PHYS 101: General Physics 1 **KOÇ UNIVERSITY** **Fall Semester 2012**

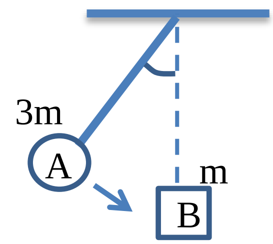
College of Arts and Sciences

Section **Quiz 8** **22 November 2012**

Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes

Name: **Student ID:** **Signature:**

Particle A of mass 3*m* is fixed at the end of a massless rod of length L, which is free to rotate about a pivot point at the ceiling. The particle is released from rest when the rod makes some angle with the vertical. As it passes from the lowest point, it makes a head-on elastic collision with particle B of mass *m* at rest. Find ratio of the tension in the rod just before and after the collision.



Solution:

Before collision: Suppose that the speed of particle A is v_{A1} at the lowest point. The tension in the rod is then $T_1 = m_A g + \frac{m_A v_{A1}^2}{L}$ (circular motion with constant speed at an instant)

After the collision: The speed of particle A is v_{A2}, and the tension is $T_2 = m_A g + \frac{m_A v_{A2}^2}{L}$

So the ratio is $\frac{T_1}{T_2} = \frac{gL + {v_{A1}}^2}{gL + v_{A2}^2}$. Thus, we need to determine the speed of A after the collision.

The collision is elastic: Momentum and kinetic energy are both conserved.

Momentum conservation: $m_A v_{A1} = m_A v_{A2} + m_B v_{B2}$

Kinetic energy conservation: $\frac{1}{2} m_A v_{A1}^2 = \frac{1}{2} m_A v_{A2}^2 + \frac{1}{2} m_B v_{B2}^2$

$m_B v_{B2}^2 = m_A v_{A1}^2 - m_A v_{A2}^2 = (m_A v_{A1} - m_A v_{A2})(m_A v_{A1} + m_A v_{A2})$

$m_B v_{B2} = m_A v_{A1} - m_A v_{A2}$

If we divide the equations side by side, we get:

$v_{B2} = v_{A1} + v_{A2}$

Substitute this back to momentum conservation to get:

$v_{A2} = \frac{m_A - m_B}{m_A + m_B} v_{A1} = \frac{v_{A1}}{2}$

Hence;

$$\frac{T_1}{T_2} = \frac{gL + {v_{A1}}^2}{gL + v_{A2}^2} = \frac{1 + \frac{{v_{A1}}^2}{gL}}{1 + \frac{{v_{A1}}^2}{4gL}}$$

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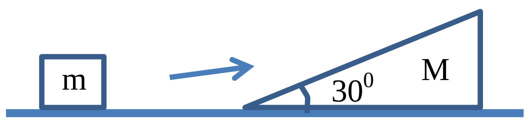
Name: Student ID: Signature:

A block of mass m moving on the ground with speed v_1 is incident on a wedge-shaped object of mass M at rest. The inclination angle is 30^0. Find the maximum distance the block can go up on the inclined surface of the wedge. Assume that all surfaces are frictionless and the interaction between the two objects is elastic.



Solution:
The block will go up on the inclined surface until its relative velocity with respect to the wedge is zero (i.e. it is not moving with respect to an observer on the wedge). At this point, however, both the block and the wedge are moving together (with the same speed) horizontally on the ground!

We apply the conservation of momentum between the initial state and when the block reached its maximum height:

$$mv_1 = mv_2 + Mv_2 = (m + M)v_2$$

Conservation of mechanical energy between these states is:

$$\frac{1}{2}mv_1^2 = \frac{1}{2}(m + M)v_2^2 + mgh$$

The first equation gives $v_2 = \frac{m}{(m+M)}v_1$. Substituting this into the second and solving for h gives:

$$h = \frac{v_1^2}{2g}\left(1 - \frac{m}{m + M}\right)$$

So, the maximum distance is $d = \frac{h}{\sin(30^0)} = \frac{v_1^2}{g}\left(1 - \frac{m}{m+M}\right)$

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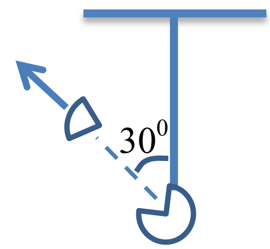
Name: Student ID: Signature:

A particle of mass 3m is fixed at the end of a massless rod of length L, which is suspended vertically from a pivot point at the ceiling. The particle explodes and one piece of mass m move in the direction as shown in the figure with speed v, whereas the other part remains attached to the rod. Find the tension in the rod when it is at half of its maximum height after the explosion.



Solution(*):
During explosion: Momentum in the x-direction is conserved. The momentum was zero before explosion. Let v'_{x1} be the x-velocity of the piece remaining at the end of the rod after explosion. Then

$0 = mv_{x1} + 2mv'_{x1}$ or $v'_{x1} = -\frac{v_{x1}}{2} = -\frac{v\sin(30^0)}{2} = -\frac{v}{4}$

Note that the minus sign just indicates that the two pieces move in opposite directions along the x-axis.

After explosion: Mechanical energy is conserved. At half of the maximum height, half of the total energy is kinetic energy. Let v'_{x2} be the x-velocity when the rod is at half of its maximum height. Then the kinetic energy is half of its value at the bottom: $K'_2 = \frac{K'_1}{2}$,

$\frac{1}{2}2mv'^2_{x2} = \frac{1}{2}(\frac{1}{2}2mv'^2_{x1})$

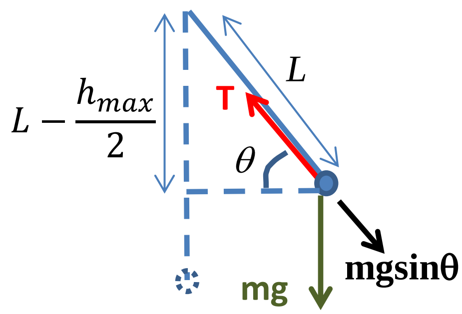
$$v'^2_{x2} = \frac{v'^2_{x1}}{2} = \frac{v^2}{32}$$

The tension in the rod found from central force:

$$T - 2mg\sin\theta = \frac{2m}{L}v'^2_{x2} = \frac{2mv^2}{32L}$$

Maximum height is found from energy conservation

$$2mgh_{max} = \frac{1}{2}2mv'^2_{x1}, \quad h_{max} = \frac{v^2}{32g}$$



The angle at half maximum:

$\sin\theta = \frac{L - h_{max}/2}{L} = 1 - \frac{h_{max}}{2L} = 1 - \frac{v^2}{64gL}$

The tension in the rod is then $T = 2m(g + \frac{v^2}{64L})$

(***** In this problem, full grade will be given if the velocity of the pendulum and its speed at half maximum are found correctly.**)**

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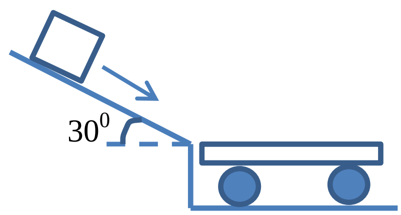
Name: **Student ID:** **Signature:**

A wooden box of mass *m*, initially at rest, slides a distance L on an inclined surface and gets transferred on a cart of mass *m*. The cart is free to move on the ground. The kinetic friction coefficient between the box and the cart surface is μ_k. Find the distance the block has travelled on the cart until it has stopped with respect to the cart.



Solution:
Conservation of energy when the block reaches the lower end of the inclined surface: $m_1 gh = m_1 g\frac{L}{2} = \frac{1}{2}mv^2$. Thus the speed of the box before transfer is $v = \sqrt{gL}$

During transfer: The x-component of the momentum is conserved. We write the momentum conservation between the states at the beginning of transfer and when the box is at rest with respect to the cart. Note: when the box is at rest with respect to the cart, they are moving together with the same speed with respect to the ground!

$$m_1 v_{1x} = m_1 v \cos(30^0) = \frac{\sqrt{3}}{2} m_1 v = (m_1 + m_2) v_{2x}$$

This gives $v_{2x} = \frac{m_1}{m_1+m_2}\frac{\sqrt{3}}{2}v = \frac{\sqrt{3}}{4}v$

We apply the law of conservation of energy between the states at the beginning of transfer and when the box is at rest with respect to the cart.

$$K_1 + W_{other} = K_2$$

Here, $W_{other} = -m_1 g\mu_k d$is the work done by the friction force on the cart surface.

$K_1 = \frac{1}{2}mv^2$ and $K_2 = \frac{1}{2}2m\left(\frac{\sqrt{3}}{4}v\right)^2 = \frac{3}{16}mv^2$

Hence:

$$d = \frac{K_1 - K_2}{m_1 g\mu_k} = \frac{5L}{16\mu_k}$$

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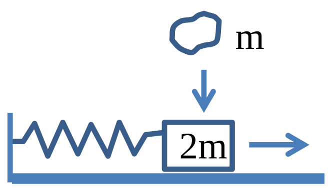
Name: **Student ID:** **Signature:**

A box of mass 2m is fixed to the end of a spring (spring constant is *k*) and it can move on a horizontal frictionless surface. The other end of the spring is fixed to the wall. The spring is compressed by a distance d from the equilibrium position and released. When the box gained its maximum speed, an object of mass m is dropped from the top in the vertical direction on the box and sticks to it. Find the maximum displacement of the spring from its initial position afterwards in terms of d.



Solutions:
Before collision: Mechanical energy conserved between the initial state and the state just before the collision: $K_1 + U_{el,1} = K_2 + U_{el,2}$
$K_1 = 0$. Since the box has maximum speed before collision it must have the maximum kinetic energy. Thus, $U_{el,2} = 0$. This gives:

$$\frac{1}{2}kd^2 = \frac{1}{2}2mv^2, \text{ or } v^2 = \frac{k}{2m}d^2.$$

During collision: Momentum is conserved in the x-direction. This is a completely inelastic collision so the velocity of the particles after collision is the same:

$$2mv = 3mv' \text{ or } v' = \frac{2}{3}v$$

After collision: Mechanical energy is conserved. We use this between the state just after collision and the state of maximum displacement after the collision. Let d' be the maximum displacement from the equilibrium position of the spring. Then

$$\frac{1}{2}3mv'^2 = \frac{1}{2}kd'^2$$

Substituting v' gives $d' = \sqrt{\frac{2}{3}}d$. The maximum displacement *from the initial position* is

$$d + d' = d(1 + \sqrt{\frac{2}{3}}).$$