

Closed book. No calculators are to be used for this quiz.

Quiz duration: 10 minutes

Name:

Student ID:

Signature:

Find the vector \vec{A} that has unit magnitude and that is perpendicular to the two vectors

$$\vec{B} = 2\hat{j} + 3\hat{k} \text{ and } \vec{C} = \hat{i} - \hat{j}.$$

Two methods:

$$a) \vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z = 0$$

$$B_y = 2 \quad B_z = 3 \Rightarrow 2A_y + 3A_z = 0 \Rightarrow A_y = \frac{-3A_z}{2}$$

$$\vec{A} \cdot \vec{C} = A_x C_x + A_y C_y + A_z C_z = 0$$

$$C_x = 1 \quad C_y = -1 \Rightarrow A_x - A_y = 0 \Rightarrow \boxed{A_x = A_y = \frac{-3A_z}{2}}$$

Let \vec{A}' be a vector in the direction of \vec{A} ; but not with unit length. Choose $A_x = 1$

$$\text{Using } \star \text{ box, } \vec{A}' = \hat{i} + \hat{j} - \frac{3}{2} \hat{k}$$

$$\text{Normalize } \vec{A}' \Rightarrow \vec{A} = \frac{\vec{A}'}{|\vec{A}'|} = \frac{\hat{i} + \hat{j} - \frac{3}{2} \hat{k}}{\sqrt{1+1+\frac{9}{4}}}$$

$$\boxed{\vec{A} = \frac{\hat{i} + \hat{j} - \frac{3}{2} \hat{k}}{\sqrt{\frac{22}{9}}}}$$

If we choose $A_x = -1$,

$$\boxed{\vec{A} = \frac{-\hat{i} - \hat{j} + \frac{3}{2} \hat{k}}{\sqrt{\frac{22}{9}}}}$$

Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes

Name:

Student ID:

Signature:

Find the angle between $(\vec{A} - \vec{B})$ and \vec{A} , for $\vec{A} = 4\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{B} = 2\hat{i} + 3\hat{j} + \hat{k}$.

$$\vec{A} - \vec{B} = 2\hat{i} - 2\hat{j} + \hat{k}$$

$$\cos \theta = \frac{(\vec{A} - \vec{B}) \cdot \vec{A}}{|\vec{A} - \vec{B}| |\vec{A}|} = \frac{8 - 2 + 2}{\sqrt{21} \sqrt{9}} = \frac{8}{3\sqrt{21}}$$

$$\theta = \arccos\left(\frac{8}{3\sqrt{21}}\right)$$

Closed book. No calculators are to be used for this quiz.

Quiz duration: 10 minutes

Name:

Student ID:

Signature:

Find a vector that is perpendicular to both $(\vec{A} - \vec{B})$ and $(\vec{A} + \vec{B})$ and has a magnitude equal to the scalar product of \vec{A} and \vec{B} where $\vec{A} = 2\hat{i} - 3\hat{j} + 3\hat{k}$ and $\vec{B} = \hat{i} - \hat{j} + 2\hat{k}$.

$$(\vec{A} - \vec{B}) = \hat{i} - 2\hat{j} + \hat{k}$$

$$(\vec{A} + \vec{B}) = 3\hat{i} - 4\hat{j} + 5\hat{k}$$

$$\vec{X}' = (\vec{A} - \vec{B}) \times (\vec{A} + \vec{B}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ 3 & -4 & 5 \end{vmatrix}$$

$$= \hat{i} \begin{vmatrix} -2 & 1 \\ -4 & 5 \end{vmatrix} - \hat{j} \begin{vmatrix} 1 & 1 \\ 3 & 5 \end{vmatrix} + \hat{k} \begin{vmatrix} 1 & -2 \\ 3 & -4 \end{vmatrix}$$

$$= -6\hat{i} - 2\hat{j} + 2\hat{k}$$

$$\vec{A} \cdot \vec{B} = 2 + 3 + 6 = 11$$

\hat{X} : unit vector in the direction of \vec{X}'

$$\hat{X} = \frac{-6\hat{i} - 2\hat{j} + 2\hat{k}}{\sqrt{36 + 4 + 4}} = \frac{-3\hat{i} - \hat{j} + \hat{k}}{\sqrt{11}}$$

$$\vec{X} = 11 \hat{X} = (-3\hat{i} - \hat{j} + \hat{k}) \sqrt{11}$$

If we used $\vec{X}' = (\vec{A} + \vec{B}) \times (\vec{A} - \vec{B})$

$$\vec{X} = (3\hat{i} + \hat{j} - \hat{k}) \sqrt{11}$$

Closed book. No calculators are to be used for this quiz.

Quiz duration: 10 minutes

Name:

Student ID:

Signature:

Find a vector \vec{A} which has magnitude 3 and is parallel to $\vec{B} = 5\hat{i} + 3\hat{j} - \hat{k}$.

\hat{A} : unit vector in the direction of \vec{A}

$$\hat{A} = \frac{\vec{B}}{|\vec{B}|} = \frac{5\hat{i} + 3\hat{j} - \hat{k}}{\sqrt{25+9+1}} = \frac{5\hat{i} + 3\hat{j} - \hat{k}}{\sqrt{35}}$$

$$\vec{A} = 3\hat{A} = \frac{3}{\sqrt{35}} (5\hat{i} + 3\hat{j} - \hat{k})$$

Closed book. No calculators are to be used for this quiz.

Quiz duration: 10 minutes

Name:

Student ID:

Signature:

Calculate the vector product of $(\vec{A} + \vec{B})$ and $(\vec{A} - \vec{B})$ where $\vec{A} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ and $\vec{B} = 3\hat{i} + 4\hat{j} - 2\hat{k}$.

$$\vec{A} + \vec{B} = 8\hat{i} + 2\hat{j} + 2\hat{k}$$

$$\vec{A} - \vec{B} = 2\hat{i} - 6\hat{j} + 6\hat{k}$$

$$(\vec{A} + \vec{B}) \times (\vec{A} - \vec{B}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & 2 & 2 \\ 2 & -6 & 6 \end{vmatrix}$$

$$= \hat{i} \begin{vmatrix} 2 & 2 \\ -6 & 6 \end{vmatrix} - \hat{j} \begin{vmatrix} 8 & 2 \\ 2 & 6 \end{vmatrix} + \hat{k} \begin{vmatrix} 8 & 2 \\ 2 & -6 \end{vmatrix}$$

$$= 24\hat{i} - 44\hat{j} - 52\hat{k}$$