KOÇ ÜNİVERSİTESİ

ENG 216

Phys 101:

1.3. Units

~~[illegible]~~ We will always refer to a physical quantity by comparing it with some reference standard.

This reference standard is called the unit.

For example: When we refer to mass we will use the unit of kilogram.

1 kg : the mass of a particular cylinder of platinium-iridium alloy located near Paris.

time, length, ... have such standards.

Unit system: A system of units which are dimensionally consistent.

We will use the SI unit system (standard unit system) (metric system)

In SI unit system:

mass	kg	(kilogram)
time	sec	(second)
length	m	(meter)
force	N	(Newton)
current	A	(ampere)
temperature	K	(kelvin)

There is also the British unit system, in this system:

length	in	(inch) , 1 inch = 2.54 cm
force	pound	

Unit prefixes:

giga $\longrightarrow 10^{9}$

mega $\longrightarrow 10^{6}$

kilo $\longrightarrow 10^{3}$

deci $\longrightarrow 10^{-1}$

centi $\longrightarrow 10^{-2}$

milli $\longrightarrow 10^{-3}$

micro $\longrightarrow 10^{-6}$

nano $\longrightarrow 10^{-9}$

for example: 1 kilogram $= 1 \times 10^{3}$ gram

1.4. Unit Conversions

Ex: ~~How many meters are Displacement~~

Distance in meters travelled by a car in 10 seconds moving at a speed of 80 km/h ?

$$d = v\,t = \underbrace{80\left(\frac{km}{h}\right)}_{\text{needs unit conversion}} \times 10\,sec = 80\left(\frac{\cancel{km}}{\cancel{h}}\right)\left(\frac{1000\,m}{1\,\cancel{km}}\right)\left(\frac{1\,\cancel{h}}{3600\,sec}\right)\times 10\,sec$$

$$= \frac{80\times10^3}{36\times10^2}\left(\frac{m}{\cancel{sec}}\right)\times 10\,(\cancel{sec}) = \frac{80\times10^4}{36\times10^2}\,(m) = \frac{8000}{36}\,(m) = \boxed{222\,m}$$

→ In doing unit conversions multiply and divide units like numbers in the equation.

__Warnings__:

1) Always convert the units to the SI unit system before calculating the results of your equations.

- You can make a dimensional check of your calculations:

$$d = v\,t = 5\left(\frac{m}{\cancel{sec}}\right)\times 2\,(\cancel{sec}) = 10\,\underline{\underline{m}}$$

$$(m) = \left(\frac{m}{\cancel{sec}}\right)(\cancel{sec}) = (m)\ \checkmark$$

2) Always indicate the unit __of your final result__ in a calculation.

Or you loose points in exams!

1.5. Uncertainty and Significant Figures:

Consider a measurement which yields:

$34.2\,m$: In this measurement, there are 3 significant figures, uncertainty is $0.1\,m = 1\,dm$

If the result was expressed as:

$34.20\,m$: the number of significant figures is 4, uncertainty is $0.01\,m = 1\,cm$

In scientific notation, you [redacted] write:

$$34.20\,m = 3.420 \times 10^{1}\,m \quad , \quad 3.4\,\mu m = 3.4 \times 10^{-6}\,m$$

→ If you perform addition or substraction:

The result should have the uncertainty of the quantity with largest uncertainty.

for example:

$$3.42\,m + 0.5\,m = 3.9\,m$$
$$3.42\,m + 0.50\,m = 3.92\,m$$
$$3.42\,m + 50\,m = 53\,m$$

→ If you perform multiplication or division

The result should have the number of significant figures ~~equal to the~~ no more than in the quantity with the fewest significant figures:

for example:

$$1.2\,m \times 5 \times 10^{1}\,m = 6 \times 10^{1}\,m^2$$
$$1.2\,m \times 50\,m = 60\,m^2 = 6.0 \times 10^{1}\,m^2$$
$$1.2\,m \times 50\,m \times 3\,m = 2 \times 10^{2}\,m^3$$
$$1.2\,m \times 50\,m \times 3.0\,m = 1.8 \times 10^{2}\,m^3$$

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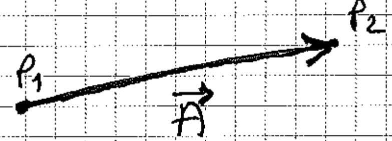
1.7. Vectors :

We use vectors to describe directional quantities.
Vectors are indispensable in physics:
velocity, acceleration, force, momentum, ...
all vectoral quantities.

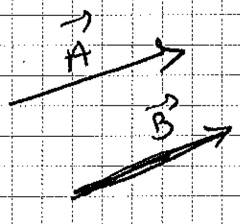
A vector is described by two parts: { - Magnitude, - Direction }

Consider the displacement vector which corresponds to the change in position between two points in space.



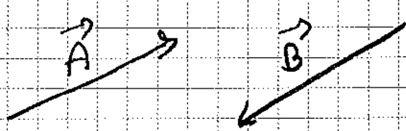
$A = |\vec{A}|$: magnitude of $\vec{A}$ is the distance between P_1 and P_2, e.g. 5m
magnitude has units.

— direction is the direction from P_1 to P_2



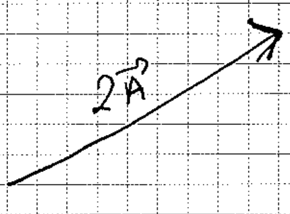
- Two vectors with the same direction are parallel to each other $\vec{A} \parallel \vec{B}$
- Two vectors with the same direction and magnitude are equal to each other. $\vec{A} = \vec{B}$

We define the negative of a vector, as the vector withe the same magnitude but opposite direction.



$\vec{B} = -\vec{A}$ ⟸ notation

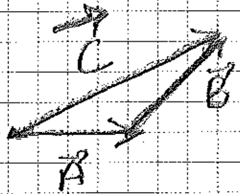
In general:



When a vector is multiplied with a positive scalar its magnitude ~~changes~~ is multiplied by the scalar factor, but its direction remains constant.

Vector Addition:

Consider two vectors $\vec{A}$ and $\vec{B}$, the addition of these vectors is defined as $\vec{C}$ below.

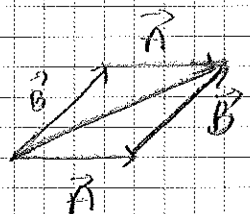


$$\vec{C} = \vec{A} + \vec{B}$$

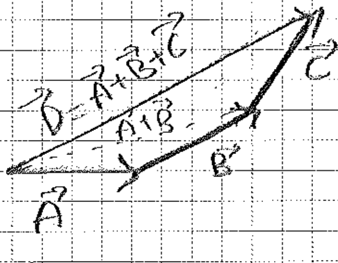
$\vec{C}$: vector from the initial point of $\vec{A}$ to the final point of $\vec{B}$.

The order is not important in vector addition.

$$\vec{C} = \vec{A} + \vec{B} = \vec{B} + \vec{A}$$



Addition of more than two vectors is found by first adding the first two vectors and adding the next vector and so on...



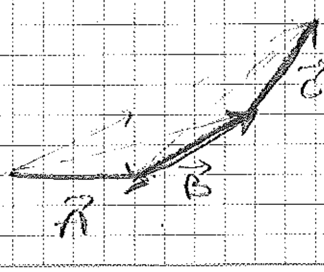
$$\vec{D} = (\vec{A} + \vec{B}) + \vec{C}$$

The order is not important in the addition of more than two vectors.

$$(\vec{A}+\vec{B})+\vec{C} = \vec{A} + (\vec{B}+\vec{C}) = \vec{B} + (\vec{A}+\vec{C}) = \ldots$$

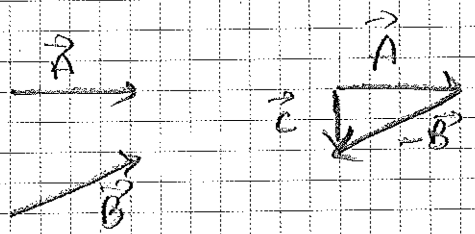
$$= \underbrace{\vec{A} + \vec{B} + \vec{C}}_{\text{without paranthesis.}}$$

You can add in any way you like.



Vector Substraction:

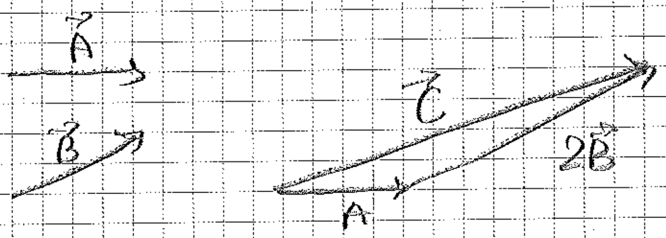
$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$, substraction is addition of the first vector with the negative of the second vector.



$\vec{C} = \vec{A} - \vec{B}$

~~Hence~~ Generally we [redacted] define $\vec{A} + \underbrace{2\vec{B}}$

vector with the same direction as ~~2~~ $\vec{B}$, and twice the magnitude.

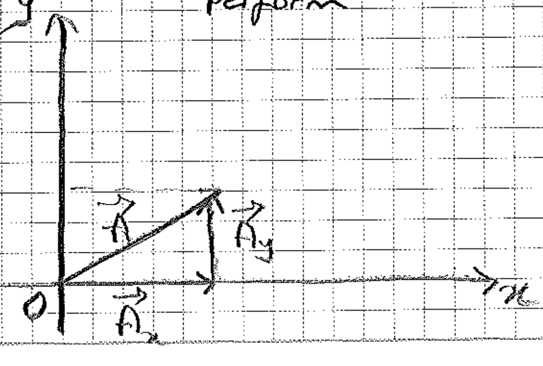


$\vec{C} = \vec{A} + 2\vec{B}$

1.8. Components of Vectors:

~~The method of~~ components of vectors ~~is a simple method used to~~ provide a great help in performing vector operations.

Basically: – You define a coordinate system, common for all vectors
– Separate the vectors into their components
– ~~Make~~ Perform the vector operations using the components.



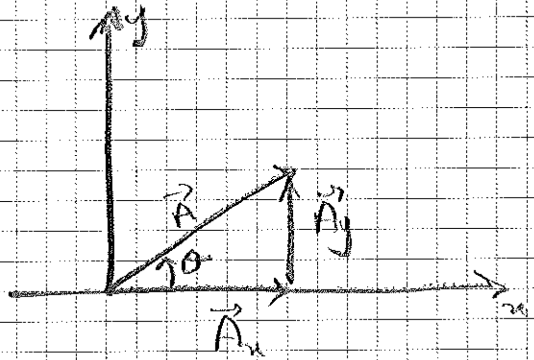
Consider the rectangular coordinate system with origin O.

Locate the vector $\vec{A}$ in this coordinate system with its tail at O.

The projection of vector $\vec{A}$ to the x and y axes are called the component vectors of $\vec{A}$. $\vec{A}_x$ is the x-component vector (projection of $\vec{A}$ to x-axis), $\vec{A}_y$ is the y-component vector (projection of $\vec{A}$ to y-axis).

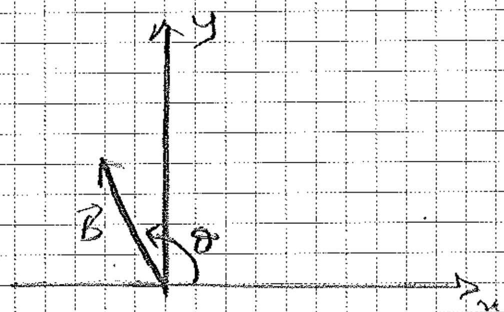
$$\vec{A} = \vec{A}_x + \vec{A}_y$$

The components of $\vec{A}$ can be found if we know its ~~the~~ magnitude and direction.



$$\boxed{\left.\begin{array}{l} A_x = A\cos\theta \\ A_y = A\sin\theta \end{array}\right\} \text{Components of } \vec{A}.}$$

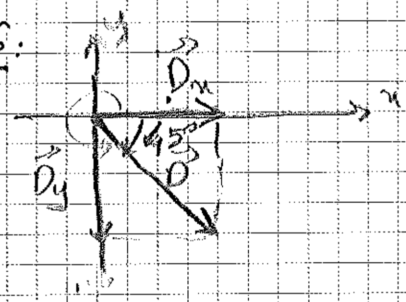
θ is measured from the $+x$ axis, rotating toward the $+y$ axis.



$B_x = B\cos\theta \leftarrow$ negative number.

$B_y = B\sin\theta$

<u>Ex 1.6:</u>



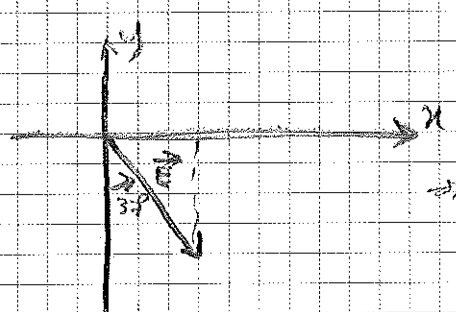
$D = 3\,m, \quad \alpha = 45^\circ$

$\Rightarrow \theta = 360^\circ - 45^\circ = 315^\circ$

components of $\vec{D}$:

$D_x = D\cos(315^\circ) = +\frac{3\sqrt{2}}{2}\,m$

$D_y = D\sin(315^\circ) = -\frac{3\sqrt{2}}{2}\,m$



$E = 4.5\,m, \quad \theta = 270^\circ + 37^\circ = 307^\circ$

$\Rightarrow$ components of $\vec{E}$

$E_x = E\cos 307^\circ = 4.5 \cdot \frac{3}{5}\,m = 2.7\,m$

$E_y = E\sin 307^\circ = -4.5 \cdot \frac{4}{5}\,m = -3.6\,m$

Using Components to Add Vectors;

Simply: if $\vec{R} = \vec{A} + \vec{B}$

the components of the result ($\vec{R}$) are given as

$$R_x = A_x + B_x \; , \quad R_y = A_y + B_y$$

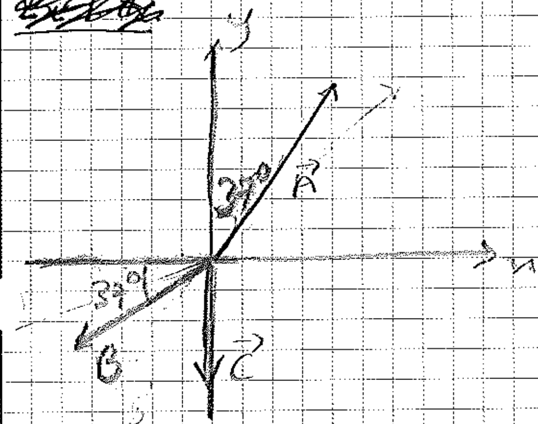
In general in a 3 dimensional addition of many vectors:

$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \dots \Rightarrow \begin{aligned} R_x &= A_x + B_x + C_x + \dots \\ R_y &= A_y + B_y + C_y + \dots \\ R_z &= A_z + B_z + C_z + \dots \end{aligned}$$

magnitude of $\vec{R}$ is: $R = \sqrt{R_x^2 + R_y^2 + R_z^2}$

Ex:

~~Ex-Prob~~



$A = 75\,m$, $B = 60\,m$, $C = 20m$

$\vec{R} = \vec{A} + \vec{B} + \vec{C} \Rightarrow$ What is the angle $\vec{R}$ makes from the x-axis?

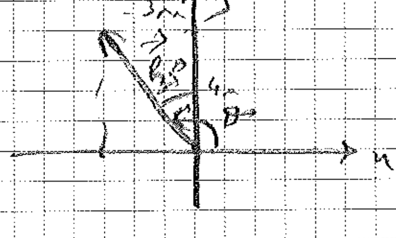
$A_x = 75 \times \frac{3}{5}\,m = 45m$, $A_y = 75 \times \frac{4}{5}\,m = 60m$

$B_x = 60 \times -\frac{4}{5}\,m = -48m$, $B_y = 60 \times -\frac{3}{5}\,m = -36m$

$C_x = 0$ $\qquad C_y = -20m$

$\Rightarrow R_x = 45m - 48m + 0m = -3m$

$R_y = 60m - 36m - 20m = 4m$

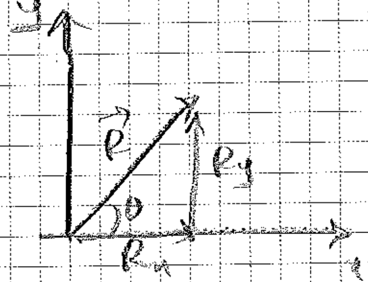


$\theta = 90° + 37° = 127°$ $\quad$ = 127

In general given R_x, R_y, θ and R can be found by the relationships:

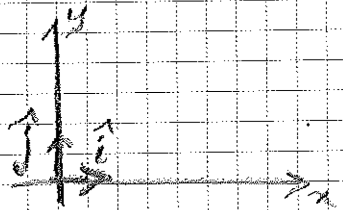
$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right), \quad R = \sqrt{R_x^2 + R_y^2}$$

$$\left(\text{in 3D} \quad R = \sqrt{R_x^2 + R_y^2 + R_z^2}\right)$$



1.9. Unit Vectors :

A unit vector is a vector that has a magnitude of 1 with no units.

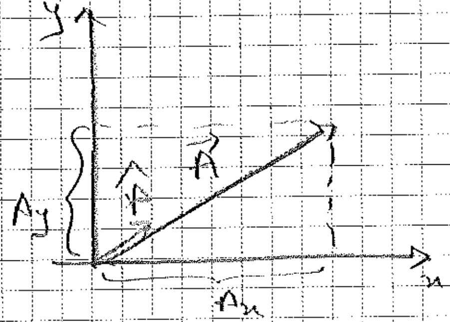


$\hat{i}$: unit vector along the x-axis

$\hat{j}$: unit vector along the y-axis

$\hat{k}$: unit vector along the z-axis

- We always denote a unit vector with a "hat" (^).
- An arbitrary vector $\vec{A}$ can be expressed by ~~unit vectors, and~~ its components and unit vectors:



$$\boxed{\vec{A} = A_x\hat{i} + A_y\hat{j}}, \quad \vec{A} = \underbrace{A}_{\text{magnitude}}\ \underbrace{\hat{A}}_{\text{direction}}$$

Consider two vectors $\vec{A}$ and $\vec{B}$,

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$\Rightarrow$ Sum of the two vectors can be expressed as:

$$\vec{R} = \vec{A} + \vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) + (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$= \underbrace{(A_x + B_x)}_{R_x}\hat{i} + \underbrace{(A_y + B_y)}_{R_y}\hat{j} + \underbrace{(A_z + B_z)}_{R_z}\hat{k}$$

//

Ex 1.9.: Given the two displacements

$$\vec{D} = (6\hat{i} + 3\hat{j} - \hat{k})\ m \quad , \quad \vec{E} = (4\hat{i} - 5\hat{j} + 8\hat{k})\ m$$

find the magnitude of the displacement $2\vec{D} - \vec{E}$.

$$2\vec{D} - \vec{E} = 2 \times (6\hat{i} + 3\hat{j} - \hat{k}) - (4\hat{i} - 5\hat{j} + 8\hat{k})$$

$$= (12\hat{i} + 6\hat{j} - 2\hat{k}) - (4\hat{i} - 5\hat{j} + 8\hat{k})$$

$$= (8\hat{i} + 11\hat{j} - 10\hat{k})\ m$$

$$\Rightarrow |2\vec{D} - \vec{E}| = \sqrt{8^2 + (11)^2 + 10^2} = \sqrt{64 + 121 + 100} = \sqrt{285} = \boxed{17\,m}$$

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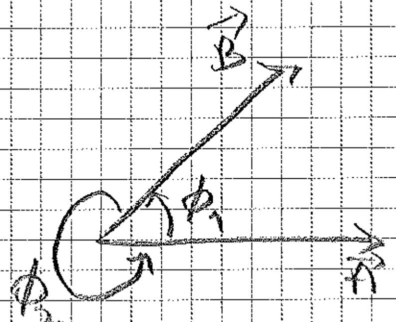
1.10. Products of Vectors:

Apart from the vector addition and substraction, we define two different vector operations:

- scalar product (dot product) $\vec{A} \cdot \vec{B}$
- vector product (cross product) $\vec{A} \times \vec{B}$

Scalar Product

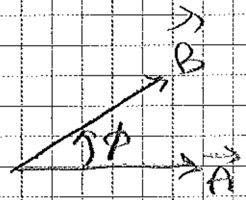
Consider two vectors $\vec{A}, \vec{B}$ are placed such that their tails coincide with each other:



The smaller of ϕ_1 and ϕ_2 is called as the angle between $\vec{A}$ and $\vec{B}$.

$$\phi = \min\{\phi_1, \phi_2\}$$

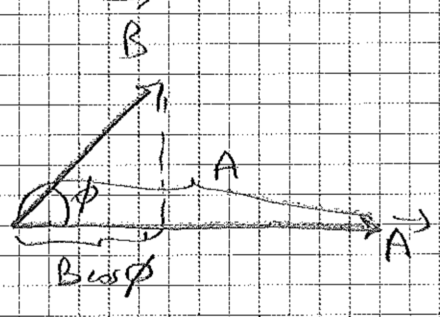
ϕ is always between 0° and 180°.



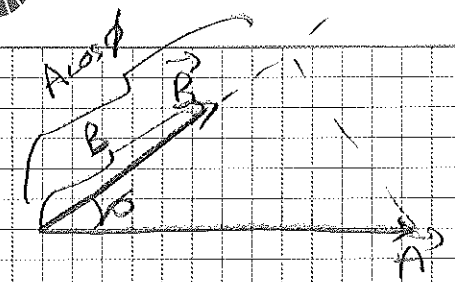
The scalar product of $\vec{A}$ and $\vec{B}$ is defined as:

$$\boxed{\vec{A} \cdot \vec{B} = A\,B\,\cos\phi}$$

A: magnitude of $\vec{A}$ — B: magnitude of $\vec{B}$ — ϕ: angle between $\vec{A}$ and $\vec{B}$



$\vec{A} \cdot \vec{B}$ is the magnitude of $\vec{A}$ multiplied by the projection of $\vec{B}$ parallel to $\vec{A}$



similarly, $\vec{A}\cdot\vec{B}$ can be viewed as the magnitude of $\vec{B}$ multiplied by the projection of $\vec{A}$ parallel to $\vec{B}$.

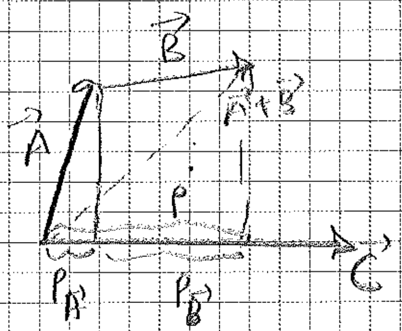
Notes:

- The result of $\vec{A}\cdot\vec{B}$ is a SCALAR, not VECTOR!
  it may be $+$, $-$ or 0.
  for $0 \leq \phi < 90$ for $90 < \phi \leq 180°$ for $\phi = 90°$
- $\phi = 90°$, $\vec{A} \perp \vec{B} \Rightarrow \vec{A}\cdot\vec{B}$, scalar product of two perpendicular vectors is zero.
- $\vec{A}\cdot\vec{A} = A^2$, ($\phi = 0$, naturally)
- For any $\vec{A}, \vec{B}$: $AB\cos\phi = BA\cos\phi \Rightarrow \vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$
  "commutative law of multiplication"

We will use the scalar product in various topics in this class.
an example is the work done by a force

$$W = \underset{\text{force}}{\vec{F}} \cdot \underset{\text{displacement}}{\vec{s}}$$

Property: $(\vec{A}+\vec{B})\cdot\vec{C} = \vec{A}\cdot\vec{C} + \vec{B}\cdot\vec{C}$ (distributive law)



p : projection of $\vec{A}+\vec{B}$ parallel to $\vec{C}$

$p_{\vec{A}}$: projection of $\vec{A}$ " " $\vec{C}$

$p_{\vec{B}}$: " " $\vec{B}$ " " $\vec{C}$

$$p = p_{\vec{A}} + p_{\vec{B}}$$

$$\Rightarrow (\vec{A}+\vec{B})\cdot\vec{C} = p \times C = p_{\vec{A}}C + p_{\vec{B}}C = \vec{A}\cdot\vec{C} + \vec{B}\cdot\vec{C}$$ //

We can use this property to benefit from the components of vectors.

$$\vec{A}\cdot\vec{B} = \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right)\cdot\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$$

$$= A_xB_x\underbrace{\hat{i}\cdot\hat{i}}_{=1} + A_xB_y\underbrace{\hat{i}\cdot\hat{j}}_{=0} + A_xB_z\underbrace{\hat{i}\cdot\hat{k}}_{=0} + \ldots$$

$$\Rightarrow \boxed{\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z}$$ A very useful formula!

Example 1.11: Find the angle between

$$\vec{A} = 2\hat{i} + 3\hat{j} + \hat{k} \quad \text{and} \quad \vec{B} = -4\hat{i} + 2\hat{j} - \hat{k}$$

we will use the property: $\vec{A}\cdot\vec{B} = AB\cos\phi$

$$\Rightarrow \cos\phi = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{A_xB_x + A_yB_y + A_zB_z}{AB}$$

$$\Rightarrow \vec{A}\cdot\vec{B} = -8 + 6 - 1 = -3$$

$$A = \sqrt{4+9+1} = \sqrt{14} \quad \Rightarrow \quad AB = \sqrt{2\times7\times2\times10} = 2\sqrt{70}$$

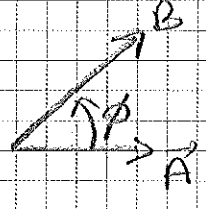
$$B = \sqrt{16+4+1} = \sqrt{20}$$

$$\Rightarrow \cos\phi = \frac{-3}{2\sqrt{70}} \quad \Rightarrow \quad \boxed{\phi = 100^\circ}$$

$$= -0.175$$

Vector Product

Consider two vectors $\vec{A}$ and $\vec{B}$ placed with their tails at the same point:



The magnitude of the vector $\vec{C}$ which is the result of cross-product operation $\vec{C} = \vec{A}\times\vec{B}$ is given as:

$$|\vec{A}\times\vec{B}| = \boxed{AB\sin\phi} \rightarrow$$ Always a positive number

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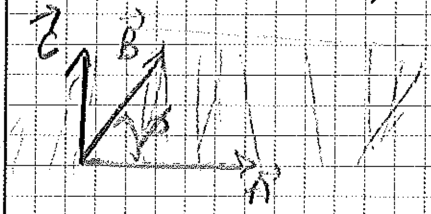
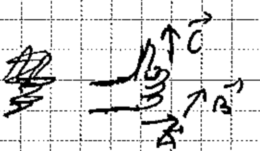
$\vec{C} = \vec{A} \times \vec{B}$, The direction of $\vec{C}$ is perpendicular to the plane defined by $\vec{A}$, $\vec{B}$ given by the right-hand rule.





<u>Properties:</u>

→ The direction of $\vec{B} \times \vec{A}$ is opposite to the direction of $\vec{A} \times \vec{B}$

$$\vec{B} \times \vec{A} = -\vec{A} \times \vec{B} \quad \text{(noncommutative)}$$

- if $\vec{A} \parallel \vec{B}$, $\phi = 0 \Rightarrow \vec{A} \times \vec{B} = 0$, similarly $\vec{A} \times \vec{A} = 0$

- if $\vec{A} \perp \vec{B} \Rightarrow |\vec{A} \times \vec{B}| = AB$, ($\phi = 90°$)

- $(\vec{A} + \vec{B}) \times \vec{C} = \vec{A} \times \vec{C} + \vec{B} \times \vec{C}$ (distributive law)

We make use of the distributive law to benefit from components of vectors.

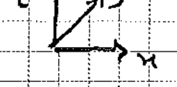
$$\vec{A} \times \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \times (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$= A_x B_x (\hat{i} \times \hat{i}) + A_x B_y \, \hat{i} \times \hat{j} + A_x B_z \, \hat{i} \times \hat{k} +$$
$$A_y B_x (\hat{j} \times \hat{i}) + A_y B_y \, \hat{j} \times \hat{j} + A_y B_z \, \hat{j} \times \hat{k} +$$
$$A_z B_x \, \hat{k} \times \hat{i} + A_z B_y \, \hat{k} \times \hat{j} + A_z B_z \, \hat{k} \times \hat{k}$$

$$= A_x B_y \hat{k} - A_x B_z \hat{j} + A_y B_x (-\hat{k}) + A_y B_z \hat{i} + A_z B_x \hat{j} + A_z B_y (-\hat{i})$$

$$= (A_y B_z - A_z B_y)\hat{i} + (A_z B_x - A_x B_z)\hat{j} + (A_x B_y - A_y B_x)\hat{k} \quad //$$

use

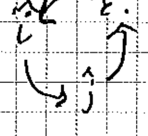


$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$

$\hat{i} \times \hat{j} = \hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$

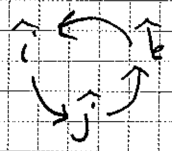
$\hat{k} \times \hat{i} = \hat{j}$



⇒ Can also express in determinant form:

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

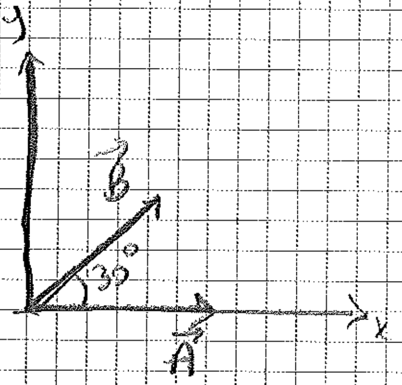
Don't learn this formula by heart.
Simply learn the rule:



We will use the cross product in various topics during this class, e.g. torque $\vec{\tau} = \vec{r}\times\vec{F}$, angular momentum $\vec{L} = \vec{r}\times\vec{p}$

Ex. 1.12: $\vec{A}$ has magnitude of 6 m in x-axis
$\vec{B}$ has magn. of 4 m and lies in the xy plane with 30° with the x-axis.

What is $\vec{A}\times\vec{B}$?



(i) $|\vec{A}\times\vec{B}| = AB\sin\phi = 6\times 4\times\sin 30° = 12\ m^2$
direction of $\vec{A}\times\vec{B}$ is $\hat{k}$
$\Rightarrow \vec{C} = \vec{A}\times\vec{B} = 12\hat{k}\ (m^2)$.

(ii) $\left.\begin{aligned} \vec{A} &= 6\hat{i} \\ \vec{B} &= 2\hat{j} + 6\tfrac{\sqrt{3}}{2}\hat{i} \end{aligned}\right\} \Rightarrow \vec{A}\times\vec{B} = (6\hat{i})\times(3\sqrt{3}\,\hat{i} + 2\hat{j})$

$= 18\sqrt{3}\,\underbrace{\hat{i}\times\hat{i}}_{0} + 12\,\underbrace{\hat{i}\times\hat{j}}_{\hat{k}} = 12\hat{k}\ m^2$