ${}^{24}_{12}\mathrm{Mg}$ atom is 23.985042 u.

43.45 • The second reaction in the proton-proton chain (see Fig. 43.16) produces a ${}^{3}_{2}\mathrm{He}$ nucleus. A ${}^{3}_{2}\mathrm{He}$ nucleus produced in this way can combine with a ${}^{4}_{2}\mathrm{He}$ nucleus:

$$ {}^{3}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{7}_{4}\mathrm{Be} + \gamma $$

Calculate the energy liberated in this process. (This is shared between the energy of the photon and the recoil kinetic energy of the beryllium nucleus.) The mass of a ${}^{7}_{4}\mathrm{Be}$ atom is 7.016929 u.

43.46 • Consider the nuclear reaction

$$ {}^{4}_{2}\mathrm{He} + {}^{7}_{3}\mathrm{Li} \rightarrow \mathrm{X} + {}^{1}_{0}\mathrm{n} $$

where X is a nuclide. (a) What are Z and A for the nuclide X? (b) Is energy absorbed or liberated? How much?

43.47 •• **CP** In a 100.0-cm^3 sample of water, 0.015% of the molecules are D_2O. Compute the energy in joules that is liberated if all the deuterium nuclei in the sample undergo the fusion reaction of Example 43.13.

PROBLEMS

43.48 • **Comparison of Energy Released per Gram of Fuel.** (a) When gasoline is burned, it releases 1.3×10^{8} J of energy per gallon (3.788 L). Given that the density of gasoline is 737 kg/m^3, express the quantity of energy released in J/g of fuel. (b) During fission, when a neutron is absorbed by a ${}^{235}\mathrm{U}$ nucleus, about 200 MeV of energy is released for each nucleus that undergoes fission. Express this quantity in J/g of fuel. (c) In the proton-proton chain that takes place in stars like our sun, the overall fusion reaction can be summarized as six protons fusing to form one ${}^{4}\mathrm{He}$ nucleus with two leftover protons and the liberation of 26.7 MeV of energy. The fuel is the six protons. Express the energy produced here in units of J/g of fuel. Notice the huge difference between the two forms of nuclear energy, on the one hand, and the chemical energy from gasoline, on the other. (d) Our sun produces energy at a measured rate of 3.86×10^{26} W. If its mass of 1.99×10^{30} kg were all gasoline, how long could it last before consuming all its fuel? (*Historical note:* Before the discovery of nuclear fusion and the vast amounts of energy it releases, scientists were confused. They knew that the earth was at least many millions of years old, but could not explain how the sun could survive that long if its energy came from chemical burning.)

43.49 •• Use conservation of mass-energy to show that the energy released in alpha decay is positive whenever the mass of the original neutral atom is greater than the sum of the masses of the final neutral atom and the neutral ${}^{4}\mathrm{He}$ atom. (*Hint:* Let the parent nucleus have atomic number Z and nucleon number A. First write the reaction in terms of the nuclei and particles involved, and then add Z electron masses to both sides of the reaction and allot them as needed to arrive at neutral atoms.)

43.50 •• Use conservation of mass-energy to show that the energy released in β^- decay is positive whenever the neutral atomic mass of the original atom is greater than that of the final atom. (See the hint in Problem 43.49.)

43.51 •• Use conservation of mass-energy to show that the energy released in β^+ decay is positive whenever the neutral atomic mass of the original atom is at least two electron masses greater than that of the final atom. (See the hint in Problem 43.49.)

43.52 •• (a) Calculate the minimum energy required to remove one proton from the nucleus ${}^{12}_{6}\mathrm{C}$. This is called the proton-removal energy. (*Hint:* Find the difference between the mass of a ${}^{12}_{6}\mathrm{C}$ nucleus and the mass of a proton plus the mass of the nucleus formed when a proton is removed from ${}^{12}_{6}\mathrm{C}$.) (b) How does the proton-removal energy for ${}^{12}_{6}\mathrm{C}$ compare to the binding energy per nucleon for ${}^{12}_{6}\mathrm{C}$, calculated using Eq. (43.10)?

43.53 •• (a) Calculate the minimum energy required to remove one neutron from the nucleus ${}^{17}_{8}\mathrm{O}$. This is called the neutron-removal energy. (See Problem 43.52.) (b) How does the neutron-removal energy for ${}^{17}_{8}\mathrm{O}$ compare to the binding energy per nucleon for ${}^{17}_{8}\mathrm{O}$, calculated using Eq. (43.10)?

43.54 •• The neutral atomic mass of ${}^{14}_{6}\mathrm{C}$ is 14.003242 u. Calculate the proton removal energy and the neutron removal energy for ${}^{15}_{7}\mathrm{N}$. (See Problems 43.52 and 43.53.) What is the percentage difference between these two energies, and which is larger?

43.55 • **BIO** **Radioactive Fallout.** One of the problems of in-air testing of nuclear weapons (or, even worse, the *use* of such weapons!) is the danger of radioactive fallout. One of the most problematic nuclides in such fallout is strontium-90 (${}^{90}\mathrm{Sr}$), which breaks down by β^- decay with a half-life of 28 years. It is chemically similar to calcium and therefore can be incorporated into bones and teeth, where, due to its rather long half-life, it remains for years as an internal source of radiation. (a) What is the daughter nucleus of the ${}^{90}\mathrm{Sr}$ decay? (b) What percentage of the original level of ${}^{90}\mathrm{Sr}$ is left after 56 years? (c) How long would you have to wait for the original level to be reduced to 6.25% of its original value?

43.56 •• **CP** Thorium ${}^{230}_{90}\text{Th}$ decays to radium ${}^{226}_{88}\text{Ra}$ by α emission. The masses of the neutral atoms are 230.033127 u for ${}^{230}_{90}\text{Th}$ and 226.025403 u for ${}^{226}_{88}\text{Ra}$. If the parent thorium nucleus is at rest, what is the kinetic energy of the emitted α particle? (Be sure to account for the recoil of the daughter nucleus.)

43.57 •• The atomic mass of ${}^{25}_{12}\text{Mg}$ is 24.985837 u, and the atomic mass of ${}^{25}_{13}\text{Al}$ is 24.990429 u. (a) Which of these nuclei will decay into the other? (b) What type of decay will occur? Explain how you determined this. (c) How much energy (in MeV) is released in the decay?

43.58 •• The polonium isotope ${}^{210}_{84}\text{Po}$ has atomic mass 209.982857 u. Other atomic masses are ${}^{206}_{82}\text{Pb}$, 205.974449 u; ${}^{209}_{83}\text{Bi}$, 208.980383 u; ${}^{210}_{83}\text{Bi}$, 209.984105 u; ${}^{209}_{84}\text{Po}$, 208.982416 u; and ${}^{210}_{85}\text{At}$, 209.987131 u. (a) Show that the alpha decay of ${}^{210}_{84}\text{Po}$ is energetically possible, and find the energy of the emitted α particle. (b) Is ${}^{210}_{84}\text{Po}$ energetically stable with respect to emission of a proton? Why or why not? (c) Is ${}^{210}_{84}\text{Po}$ energetically stable with respect to emission of a neutron? Why or why not? (d) Is ${}^{210}_{84}\text{Po}$ energetically stable with respect to β^- decay? Why or why not? (e) Is ${}^{210}_{84}\text{Po}$ energetically stable with respect to β^+ decay? Why or why not?

43.59 •• **BIO** **Irradiating Ourselves!** The radiocarbon in our bodies is one of the naturally occurring sources of radiation. Let's see how large a dose we receive. ${}^{14}\text{C}$ decays via β^- emission, and 18% of our body's mass is carbon. (a) Write out the decay scheme of carbon-14 and show the end product. (A neutrino is also produced.) (b) Neglecting the effects of the neutrino, how much kinetic energy (in MeV) is released per decay? The atomic mass of ${}^{14}\text{C}$ is 14.003242 u. (c) How many grams of carbon are there in a 75-kg person? How many decays per second does this carbon produce? (*Hint:* Use data from Example 43.9.) (d) Assuming that all the energy released in these decays is absorbed by the body, how many MeV/s and J/s does the ${}^{14}\text{C}$ release in this person's body? (e) Consult Table 43.3 and use the largest appropriate RBE for the particles involved. What radiation dose does the person give himself in a year, in Gy, rad, Sv, and rem?

43.60 •• **BIO** **Pion Radiation Therapy.** A neutral pion (π^0) has a mass of 264 times the electron mass and decays with a lifetime of 8.4×10^{-17} s to two photons. Such pions are used in the radiation treatment of some cancers. (a) Find the energy and wavelength of these photons. In which part of the electromagnetic spectrum do they lie? What is the RBE for these photons? (b) If you want to deliver a dose of 200 rem (which is typical) in a single treatment to 25 g of tumor tissue, how many π^0 mesons are needed?

43.61 • Gold, ${}^{198}_{79}\text{Au}$, undergoes β^- decay to an excited state of ${}^{198}_{80}\text{Hg}$. If the excited state decays by emission of a γ photon with energy 0.412 MeV, what is the maximum kinetic energy of the electron emitted in the decay? This maximum occurs when the antineutrino has negligible energy. (The recoil energy of the ${}^{198}_{80}\text{Hg}$ nucleus can be ignored. The masses of the neutral atoms in their ground states are 197.968225 u for ${}^{198}_{79}\text{Au}$ and 197.966752 u for ${}^{198}_{80}\text{Hg}$.)

43.62 •• Calculate the mass defect for the β^+ decay of ${}^{11}_{6}\text{C}$. Is this decay energetically possible? Why or why not? The atomic mass of ${}^{11}_{6}\text{C}$ is 11.011434 u.

43.63 •• Calculate the mass defect for the β^+ decay of ${}^{13}_{7}\text{N}$. Is this decay energetically possible? Why or why not? The atomic mass of ${}^{13}_{7}\text{N}$ is 13.005739 u.

43.64 •• The results of activity measurements on a radioactive sample are given in the table. (a) Find the half-life. (b) How many radioactive nuclei were present in the sample at $t = 0$? (c) How many were present after 7.0 h?

Time (h)	Decays/s
0	20,000
0.5	14,800
1.0	11,000
1.5	8,130
2.0	6,020
2.5	4,460
3.0	3,300
4.0	1,810
5.0	1,000
6.0	550
7.0	300

43.65 •• **BIO** A person ingests an amount of a radioactive source with a very long lifetime and activity 0.63 μCi. The radioactive material lodges in the lungs, where all of the 4.0-MeV α particles emitted are absorbed within a 0.50-kg mass of tissue. Calculate the absorbed dose and the equivalent dose for one year.

43.66 •• **Measuring Very Long Half-Lives.** Some radioisotopes such as samarium (${}^{149}\text{Sm}$) and gadolinium (${}^{152}\text{Gd}$) have half-lives that are much longer than the age of the universe, so we can't measure their half-lives by watching their decay rate decrease. Luckily, there is another way of calculating the half-life, using Eq. (43.16). Suppose a 12.0-g sample of ${}^{149}\text{Sm}$ is observed to decay at a rate of 2.65 Bq. Calculate the half-life of the sample in years. (*Hint:* How many nuclei are there in the 12.0-g sample?)

43.67 • **We Are Stardust.** In 1952 spectral lines of the element technetium-99 (${}^{99}\text{Tc}$) were discovered in a red giant star. Red giants are very old stars, often around 10 billion years old, and near the end of their lives. Technetium has *no* stable isotopes, and the half-life of ${}^{99}\text{Tc}$ is 200,000 years. (a) For how many half-lives has the ${}^{99}\text{Tc}$ been in the red-giant star if its age is 10 billion years? (b) What fraction of the original ${}^{99}\text{Tc}$ would be left at the end of that time? This discovery was extremely important because it provided convincing evidence for the theory (now essentially known to be true) that most of the atoms heavier than hydrogen and helium were made inside of stars by thermonuclear fusion and other nuclear processes. If the ${}^{99}\text{Tc}$ had been part of the star since it was born, the amount remaining after 10 billion years would have been so minute that it would not have been detectable. This knowledge is what led the late astronomer Carl Sagan to proclaim that "we are stardust."

43.68 • **BIO** A 70.0-kg person experiences a whole-body exposure to α radiation with energy 4.77 MeV. A total of $6.25 \times 10^{12} \alpha$ particles are absorbed. (a) What is the absorbed dose in rad? (b) What is the equivalent dose in rem? (c) If the source is 0.0320 g of ${}^{226}\text{Ra}$ (half-life 1600 y) somewhere in the body, what is the activity of this source? (d) If all the alpha particles produced are absorbed, what time is required for this dose to be delivered?

43.69 •• Measurements indicate that 27.83% of all rubidium atoms currently on the earth are the radioactive ${}^{87}\text{Rb}$ isotope. The rest are the stable ${}^{85}\text{Rb}$ isotope. The half-life of ${}^{87}\text{Rb}$ is 4.75×10^{10} y. Assuming that no rubidium atoms have been formed since, what percentage of rubidium atoms were ${}^{87}\text{Rb}$ when our solar system was formed 4.6×10^9 y ago?

43.70 •• A ${}^{186}_{76}\text{Os}$ nucleus at rest decays by the emission of a 2.76-MeV α particle. Calculate the atomic mass of the daughter nuclide produced by this decay, assuming that it is produced in its ground state. The atomic mass of $^{186}_{76}Os$ is 185.953838 u.

43.71 •• **BIO** A ^{60}Co source with activity 2.6×10^{-4} Ci is embedded in a tumor that has mass 0.200 kg. The source emits γ photons with average energy 1.25 MeV. Half the photons are absorbed in the tumor, and half escape. (a) What energy is delivered to the tumor per second? (b) What absorbed dose (in rad) is delivered per second? (c) What equivalent dose (in rem) is delivered per second if the RBE for these γ rays is 0.70? (d) What exposure time is required for an equivalent dose of 200 rem?

43.72 • The nucleus $^{15}_{8}O$ has a half-life of 122.2 s; $^{19}_{8}O$ has a half-life of 26.9 s. If at some time a sample contains equal amounts of $^{15}_{8}O$ and $^{19}_{8}O$, what is the ratio of $^{15}_{8}O$ to $^{19}_{8}O$ (a) after 4.0 minutes and (b) after 15.0 minutes?

43.73 • A bone fragment found in a cave believed to have been inhabited by early humans contains 0.29 times as much ^{14}C as an equal amount of carbon in the atmosphere when the organism containing the bone died. (See Example 43.9 in Section 43.4.) Find the approximate age of the fragment.

43.74 •• **An Oceanographic Tracer.** Nuclear weapons tests in the 1950s and 1960s released significant amounts of radioactive tritium ($^{3}_{1}H$, half-life 12.3 years) into the atmosphere. The tritium atoms were quickly bound into water molecules and rained out of the air, most of them ending up in the ocean. For any of this tritium-tagged water that sinks below the surface, the amount of time during which it has been isolated from the surface can be calculated by measuring the ratio of the decay product, $^{3}_{2}He$, to the remaining tritium in the water. For example, if the ratio of $^{3}_{2}He$ to $^{3}_{1}H$ in a sample of water is 1:1, the water has been below the surface for one half-life, or approximately 12 years. This method has provided oceanographers with a convenient way to trace the movements of subsurface currents in parts of the ocean. Suppose that in a particular sample of water, the ratio of $^{3}_{2}He$ to $^{3}_{1}H$ is 4.3 to 1.0. How many years ago did this water sink below the surface?

43.75 •• Consider the fusion reaction $^{2}_{1}H + ^{2}_{1}H \rightarrow ^{3}_{2}He + ^{1}_{0}n$. (a) Estimate the barrier energy by calculating the repulsive electrostatic potential energy of the two $^{2}_{1}H$ nuclei when they touch. (b) Compute the energy liberated in this reaction in MeV and in joules. (c) Compute the energy liberated *per mole* of deuterium, remembering that the gas is diatomic, and compare with the heat of combustion of hydrogen, about 2.9×10^{5} J/mol.

43.76 •• **BIO** In the 1986 disaster at the Chernobyl reactor in the Soviet Union (now Ukraine), about $\frac{1}{8}$ of the ^{137}Cs present in the reactor was released. The isotope ^{137}Cs has a half-life for β decay of 30.07 y and decays with the emission of a total of 1.17 MeV of energy per decay. Of this, 0.51 MeV goes to the emitted electron and the remaining 0.66 MeV to a γ ray. The radioactive ^{137}Cs is absorbed by plants, which are eaten by livestock and humans. How many ^{137}Cs atoms would need to be present in each kilogram of body tissue if an equivalent dose for one week is 3.5 Sv? Assume that all of the energy from the decay is deposited in that 1.0 kg of tissue and that the RBE of the electrons is 1.5.

43.77 •• **CP** (a) Prove that when a particle with mass m and kinetic energy K collides with a stationary particle with mass M, the total kinetic energy K_{cm} in the center-of-mass coordinate system (the energy available to cause reactions) is

$$K_{cm} = \frac{M}{M+m}K$$

Assume that the kinetic energies of the particles and nuclei are much lower than their rest energies. (b) If K_{th} is the minimum, or threshold, kinetic energy to cause an endoergic reaction to occur in the situation of part (a), show that

$$K_{th} = -\frac{M+m}{M}Q$$

43.78 • Calculate the energy released in the fission reaction $^{235}_{92}U + ^{1}_{0}n \rightarrow ^{140}_{54}Xe + ^{94}_{38}Sr + 2^{1}_{0}n$. You can ignore the initial kinetic energy of the absorbed neutron. The atomic masses are $^{235}_{92}U$, 235.043923 u; $^{140}_{54}Xe$, 139.921636 u; and $^{94}_{38}Sr$, 93.915360 u.

CHALLENGE PROBLEMS

43.79 ••• The results of activity measurements on a mixed sample of radioactive elements are given in the table. (a) How many different nuclides are present in the mixture? (b) What are their half-lives? (c) How many nuclei of each type are initially present in the sample? (d) How many of each type are present at $t = 5.0$ h?

Time (h)	Decays/s
0	7500
0.5	4120
1.0	2570
1.5	1790
2.0	1350
2.5	1070
3.0	872
4.0	596
5.0	414
6.0	288
7.0	201
8.0	140
9.0	98
10.0	68
12.0	33

43.80 ••• **Industrial Radioactivity.** Radioisotopes are used in a variety of manufacturing and testing techniques. Wear measurements can be made using the following method. An automobile engine is produced using piston rings with a total mass of 100 g, which includes 9.4 μCi of ^{59}Fe whose half-life is 45 days. The engine is test-run for 1000 hours, after which the oil is drained and its activity is measured. If the activity of the engine oil is 84 decays/s, how much mass was worn from the piston rings per hour of operation?

Answers

Chapter Opening Question ?

When an organism dies, it stops taking in carbon from atmospheric CO_2. Some of this carbon is radioactive ^{14}C, which decays with a half-life of 5730 years. By measuring the proportion of ^{14}C that remains in the specimen, scientists can determine how long ago the organism died. (See Section 43.4.)

Test Your Understanding Questions

43.1 Answers: (a) (iii), (b) (v) The radius R is proportional to the cube root of the mass number A, while the volume is proportional to R^3 and hence to $(A^{1/3})^3 = A$. Therefore, doubling the volume requires increasing the mass number by a factor of 2; doubling the radius implies increasing both the volume and the mass number by a factor of $2^3 = 8$.