PHYS 101: General Physics 1 KOÇ UNIVERSITY Fall Semester 2012

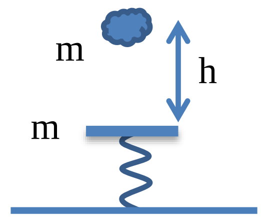
College of Arts and Sciences

Section Quiz 27 December 2012

Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes

Name: Student ID: Signature:

An object of mass m is dropped from a height h on to a platform of mass m, which is atop of a spring as shown in the figure. The spring constant is k and the mass of the spring is negligible. Assume that the object sticks to the platform during the collision. Find the amplitude and period of oscillation that the system makes after the collision, in terms of given parameters (h, g, k, m).



Solution:

The combined system will make a simple harmonic motion. The period is $T = 2\pi\sqrt{\frac{2m}{k}}$

Determining the amplitude:

Before collision: Energy conservation for the dropped particle gives $v_1 = \sqrt{2gh}$

During collision: Momentum conservation: $mv_1 = 2mv_2$ so $v_2 = \sqrt{\frac{gh}{2}}$

After collision: Let's call the initial speed of the mass+platform system $v_0 \equiv v_2$.

If we can determine the initial displacement of the mass+platform system from the new equilibrium position of the spring with mass+platform attached we can apply the formula $A = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}}$. Here, $\omega = \sqrt{\frac{k}{2m}}$ is the angular frequency of the mass+platform.

Let the old equilibrium position (platform only) be x = 0. This is the position of the mass + platform just after the collision.

The new equilibrium position (mass+platform) will be $x' = \frac{mg}{k}$ below the old equilibrium position. So the initial displacement of the mass+platform is $x_0 = \frac{mg}{k}$. Substituting these values:

$$A = \sqrt{\left(\frac{mg}{k}\right)^2 + \frac{2mgh}{2k}} = \frac{mg}{k}\sqrt{1 + \frac{kh}{mg}}$$

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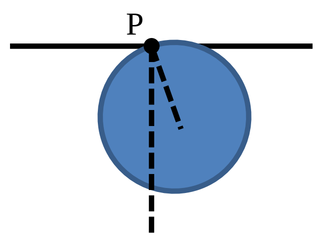
Name: Student ID: Signature:

A physical pendulum consists of a disk of uniform mass m, radius R, which is suspended from a pivoting point P at its end as shown in the figure. The disk makes simple harmonic motion about the vertical dotted line. If the amplitude of the center of mass displacement is equal to $A = \sqrt{\frac{2}{3}}\frac{R}{10}$, determine the angular frequency and the maximum angular speed in terms of given parameters (m, g, R). (The moment of inertia of the disk of mass m, radius R about an axis through its center is $I = \frac{1}{2}mR^2$)



Solution:

Since this is a physical pendulum, the angular frequency is given by $\omega = \sqrt{\frac{mgd}{I}}$

By parallel axis theorem: $I_P = I_{CM} + mR^2 = \frac{3}{2}mR^2$

The distance of the center of mass to the pivoting point: $d = R$

$$\omega = \sqrt{\frac{2g}{3R}}$$

The angular speed is $\omega' = \frac{d\theta}{dt}$. Its maximum value occurs when the center of mass passes from the equilibrium point. We can use the conservation of energy by taking the gravitational potential zero at the equilibrium position:

$$mgR(1 - \cos\theta) = \frac{1}{2}I_p\omega_{max}^2$$

Hence $\omega'_{max} = \sqrt{\frac{2}{3}gR(1 - \cos\theta)}$.

The amplitude is $A = R\theta$ hence $\omega'_{max} = \sqrt{\frac{2}{3}gR\left[1 - \cos\left(\frac{1}{10}\sqrt{\frac{2}{3}}\right)\right]}$

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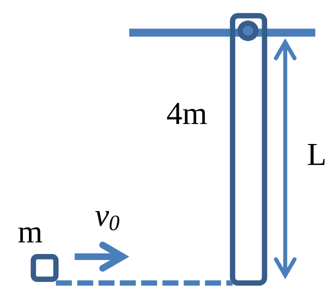
Name: Student ID: Signature:

A particle of mass m moving horizontally with speed v_0 collides and attaches to the free end of a vertically suspended rod of mass 4m, length L. The rod is free to rotate about the other end, so the rod-particle pendulum makes simple harmonic motion after the collision Find the period of oscillations and the maximum angular speed in terms of given parameters. The moment of inertia of a rod (mass M, length L) about an axis from one of its end is equal to $\frac{ML^2}{12}$. (Hint: Pay attention to the type of momentum conservation that is valid during this collision.)



Solution:

(In the question, the moment of inertia of the rod about its end is given incorrect. The solutions using both $\frac{ML^2}{12}$ and $\frac{ML^2}{3}$ will be graded equally. The following solution uses the correct value $\frac{ML^2}{3}$)

After the collision, the system is a physical pendulum consisting of the rod and the particle attached at the end of the rod. The period is given by $T = 2\pi\sqrt{\frac{I}{Mgd}}$, where M is the total mass, d is the distance of the center of mass to the pivot and I is the moment of inertia with respect to the pivot point.

$$I = I_{particle} + I_{rod} = \frac{4mL^2}{3} + mL^2 = \frac{7mL^2}{3}$$

$$d = \frac{4m\frac{L}{2} + mL}{5m} = \frac{3}{5}L$$

$$T = \frac{2\pi}{3}\sqrt{35\frac{L}{g}}$$

Linear momentum is *not* conserved! Apply conservation of angular momentum during collision: $mv_0L = (I_{particle} + I_{rod})\omega'$, where ω' is the common angular speed of the rod and particle after the collision. This gives the maximum angular speed of the system:

$$\omega' = \frac{3}{7}\frac{v_0}{L}.$$

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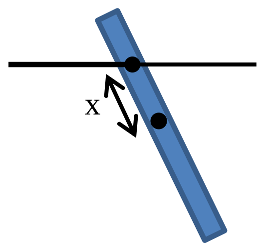
Name: Student ID: Signature:

A uniform rod of mass m, length L is pivoted at a point which is x away from its center (x < L/2) as shown in the figure. The rod makes small amplitude oscillations about this point. Find the period of oscillations. (Note: for small angles, $\sin\theta \approx \theta$)



Solution:
The moment of inertia of the rod with respect to the pivot point is found by using parallel axis theorem:

$$I_P = I_{CM} + mx^2 = \frac{mL^2}{12} + mx^2$$

Newton's 2nd law applied to rotational motion: $\tau = mgx\theta = -I_P \frac{d^2\theta}{dt^2}$. Note that the torque and the angular displacement (θ) are in opposite directions. That's why there is a minus sign!

$$\frac{d^2\theta}{dt^2} = -\frac{gx}{\left(\frac{L^2}{12} + x^2\right)}\theta \equiv -\omega^2\theta$$

Hence, the period is $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\frac{L^2}{12}+x^2}{gx}}$

Alternate solution: The period of a physical pendulum is given by $T = 2\pi\sqrt{\frac{I}{Mgd}}$, where M is the mass, d is the distance of the center of mass to the pivot and I is the moment of inertia with respect to the pivot point. Hence, using $d = x$ and I_P above gives the period.

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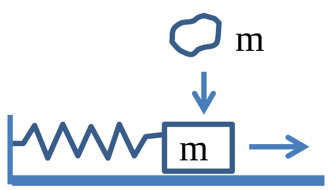
A box m is fixed to the end of a spring (spring constant is k) and it can move on a horizontal frictionless surface. The other end of the spring is fixed to the wall. The spring is compressed by a distance A from the equilibrium position and released. At a point where the kinetic and potential energies of the box are equal, an object of mass m is dropped from the top in the vertical direction onto the box and sticks to it. Find the new amplitude and frequency of oscillations of the combined system.



Solution:

Find the speed of oscillating box before the collision instant. By conservation of energy:

$$E = \frac{1}{2}kA^2 = K + U = 2K = 2\frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{\frac{k}{2m}}A$$

During collision, the x-momentum is conserved:

$$mv = 2mv' \quad \Rightarrow \quad v' = \frac{v}{2}$$

After the collision, energy is conserved. Note that just after the collision, the elastic potential energy in the spring is still half of the total initial energy.

$$U_{el} = \frac{E}{2} = \frac{1}{4}kA^2$$

So the new amplitude A' is found by calculating the total energy in the system:

$$\frac{1}{4}kA^2 + \frac{1}{2}2mv'^2 = \frac{1}{2}kA'^2$$

This gives $A' = \frac{\sqrt{3}}{2}A$. Note that the amplitude and hence the total energy is decreased because of the inelastic collision.

The new frequency of oscillations is $f = \frac{1}{2\pi}\sqrt{\frac{k}{2m}}$