

Closed book. No calculators are to be used for this quiz.

Quiz duration: 10 minutes

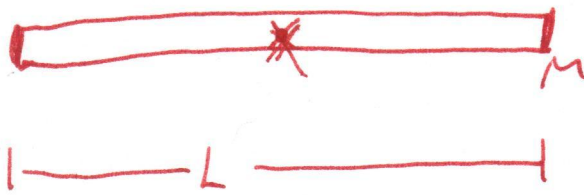
Name:

Student ID:

Signature:

A rod of mass $M = 1$ kg and length 1 m connects 2 small 1 kg masses. Find the moment of inertia of the composite system about an axis through the center, perpendicular to the rod.

o For an uniform rod free to rotate by an axis passing through center of mass:



$$I_{rod} = \frac{1}{12} ML^2$$

$$= \frac{M}{L} \int_{-L/2}^{+L/2} x^2 dx$$

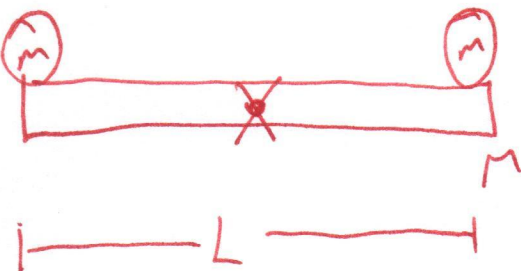
o Two equidistant small masses, rotating through same center:



$$I_{masses} = m \left(\frac{L}{2}\right)^2 + m \left(\frac{L}{2}\right)^2$$

$$= \frac{2mL^2}{4} = \frac{mL^2}{2}$$

o Combine/superpose these:



$$I_{sys} = \frac{ML^2}{12} + \frac{mL^2}{2}$$

$$= (M + 6m) \frac{L^2}{12}$$

$$I_{sys} = \frac{(1 \text{ kg} + 6 \times 1 \text{ kg}) (1 \text{ meter})^2}{12} = \frac{7}{12} [\text{kg} \cdot \text{m}^2]$$

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Name:

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How much energy is dissipated when a $M = 2$ kg wheel of radius $R = 2$ m is brought to rest from an initial velocity of 30 revolutions per minute (rpm).

(Moment of inertia of the disk is $I = MR^2/2$. Take $\pi = 3$).



$$E_{\text{initial}} = K_{\text{initial}}$$

$$K_{\text{initial}} = \frac{1}{2} I \omega_{\text{in}}^2$$

$$E_{\text{final}} = 0$$

$$E_{\text{dissipated}} = E_{\text{initial}} - E_{\text{final}}$$

$$= \frac{1}{2} I \omega_{\text{in}}^2$$

$$= \frac{1}{2} (4 \text{ kg} \cdot \text{m}^2) (3 \text{ rad/s})^2$$

$$E_{\text{dis}} = 18 \text{ kg} \cdot \text{m}^2/\text{s}^2 \equiv 18 \text{ Joule}$$

$$\omega_{\text{initial}} = 30 \text{ rev/min}$$

$$1 \text{ rev} \equiv 2\pi \text{ rad}$$

$$1 \text{ min} \equiv 60 \text{ s}$$

$$\omega_{\text{initial}} = 30 \frac{2\pi \text{ rad}}{60 \text{ s}}$$

$$= \frac{60\pi \text{ rad}}{60 \text{ s}} \approx 3 \text{ rad/s}$$

$$I = \frac{(2 \text{ kg})(2 \text{ meter})^2}{2}$$

$$= 4 [\text{kg} \cdot \text{m}^2]$$

Section 3

Quiz 10

06 December 2012

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Quiz duration: 10 minutes

$1 \text{ rev} \equiv 2\pi \text{ rad}$

$1 \text{ min} \equiv 60 \text{ s}$

Name: $R = 0.1 \text{ m}$

Student ID:

Signature:

A disk of radius 10 cm starts to rotate, about a fixed perpendicular axis through its center, from rest and accelerates for 2 s (no acceleration after 2 s) with constant angular acceleration to an angular speed of 30 revolutions per minute (rpm). (i) Find the (radial and tangential) components of the linear acceleration of a point on the edge 1 s after the rotation is turned on, and (ii) 3 s after the rotation is turned on. (Take $\pi = 3$).

for $\alpha(t) = \alpha \rightarrow \omega(t) = \omega_0 + \alpha t \rightarrow \theta(t) = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$

and we can choose our referencing angle to be zero, $\theta_0 = 0$

Initially; $\omega_0 = 0$ at rest $\rightarrow \omega(t=2\text{s}) = \alpha t' = \omega_{t=2\text{s}} = 30 \text{ rpm}$

$\omega_{t=2\text{s}} = 30 \frac{\text{rev}}{\text{min}} = 30 \frac{2\pi \text{ rad}}{60 \text{ s}} \approx 3 \text{ rad/s} = \alpha \cdot (2 \text{ s})$

$\rightarrow \alpha = \frac{3}{2} \text{ rad/s}^2$: amount of constant angular acceleration : $a = \frac{3}{2} \times 10^{-1} \text{ m/s}^2$ $a = \alpha R$

(i) $\omega(t=1\text{s}) = (\frac{3}{2} \text{ rad/s}^2) \cdot (1\text{s}) = \frac{3}{2} \text{ rad/s}$

$a_{\text{rad}}(t=1\text{s}) = \frac{a_{\text{rad}}(t=1\text{s})}{R} = \frac{v^2(t=1\text{s})}{R^2}$
 $= \omega^2(t=1\text{s}) \cdot R$

$a_{\text{rad}}(t=1\text{s}) = (\frac{3}{2} \text{ rad/s})^2 \cdot (0.1 \text{ m})$
 $= \frac{9}{4} \times 10^{-1} \text{ m/s}^2$

~~$a_{\text{tan}}(t=1\text{s}) = \alpha R$~~
 $= \alpha = \frac{3}{2} \times 10^{-1} \text{ m/s}^2$

(ii) $\omega(t=3\text{s}) = (\frac{3}{2} \text{ rad/s}^2) \cdot (2\text{s}) = 3 \text{ rad/s}$

$a_{\text{rad}}(t=3\text{s}) = \omega^2(t=3\text{s}) \cdot R$
 $= (3 \text{ rad/s})^2 \cdot (0.1 \text{ m}) = 9 \times 10^{-1} \text{ m/s}^2$

$a_{\text{tan}}(t=3\text{s}) = 0 \text{ m/s}^2$

\rightarrow Because, no acceleration after $t=2\text{s}$

Section 4

Quiz 10

06 December 2012

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Quiz duration: 10 minutes

$$1 \text{ rev} = 2\pi \text{ rad}$$

$$1 \text{ min} = 60 \text{ s}$$

Name:

Student ID:

Signature:

A disk starts to rotate, about a fixed perpendicular axis through its center, from rest and accelerates with constant angular acceleration to an angular speed of 30 revolutions per minute (rpm) in 2 s. (i) Find the angular acceleration in rad/s^2 . (ii) Find the angle turned through in degrees. (Take $\pi = 3$).

$$\omega(t=2\text{s}) = 30 \frac{\text{rev}}{\text{min}} = 30 \frac{2\pi \text{ rad}}{60\text{s}} \approx 3 \text{ rad/s} ; \boxed{\omega_0 = 0 \text{ rest}}$$

$$\text{for } \alpha(t) = \alpha \rightarrow \omega(t) = \omega_0 + \alpha t \rightarrow \theta(t) = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

where referencing angle can chosen to be zero, $\boxed{\theta_0 = 0}$.

$$\omega(t=2\text{s}) = 3 \text{ rad/s} = \alpha \cdot (2\text{s}) ; \boxed{\alpha = \frac{3}{2} \frac{\text{rad}}{\text{s}^2}} \text{ (i)}$$

$$\text{(ii) } \theta(t=2\text{s}) = \frac{1}{2} \alpha (t=2\text{s})^2 = \frac{1}{2} \left(\frac{3}{2} \frac{\text{rad}}{\text{s}^2} \right) (2\text{s})^2$$

$$\rightarrow \theta(t=2\text{s}) = 3 \text{ rad} \approx \pi \text{ rad} = 180^\circ \text{ s turned}$$

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Quiz duration: 10 minutes

$$1 \text{ rev} = 2\pi \text{ rad}$$

Name:

Student ID:

Signature: *(Signature)*

A disk of radius 10 cm rotates, about a fixed perpendicular axis through its center, at an angular speed of 30 revolutions per minute (rpm). It decelerates uniformly to a stop in 2 complete revolutions. (i) Find the angular acceleration in rad/s^2 . (ii) Find the time it takes to come to a stop. (Take $\pi = 3$).

for $\alpha(t) = \alpha \rightarrow \omega(t) = \omega_0 + \alpha t \rightarrow \theta(t) = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$

where referencing angle θ_0 can be chosen to be zero $\boxed{\theta_0 = 0}$

With initial speed $\omega_0 = 30 \text{ rev/min} = 30 \frac{2\pi \text{ rad}}{60 \text{ s}} \approx 3 \text{ rad/s}$ & $\alpha(t) = -\alpha$ constant deceleration

$$\omega(t) = (\omega_0 - \alpha t); \theta(t) = \omega_0 t - \frac{1}{2} \alpha t^2$$

(i) $\omega(t = t_{\text{stop}}) = \omega_0 - \alpha t_{\text{stop}} = 0 \rightarrow \alpha = \frac{\omega_0}{t_{\text{stop}}}$

$$\theta(t = t_{\text{stop}}) = \omega_0 t_{\text{stop}} - \frac{1}{2} \alpha t_{\text{stop}}^2 = 2 \text{ rev} = 4\pi \text{ rad} \approx 12 \text{ rad}$$

given $\omega_0 t_{\text{stop}} / 2 = 12 \text{ rad}$

$$\rightarrow \omega_0 t_{\text{stop}} - \frac{1}{2} \left(\frac{\omega_0}{t_{\text{stop}}} \right) t_{\text{stop}}^2 = 12 \text{ rad} \rightarrow$$

$$t_{\text{stop}} = \frac{2 \times 12 \text{ rad}}{(3 \text{ rad/s})} = 8 \text{ s}$$

(i) $\alpha = \frac{[3 \text{ rad/s}]}{[8 \text{ s}]}$

$$\rightarrow \alpha = \frac{3}{8} \text{ rad/s}^2$$