

Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes

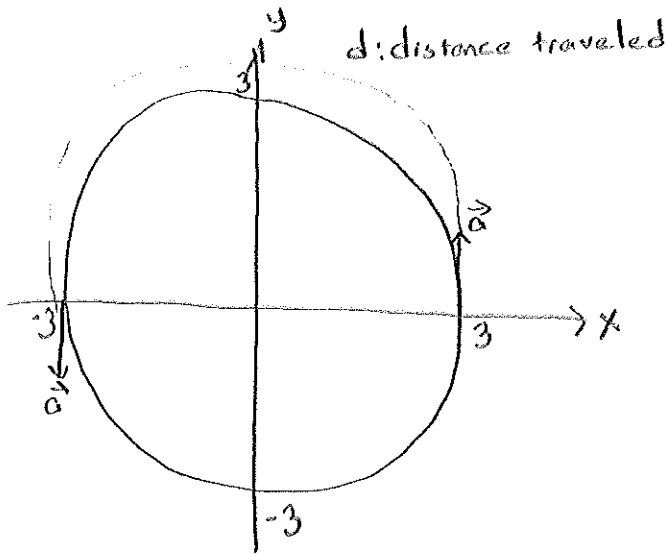
Name:

Student ID:

Signature:

A particle at rest on the positive x-axis starts to move with constant tangential acceleration counterclockwise on a circle of radius 3.0 m centered at the origin. Suppose that the particle completes the first half of the circle at $t = 2$ seconds. (Take $\pi = 3$ in calculations).

- (a) What is the average acceleration vector of the particle in this time interval? Use unit vectors to express the acceleration vector.
(b) What is the magnitude of the radial acceleration of the particle at $t = 1$ s?



$$d = \pi r = 3 \cdot 3 = 9 \text{ m. in } 2 \text{ sec.}$$

$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

$$v_1 = 0$$

$$t_1 = 0$$

$$t_2 = 2 \text{ s.}$$

$$r = r_0 + v_0 t + \frac{1}{2} a t^2$$

$$r - r_0 = \frac{1}{2} a t^2$$

$$9 = \frac{1}{2} a \cdot 4$$

$$a = 4.5 \text{ m/s}^2$$

$$v_2 = v_1 + a \cdot t$$

$$\vec{v}_2 = -9 \text{ m/s } \hat{j}$$

$$b) a_{rad} = \frac{v^2}{R}$$

$$v(t=1) = v_0 + a \cdot t = 4.5 \cdot 1 = 4.5 \text{ m/s}$$

$$a_{rad}(t=1) = \frac{(4.5)^2}{3}$$

$$\vec{a}_{av} = \frac{-9 - 0}{2} = -4.5 \text{ m/s}^2 \hat{j}$$

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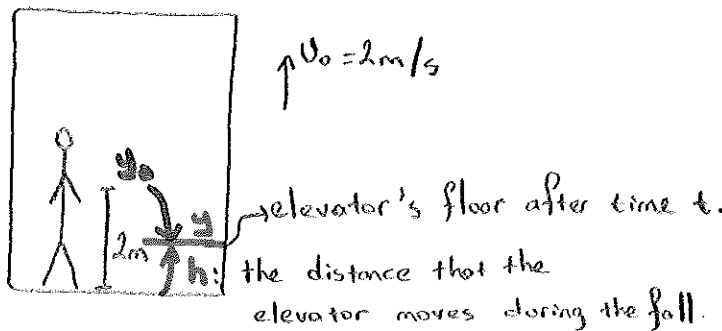
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An elevator is moving up at a constant speed $v_0 = 2\text{ m/s}$. A passenger in the elevator drops a pen at a distance $h = 2.0\text{ m}$ above the elevator floor. Find the speed of the pen just as it hits the floor

(a) according to an observer in the elevator.

(b) according to an observer in the ground floor corridor, walking towards the elevator entrance with constant speed 1 m/s .

(Take $g = 10\text{ m/s}^2$)



b) The distance that the pen will travel is $(2-h)\text{ m}$, where h is the distance that the elevator moves up with $v_0 = 2\text{ m/s}$.

$$h = 2 \cdot t \quad \rightarrow \text{initial velocity of the pen}$$

$$y - y_0 = v_{0y} \cdot t + \frac{1}{2} a_y t^2$$

\downarrow \downarrow \downarrow \downarrow
 $2t$ 2 2 -10

$$2t - 2 = 2t - 5t^2$$

$$t^2 = 2/5 \Rightarrow t = \sqrt{2/5}$$

$$v_{\text{pen}} = v_0 + a_y \cdot t$$

$$v_{\text{pen}} = 2 - \frac{10\sqrt{2}}{\sqrt{5}}$$

$$v_{\text{pen}} = 2 - 2\sqrt{10}$$

a) Since we are asked to find the speed of the pen according to an observer in the elevator, and since the elevator moves with constant velocity, the observer cannot know whether the elevator is moving or at rest. So, this is simply a free-fall.

$$v_{\text{pen}}^2 = v_{\text{pen}0}^2 + 2a_y(y - y_0)$$

\downarrow \downarrow \downarrow
 0 -10 -2

$$v_{\text{pen}}^2 = 40$$

$$v_{\text{pen}} = \sqrt{40}$$

According to the observer on the ground!

$$\vec{v}_{\text{obs}} = \vec{v}_{\text{pen}} - \vec{v}_1$$

$$v_{\text{obs}} = \sqrt{(2 - 2\sqrt{10})^2 + 1}$$

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A particle located at the origin starts to move in the x-y plane at $t = 0$. The velocity vector of the particle is given by $\vec{v} = (a - bt^2)\hat{i} + ct\hat{j}$, where a , b , and c are nonzero positive constants and t is the time coordinate.

- (a) Calculate the position and acceleration vectors as a function of time.
(b) What is the y-coordinate of the particle as it passes through the $x = 0$ line for the first time after $t = 0$?

$$a) \quad \vec{a} = \frac{d\vec{v}}{dt} = (-2bt)\hat{i} + c\hat{j}$$

$$\vec{r} = \int \vec{v} \cdot dt = \int (a - bt^2) dt \hat{i} + \int ct dt \hat{j}$$

$$= \left(at - \frac{bt^3}{3} + C_1 \right) \hat{i} + \left(\frac{ct^2}{2} + C_2 \right) \hat{j}$$

at $t=0$, particle is at the origin.

when $t=0$, $C_1 = C_2 = 0$

so that $\vec{r} = 0$ at $t=0$.

$$\vec{r} = \left(at - \frac{bt^3}{3} \right) \hat{i} + \frac{ct^2}{2} \hat{j}$$

integration constants.

to be determined by initial conditions.

b) x-component of \vec{r} should be zero, when the particle passes through $x=0$ line

$$at - \frac{bt^3}{3} = 0 \quad \left(t=0 \text{ satisfies, but we are asked the first time after } t=0 \right)$$

$$at = \frac{bt^3}{3}$$

$$a = \frac{bt^2}{3}$$

$$t^2 = \frac{3a}{b}$$

$$t = \sqrt{\frac{3a}{b}}$$

$$\vec{r}(x=0, y) = \frac{c}{2} \frac{3a}{b} \hat{j}$$

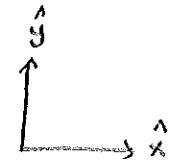
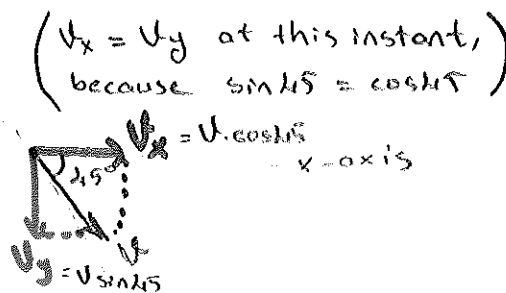
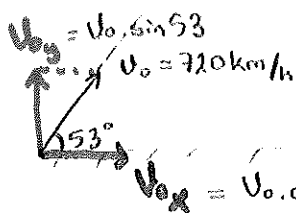
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A missile is thrown at an angle 53° into air from the ground with initial speed $v_0 = 720$ km/h (there is no air resistance). Find the altitude of the missile from the ground at the instant when its velocity vector makes an angle -45° with the x-axis. (Hint: $\tan(53^\circ) = 4/3$, $g = 10\text{m/s}^2$).



$$\vec{a}_x = 0$$

$$\vec{a}_y = (-10 \text{ m/s}^2) \hat{j}$$

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$(v_0 \sin 53)^2 \quad -10 \quad 0$$

$$v_y^2 = v_x^2 = v_{0x}^2 = (v_0 \cos 53)^2$$

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$$

$$0$$

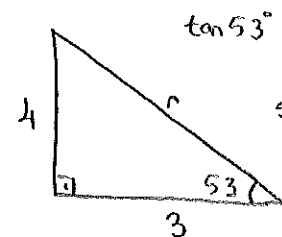
v_x is constant along the motion,
since there is no acceleration on the x-axis.

$$v_0^2 \cos^2 53^\circ = v_0^2 \sin^2 53^\circ - 2(10)(y - 0)$$

$$\left(720 \text{ km/h} = 720 \text{ km/h} \cdot 1000 \cdot \text{m/km} \cdot \frac{1}{3600} \cdot \frac{\text{h}}{\text{s}}\right)$$

"unit conversion from km/h to m/s"

$$720 \text{ km/h} = 200 \text{ m/s}$$



$$\tan 53^\circ = \frac{4}{3}$$

$$\text{so } r = 5$$

$$\text{so } \sin 53^\circ = \frac{4}{5}$$

$$\text{so } \cos 53^\circ = \frac{3}{5}$$

$$(200 \text{ m/s})^2 \cdot \left(\frac{3}{5}\right)^2 = (200 \text{ m/s})^2 \cdot \left(\frac{4}{5}\right)^2 - 20y$$

$$20y = (200)^2 \cdot \frac{7}{25} \Rightarrow \boxed{y = 560 \text{ m}}$$