

Closed book. No calculators are to be used for this quiz.

Quiz duration: 10 minutes

Name:

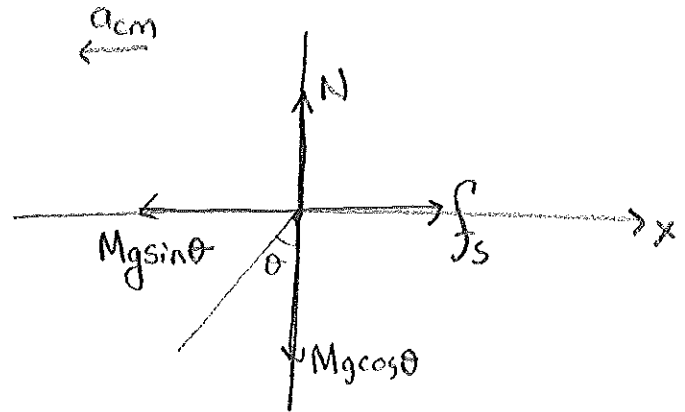
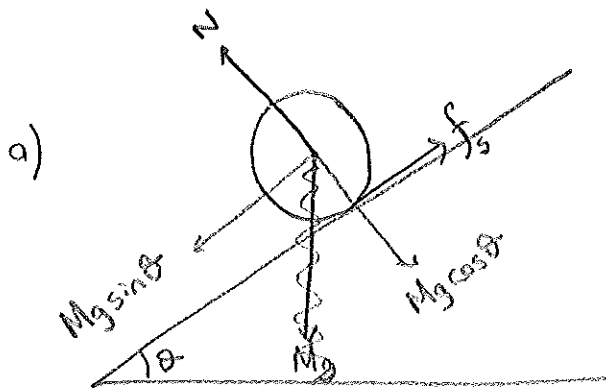
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A bowling ball of mass  $M$  and radius  $R$  rolls without slipping *down* a ramp, which is inclined at an angle  $\theta$  to the horizontal.

- Draw the free body diagram for the bowling ball.
  - What is the acceleration of the center of mass of the bowling ball?
  - What is the magnitude of the friction force on the bowling ball?
- Your results should be in terms of the given quantities ( $M, g, \theta$ )

$$I(\text{sphere}) = \frac{2}{5}MR^2$$



$$b) Mg \sin \theta - f = Ma_{cm}$$

$$\sum \tau = fR = I_{cm} \alpha = \frac{2}{5}MR^2 \frac{a_{cm}}{R}$$

$$(1) fR = \frac{2}{5}MRa_{cm}$$

$$(2) Mg \sin \theta - f = Ma_{cm}$$

$$\text{From eq. 2: } f = M(g \sin \theta - a_{cm})$$

insert in eq. 1

$$MR(g \sin \theta - a_{cm}) = \frac{2}{5}MRa_{cm}$$

$$g \sin \theta = \left(\frac{2}{5} + 1\right)a_{cm} = \frac{7}{5}a_{cm}$$

$$a_{cm} = \frac{5}{7}g \sin \theta$$

$$c) f = M(g \sin \theta - \frac{5}{7}g \sin \theta) = \frac{2}{7}Mg \sin \theta //$$

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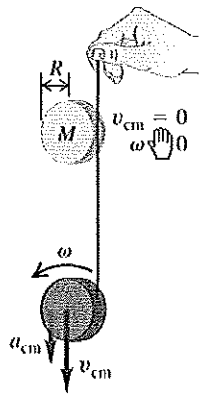
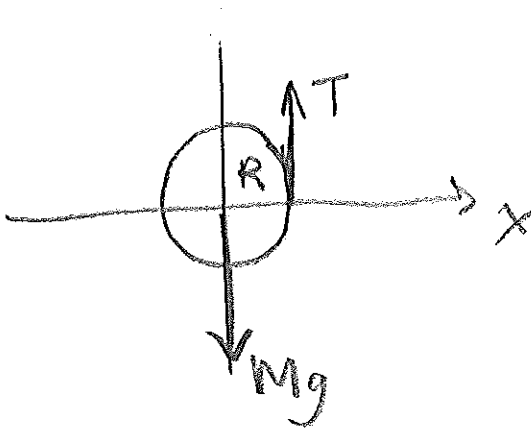
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A primitive yo-yo is made by wrapping a massless string around a solid cylinder of mass  $M$  and radius  $R$ . You hold the free end of the string stationary and release the cylinder (yo-yo) from rest. The string unwinds but does not slip or stretch as the cylinder (yo-yo) descends and rotates.

- Draw the free body diagram for the yo-yo.
- What is the acceleration of the cylinder (yo-yo)?
- What is the tension in the string?

Your results should be in terms of the given quantities ( $M, g$ ).



translational motion:  $\sum F_y = Mg - T = Ma_{cm,y}$   
 rotational motion:  $\sum \tau_z = TR = I_{cm} \alpha_z = \frac{1}{2} MR^2 \alpha_z$   
 $\alpha_z = \frac{a_{cm,y}}{R}$

$$TR = \frac{1}{2} MR^2 \frac{a_{cm,y}}{R}$$

$T = \frac{1}{2} M a_{cm,y}$ $Mg - T = M a_{cm,y}$	$= a_{cm,y} = \frac{2}{3} g //$ $T = \frac{1}{3} Mg //$
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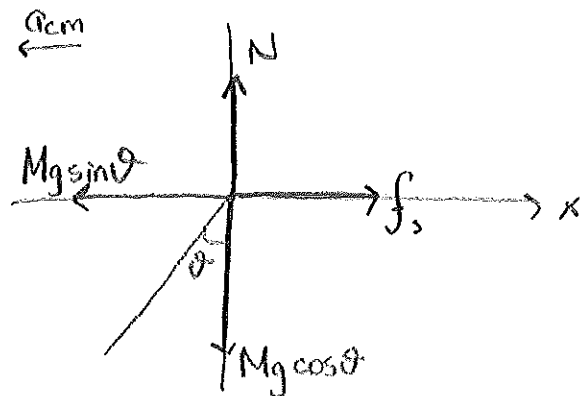
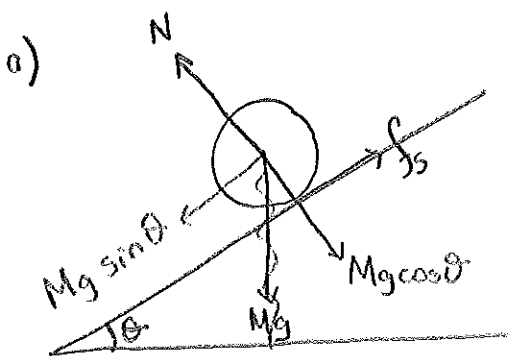
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A hollow, spherical shell with mass 3 kg rolls without slipping down a ramp, which is inclined at an angle  $30^\circ$  to the horizontal.

- Draw the free body diagram for the spherical shell.
- What is the acceleration of the center of mass of the spherical shell?
- What is the magnitude of the friction force on the spherical shell?

$I(\text{hollow, spherical shell}) = \frac{2}{3}MR^2$ ,  $(\sin 30^\circ = \cos 60^\circ = 1/2)$ ,  $g = 10 \text{ m/s}^2$ .



$$b) \quad Mg \sin \theta - f = Ma_{cm}$$

$$\tau = I\alpha$$

$$fR = \left(\frac{2}{3}MR^2\right) \frac{a_{cm}}{R}$$

$$f = \frac{2}{3}Ma_{cm}$$

$$Mg \sin \theta - \frac{2}{3}Ma_{cm} = Ma_{cm}$$

$$a_{cm} = \frac{3g \sin \theta}{5}$$

$$a_{cm} = \frac{3 \cdot 10 \cdot \frac{1}{2}}{5} = \frac{3 \text{ m/s}^2}{1}$$

$$c) \quad f = \frac{2}{3}Ma_{cm} = \frac{2}{3} \cdot 3 \cdot 3 = \frac{6 \text{ N}}{1}$$

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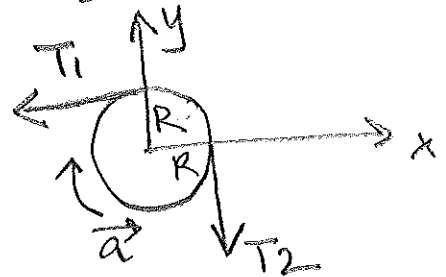
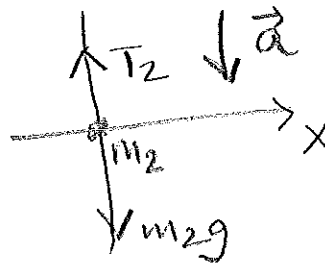
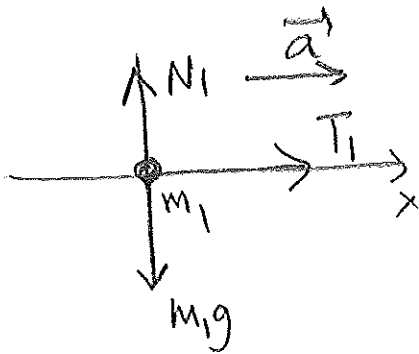
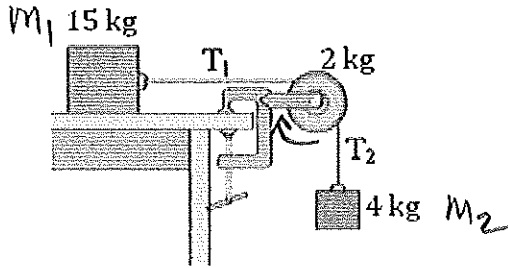
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$m_1=15$  kg box is resting on a horizontal, frictionless surface is attached to a  $m_2=4$  kg weight by a thin, light wire that passes over a frictionless pulley. The pulley has the shape of a uniform solid disk of radius  $R$  and mass  $M=2$ kg. System is released from rest.

- Draw the free body diagram and show the forces acting on box  $m_1$ , weight  $m_2$ , and the pulley with mass  $M$ .
- Find the tension in the wire on both sides of the pulley.
- Find the acceleration of the box.

For the pulley let clockwise rotation be positive.

Take  $I=(1/2)MR^2$



$\sum \vec{F} = m\vec{a}$  to each mass.

$\sum \tau_z = I\alpha_z$  to the pulley.

$a = R\alpha$

$T_1 = m_1 a$

$m_2 g - T_2 = m_2 a$

$T_2 R - T_1 R = \frac{1}{2} M R^2 \alpha$ ,  $(T_2 - T_1) R = \frac{1}{2} M R^2 \frac{a}{R}$ ,  $T_2 - T_1 = \frac{1}{2} M a$

Add 3 equations side by side

$$\left. \begin{aligned} T_1 &= m_1 a \\ m_2 g - T_2 &= m_2 a \\ T_2 - T_1 &= \frac{1}{2} M a \end{aligned} \right\} a = \frac{m_2 g}{m_1 + m_2 + \frac{M}{2}} = 2 m/s^2$$

$T_1 = (15)(2) = 30 \text{ N}, T_2 = 32 \text{ N}$

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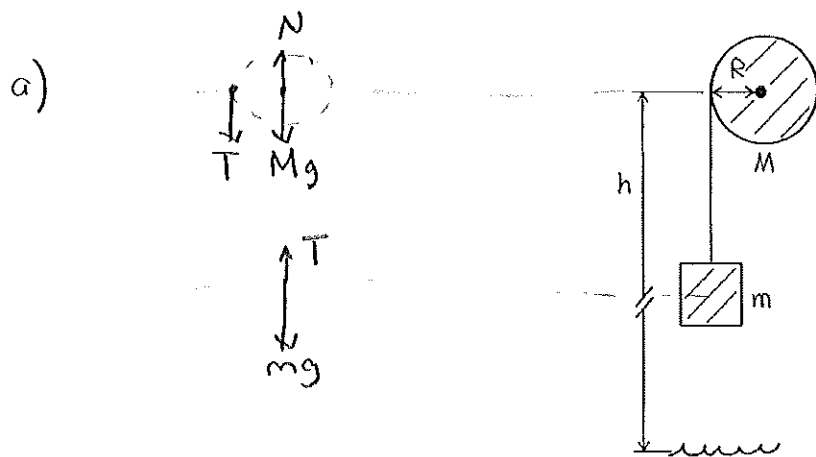
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We wrap a light, non-stretching cable around a solid cylinder with mass  $M$  and radius  $R$ . The cylinder rotates with negligible friction about a stationary horizontal axis. We tie the free end of the cable to a block of mass  $m$  and release the block from rest at a distance  $h$  above the floor. As the block falls, the cable unwinds without stretching or slipping.

- Draw the free body diagram for the block and the cylinder.
- What is the acceleration of the falling block?
- What is the tension in the cable?

Your results should be in terms of the given quantities ( $M, m, g$ )

$$I(\text{cylinder}) = \frac{1}{2}MR^2.$$



$$b) \sum \vec{F} = m\vec{a}$$

$$mg + (-T) = ma_y \quad *$$

$$\sum \vec{\tau} = I\vec{\alpha}_z, \quad a_y = R\alpha_z$$

$$RT = I\alpha_z = \left(\frac{1}{2}MR^2\right)\alpha_z$$

$$T = \frac{1}{2}MR \underbrace{\alpha_z}_{a_y} = \frac{1}{2}Ma_y$$

From (\*)

$$mg - \frac{1}{2}Ma_y = ma_y$$

$$a_y = \frac{g}{1 + M/2m} //$$

$$c) T = mg - ma_y$$

$$= mg - m \left( \frac{g}{1 + M/2m} \right)$$

$$= \frac{mg}{1 + 2m/M} //$$