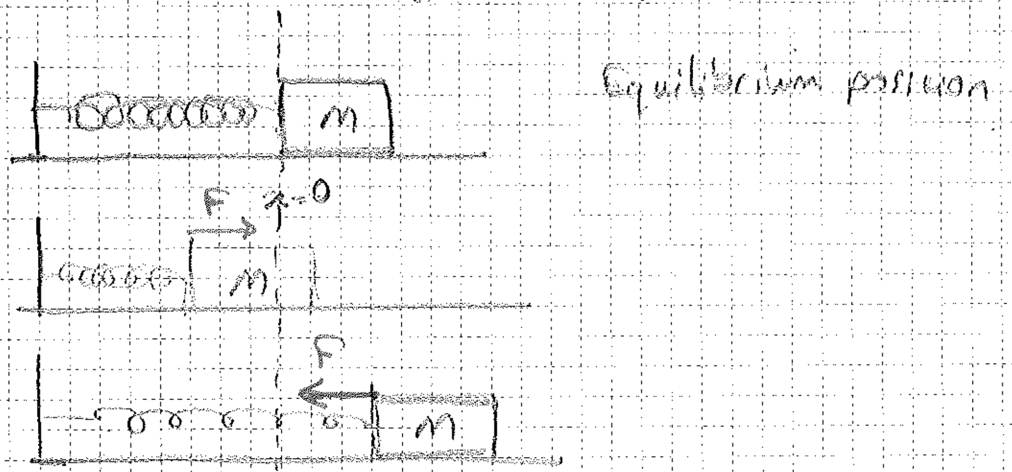
Chap. 14 : Periodic Motion

Periodic motion is in the foundations of various physical phenomena:

- swinging pendulum
- motion of a mass connected to a spring
- pistons in a car engine
- sound waves
- alternating electric currents
- and LIGHT

⇒ Periodic motion is a very fundamental and IMPORTANT type of motion.

Consider a Spring-Mass System:



A body with mass m moves on a frictionless horizontal track. When the body is a distance away from the equilibrium position, restoring force applied by the spring tries to bring the body to the equilibrium position. ⟶ Oscillation

Some Definitions:

Amplitude of the motion: Maximum magnitude of displacement

$$A = \max(|x|)$$

The period, T: Time elapsed during one complete round trip (cycle)
SI Units of the period is seconds.

The frequency, f: Number of cycles in a unit time.
SI Units is hertz, 1 hertz = 1 Hz = 1 s^{-1}

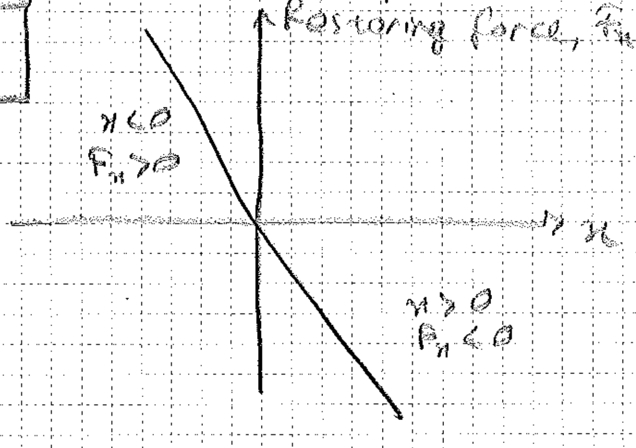
Angular frequency, ω: $\omega = 2\pi f$, units $(\omega) = (rad/s)$
Shortly, It will prove useful.

$$\boxed{f = \frac{1}{T}, \qquad \omega = \frac{2\pi}{T}}$$

Simple Harmonic Motion

This is the simplest kind of periodic motion when the restoring force is directly proportional to the displacement from equilibrium position.

$$\boxed{F_x = -kx}$$



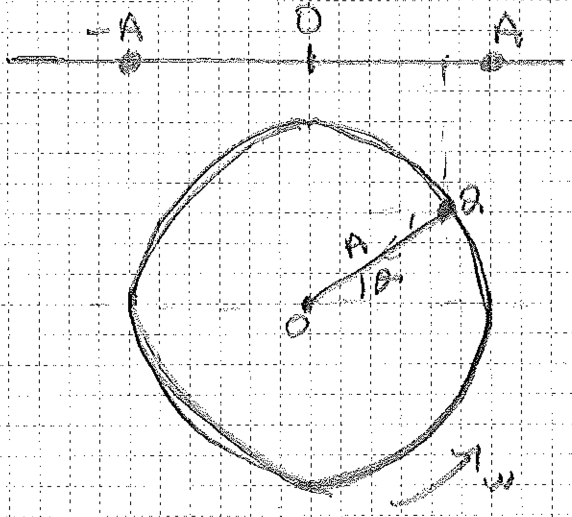
For the simple harmonic motion:

$$a_x = \frac{F_x}{m} = -\frac{k}{m}x \Rightarrow \boxed{\frac{d^2x}{dt^2} = -\frac{k}{m}x}$$

This is the differential equation, giving the position as a function of time.

Unfortunately, the solution is not as simple as the motion with constant acceleration.

We will show that simple harmonic motion is equivalent to the motion of the shadow of a particle undergoing uniform circular motion with radius A.



x component of the vector OQ is; $x = A\cos\theta$ ← changes with time

$\theta = \omega t$ ⟸ uniform circular motion

$x = A\cos(\omega t)$

Acceleration of the uniform circular motion:

$$|\vec{a}_Q| = \frac{v^2}{A}$$, in the radial direction

$$= \omega^2 A$$

⇒ x component of $\vec{a}_Q$

$$a_x = -\omega^2 A\cos\theta$$

$$\boxed{a_x = -\omega^2 x}$$ ⟸ This is the equation of motion of simple harmonic motion

⇕

$a_x = -\frac{k}{m}x$ ⇒ $\omega^2 = \frac{k}{m}$ ⇒ $\boxed{\omega = \sqrt{\frac{k}{m}}}$

∴ Angular speed in the uniform circular motion corresponds to the angular frequency of the simple harmonic motion.

$$\omega = \frac{2\pi}{T} \Rightarrow \boxed{T = 2\pi\sqrt{\frac{m}{k}}, \quad f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$$

period and frequency of simple harmonic motion.

T and f do not depend on the amplitude of the oscillation A.

Ex. 13.2: On a spring-mass system, a force $F_x = 6N$ causes a displacement of 0.03m.
A 0.5kg body is attached to the end of the spring and stretched a distance of 0.02m, and released.

a) What is k of the spring? $6N = k \cdot 0.03m \Rightarrow k = 200\,N/m$

b) Angular freq, freq, and period of the oscillation?

$\omega = \sqrt{\frac{m}{k}} = 20\,rad/s$, $T = \frac{2\pi}{\omega} = 0.31s$, $f = \frac{\omega}{2\pi} = 3.2Hz$

Displacement, Velocity and Acceleration in Simple Harmonic Motion:

We had obtained:

$x = A\cos\theta$, θ is the angle from x-axis of a body undergoing uniform circular motion, with angular speed ω.

$\Rightarrow \theta = \omega t + \phi$

$$\Rightarrow \boxed{x(t) = A\cos(\omega t + \phi)}$$

displacement in simple harmonic motion.

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ϕ: is the phase angle. It is determined by the initial displacement in simple harmonic motion.

$$\boxed{x_0 = A\cos\phi}$$

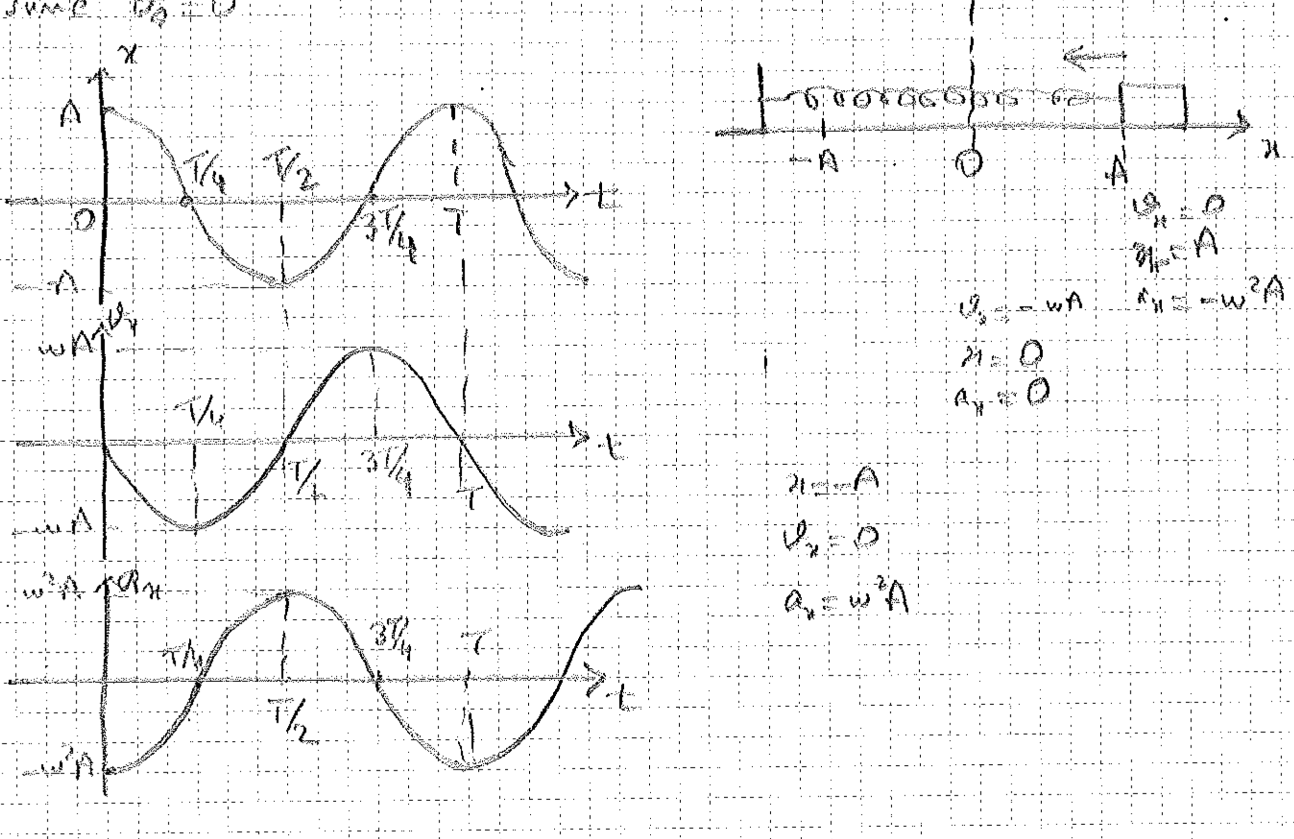
velocity and accelerations are given as:

$$\boxed{\begin{aligned} v_x &= \frac{dx}{dt} = -\omega A \sin(\omega t + \phi) \\ a_x &= \frac{dv_x}{dt} = -\omega^2 A \cos(\omega t + \phi) \end{aligned}}$$

check: $a_x \stackrel{?}{=} -\frac{k}{m}x$

$$-\omega^2 \cancel{A\cos(\omega t+\phi)} \stackrel{?}{=} -\frac{k}{m}\cancel{A\cos(\omega t+\phi)} \Rightarrow \omega^2 = \frac{k}{m} \checkmark$$

Assume $\phi_0 = 0$



If we are given the initial position x_0 and velocity v_0 we can fully characterize the simple harmonic motion.

$x_0 = A\cos\phi \quad , \quad v_{0x} = -\omega A \sin\phi$

$$\Rightarrow \quad \cos^2\phi + \sin^2\phi = 1 \Rightarrow \frac{x_0^2}{A^2} + \frac{v_0^2}{\omega^2 A^2} = 1$$

$$\Rightarrow \boxed{A = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}}}$$

Ex 13.3: $k = 200\,N/m$, $m = 0.5\,kg$

Consider an initial displacement $x_0 = 0.015\,m$ and initial velocity $v_0 = 0.40\,m/s$

a) Period, amplitude and phase angle of the motion

$$T = 2\pi\sqrt{\frac{m}{k}} = 0.31\,s$$

$$A = \sqrt{x_0^2 + \frac{v_0^2}{\omega}} = 0.025\,m$$

$$A\cos\phi = x_0 \Rightarrow (0.025\,m)\cos\phi = 0.015\,m \Rightarrow \cos\phi = \frac{0.015}{0.025} \Rightarrow \phi = 53^\circ \text{ or } -53^\circ$$

$$\sin\phi = -\frac{v_0}{\omega A} < 0 \Rightarrow \boxed{\phi = -53^\circ} = -0.93\,rad$$

b) Write equations for the displacement, velocity and acceleration as functions of time? $\quad \omega = \frac{2\pi}{0.31\,s} = 20\,rad/s$

$$x(t) = 0.025\,m \cos\left(20\,rad/s\; t - 0.93\right)$$

$$v_x(t) = -\left(20\,rad/s\right) 0.025\,m \sin\left(20\,rad/s\; t - 0.93\right)$$

$$a_x(t) = -\left(20\,rad/s\right)^2 0.025\,m \cos\left(20\,rad/s\; t - 0.93\right)$$

Ex: $x(t) = 0.05\,m \cos\left(290\,rad/s\ t + 2.5\,rad\right)$

What is A, T, ϕ, x_0?

$A = 0.05\,m$

$T = \dfrac{2\pi}{290\,rad/s}$ $\quad \phi = 2.5\,rad, \quad x_0 = 0.05\,m \cos(2.5\,rad)$

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13.3 Energy in Simple Harmonic Motion:

Spring force $F_x = -kx$ is a conservative force.

$$W_{F_x}(\Delta x) = -(U(x_2) - U(x_1)), \text{ with } U = \frac{1}{2}kx^2$$

⇒ In simple harmonic motion, total energy is conserved.

$$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \text{ is conserved.}$$

at $x = A$ (amplitude) $v = 0$

$$\Rightarrow E = \frac{1}{2}kA^2$$

This can also be verified by substituting the expressions for $x(t)$ and $v(t)$ in SHM into the energy conservation equation:

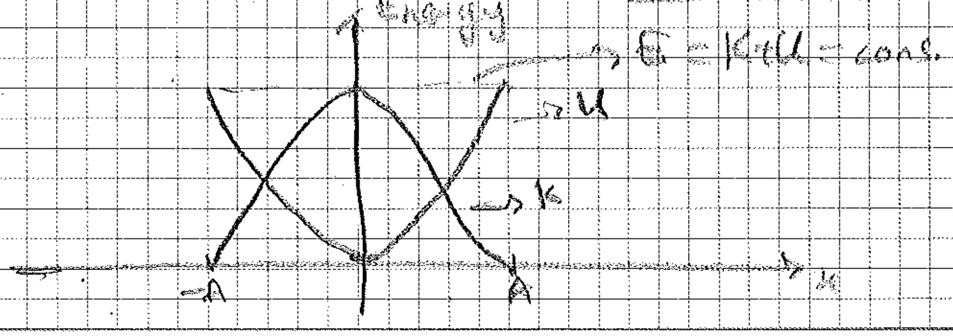
$$E = \frac{1}{2}m\left[-\omega A\sin(\omega t+\phi)\right]^2 + \frac{1}{2}k\left[A\cos(\omega t+\phi)\right]^2$$

$$= \frac{1}{2}A^2\left\{\underbrace{m\omega^2}_{=k}\sin^2(\omega t+\phi) + k\cos^2(\omega t+\phi)\right\}$$

$$= \frac{1}{2}kA^2\left\{\sin^2(\omega t+\phi) + \cos^2(\omega t+\phi)\right\} = \frac{1}{2}kA^2$$

Velocity of the body at a given displacement:

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \Rightarrow \boxed{v = \pm\sqrt{\frac{k}{m}}\sqrt{A^2 - x^2}}$$



Maximum velocity in SHM:

$$\frac{1}{2}kA^2 = \frac{1}{2}m\vartheta_{max}^2 \Rightarrow \vartheta_{max} = \sqrt{\frac{k}{m}}\,A$$

Maximum acceleration:

$$a_x = -\frac{k}{m}x, \quad a_{max} = -\frac{k}{m}(-A) = \frac{kA}{m}$$

Example 13.5:

A block with mass M is moving with simple harmonic motion having amplitude A_I, attached to a spring with force constant k.

At the instant when the block passes through its equilibrium position, a mass m is dropped on to the block.

a) New amplitude and period?

b) What happens if m was dropped at $x = A_I$?

(a) At the instant the mass is dropped:

$$M\vartheta_1 = (m+M)\vartheta_2 \quad \text{momentum cons.}$$

$$\frac{1}{2}kA^2 = \frac{1}{2}M\vartheta_1^2 \Rightarrow \vartheta_1 = \sqrt{\frac{k}{M}}\,A \Rightarrow \vartheta_2 = \frac{M}{m+M}\sqrt{\frac{k}{M}}\,A$$

new amplitude: $\frac{1}{2}(M+m)\vartheta_2^2 = \frac{1}{2}kA_2^2$

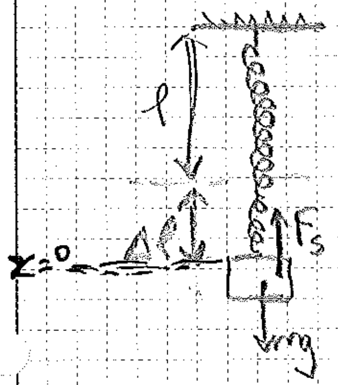
$$\Rightarrow A_2^2 = \frac{\cancel{M+m}}{\cancel{k}} \cdot \frac{M^{\cancel{2}}}{(M+m)^{\cancel{2}}} \cdot \frac{\cancel{k}}{\cancel{M}} A_1^2 \Rightarrow \boxed{A_2 = \sqrt{\frac{M}{M+m}}\,A_1}$$

$$T_2 = \sqrt{\frac{M+m}{k}}$$

(b) Nothing would have changed!

13.4. Applications of Simple Harmonic Motion

Vertical SHM:



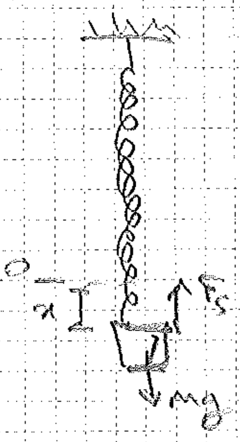
Suppose a body with mass m is attached to a spring standing vertically.

At the new equilibrium position:

$$F_s = mg \Rightarrow k\Delta l = mg$$

$l + \Delta l$ is the new equilibrium position.

Analyze the oscillation around this new equilibrium position.

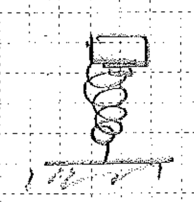


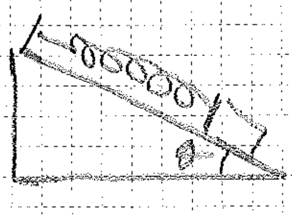
$$-F_{net} = F_s - mg = k(\Delta l + x) - mg$$
$$= kx \Rightarrow \boxed{F_{net} = -kx}$$

around the new equilibrium pos.

* Vertical SHM does not differ from horizontal SHM in an essential way. The only change is the position of the equilibrium position.

* Same is valid if a body of mass m is placed on the top of a compressible spring.

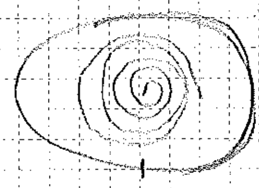




or for a spring-mass system inclined by angle θ.

Angular SHM:

Consider a balance wheel.



A coil spring exerts a restoring torque τ_z proportional to the angular displacement θ from the equilibrium position.

$$\tau_z = -\kappa\theta$$

In rotational motion:

$$\sum \tau_z = -\kappa\theta = I\alpha = I\frac{d^2\theta}{dt^2} \Rightarrow \frac{d^2\theta}{dt^2} = -\frac{\kappa}{I}\alpha$$

moment of inertia

similar to $a = -\frac{k}{m}x$

$\Rightarrow$ angular SHM will have the same properties as linear SHM.

$$\omega = \sqrt{\frac{k}{m}} \Leftrightarrow \omega = \sqrt{\frac{\kappa}{I}}$$

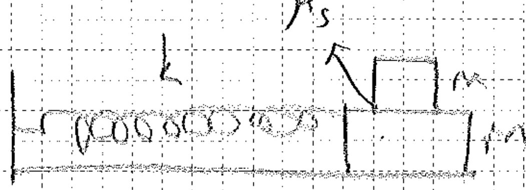
$x = A\cos(\omega t + \phi)$ $\qquad$ $\theta = \Theta\cos(\omega t + \phi)$

magnitude of oscillation

$v = -\omega A\sin(\omega t + \phi)$ $\qquad$ $\frac{d\theta}{dt} = -\omega\Theta\sin(\omega t + \phi)$

$a = -\omega^2 A\cos(\omega t + \phi)$ $\qquad$ $\frac{d^2\theta}{dt^2} = -\omega^2\Theta\cos(\omega t + \phi)$

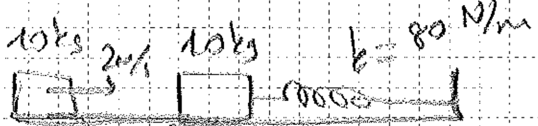
Prob. 13.63



what is the Maximum amplitude osc. such that the top block will not slip on the bottom block.

maximum force : $k\cancel{A} = \mu_s Mg \Rightarrow A = \frac{\mu_s Mg}{k}$

Problem 13.64 : A 10kg mass is traveling to the right with a speed of 2 m/s



Two bodies collide and stick together.

a) Freq, amp., and period of the osc.?

$10kg \cdot 2m/s = 20kg\, v_1 \Rightarrow v_1 = 1\, m/s$

$\cancel{\frac{1}{2}}\, 20kg\, (1\, m/s)^2 = \cancel{\frac{1}{2}}\, 80\, A^2 \Rightarrow A^2 = \frac{1}{4} \Rightarrow \boxed{A = \frac{1}{2} m}$

$w = \sqrt{\frac{k}{m}} \Rightarrow w = \sqrt{\frac{80}{20}} = 2\, rad/s \quad 2\pi f = 2\, rad/s \Rightarrow f = \frac{1}{\pi}\ 1/s$

$T = \pi$ sec

b) How long does it take the sys. to return the first time to the equilibrium pos?

$\frac{T}{2} = \frac{\pi}{2}$ sec.

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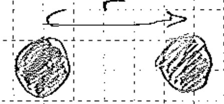
Vibrations of Molecules:



Consider the van der Waals interaction between two atoms.

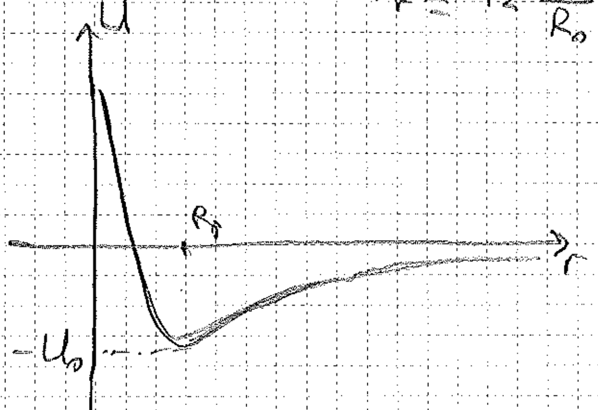
From experiments, van der Walls interaction between 2 atoms can be described as:

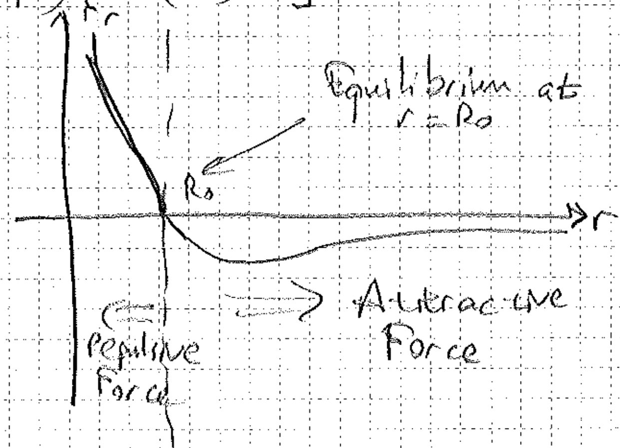
$$U = U_0\left[\left(\frac{R_0}{r}\right)^{12} - 2\left(\frac{R_0}{r}\right)^{6}\right]$$, R_0: equilibrium distance between the atoms.

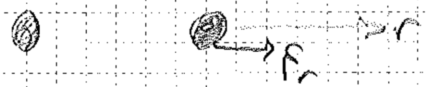
Potential energy function $\nearrow U$; U_0: a constant

The force is: $F_r = -\frac{dU}{dr} = U_0\left[\frac{12R_0^{12}}{r^{13}} - 2\,\frac{6R_0^{6}}{r^{7}}\right]$

$$F_r = 12\frac{U_0}{R_0}\left[\left(\frac{R_0}{r}\right)^{13} - \left(\frac{R_0}{r}\right)^{7}\right]$$







What is the nature of the motion of one atom with respect to the other atom around the equilibrium position?

$$F_r = 12\frac{U_0}{R_0}\left[\left(\frac{R_0}{r}\right)^{13} - \left(\frac{R_0}{r}\right)^{7}\right]$$

Consider $r = R_0 + x \Rightarrow F_r = 12\frac{U_0}{R_0}\left[\left(\frac{R_0}{R_0+x}\right)^{13} - \left(\frac{R_0}{R_0+x}\right)^{7}\right]$

$$= 12\frac{U_0}{R_0}\left[\left(\frac{1}{1+x/R_0}\right)^{13} - \left(\frac{1}{1+x/R_0}\right)^{7}\right]$$

$$\Rightarrow F_r = 12\frac{U_0}{R_0}\left[\left(1+\frac{x}{R_0}\right)^{-13} - \left(1+\frac{x}{R_0}\right)^{-7}\right]$$

Binomial Thm; $(1+u)^n = 1 + nu + \frac{n(n-1)}{2!}u^2 + \frac{n(n-1)(n-2)}{3!}u^3 + \dots$

$$\Rightarrow \left(1+\frac{x}{R_0}\right)^{-13} = 1 - 13\left(\frac{x}{R_0}\right) + \frac{13(14)}{2!}\left(\frac{x}{R_0}\right)^2 + \dots$$

for $x \ll R_0$, small oscillations

$$\Rightarrow \left(1+\frac{x}{R_0}\right)^{-13} \approx 1 - 13\left(\frac{x}{R_0}\right)$$

Similarly $\left(1+\frac{x}{R_0}\right)^{-7} \approx 1 - 7\left(\frac{x}{R_0}\right)$

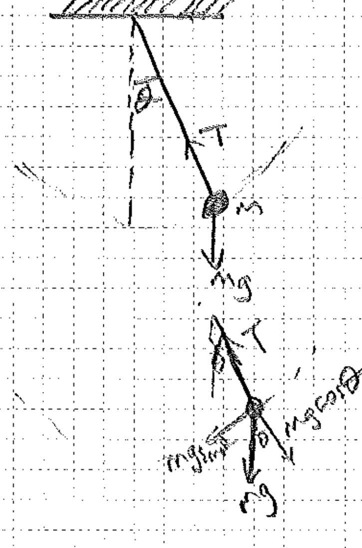
$$\Rightarrow F_r \approx 12\frac{U_0}{R_0}\left[\cancel{1} - 13\left(\frac{x}{R_0}\right) - \cancel{1} + 7\frac{x}{R_0}\right] = -\left(72\frac{U_0}{R_0^2}\right)x$$

looks like $F = -kx$

with $k = \frac{72U_0}{R_0^2}$!

Simple Harmonic Motion.

13.5 The Simple Pendulum:



A body with mass m attached to the end of a massless rope.

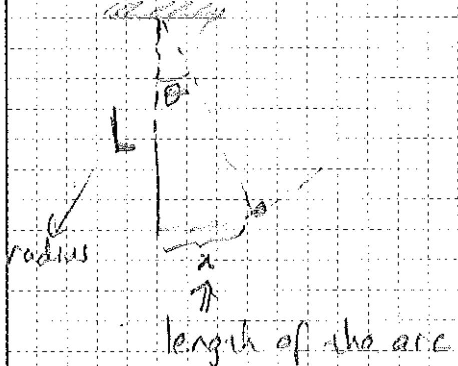
What is the equation of motion in the θ-direction?

No motion along the r-direction ⇒

$T = mg\cos\theta$

Motion along θ direction;

$-mg\sin\theta = F_\theta$



$\theta = \frac{x}{L}$, Consider small oscillations.
$\Rightarrow \theta$ a small angle,

Then $\sin\theta \approx \theta$

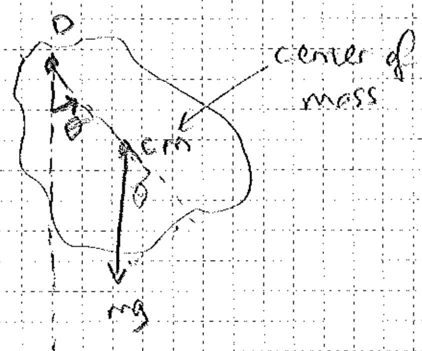
ma_θ $\leftarrow F_\theta \approx -mg\theta = -mg\frac{x}{L}$

$mL\frac{d^2\theta}{dt^2}$

$F_\theta = m\frac{d^2x}{dt^2} = -\underbrace{\frac{mg}{L}}_{k}x$

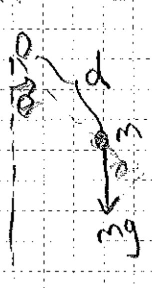
So, $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{g\cancel{m}}{L\cancel{m}}} = 2\pi f \Rightarrow \boxed{f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}}$, $\boxed{T = \frac{1}{f} = 2\pi\sqrt{\frac{L}{g}}}$

13.6. The Physical Pendulum:



Consider a body of arbitrary shape rotating about an axis through point O.

Represent the body as a point mass m at the center of mass.



Use the equation of motion in angular motion:

$\sum \tau_z = I\alpha_z$ → moment of inertia about the axis of rotation through O.

$\sum \tau_z = -mg\sin\theta\, d$

For small θ $\sin\theta \approx \theta \Rightarrow \sum \tau_z \approx -mgd\theta$

$\Rightarrow \sum \tau_z = -mgd\theta = I\frac{d^2\theta}{dt^2} \Rightarrow \frac{d^2\theta}{dt^2} = -\frac{mgd}{I}\theta$

In simple harmonic motion $a_x = -\frac{k}{m} x$

In physical pendulum $\frac{d^2\theta}{dt^2} = -\frac{mgd}{I} \theta$ ← Simple harmonic motion with $\frac{k}{m} = \frac{mgd}{I}$

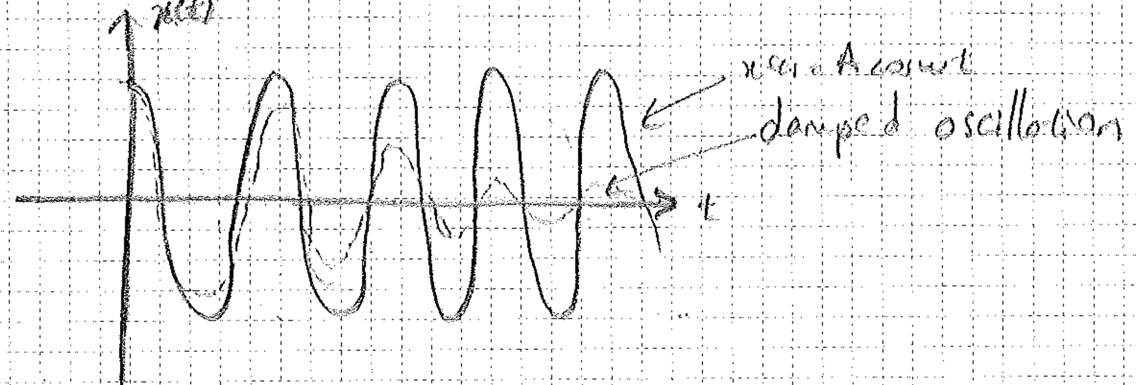
$\therefore$ $w = \sqrt{\frac{k}{m}} = \sqrt{\frac{mgd}{I}} = 2\pi f \Rightarrow \boxed{f = \frac{1}{2\pi} \sqrt{\frac{mgd}{I}}}$ $\boxed{T = 2\pi \sqrt{\frac{I}{mgd}}}$

Physical pendulum can be used to determine experimentally the moment of inertia I, about an axis of rotation.

13.7. Damped Oscillations:

In reality, practical oscillating systems do not oscillate forever. Real world systems always have dissipative forces, and oscillations die out with time.

Damped oscillations: Oscillations which have decreasing amplitudes caused by dissipative forces.



Mathematical model of damped oscillations:

$$\sum F_x = -kx - \underbrace{b v_x}_{\text{Additional force due to friction}}$$

$v_x \rightarrow \frac{dx}{dt}$

b: damping cons: describes the strength of the damping force

$\Rightarrow$ Newton's Second Law; $-kx - bv_x = ma_x$

$$\boxed{\Rightarrow -kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}}$$ Second order differential equation.

The solution for the differential equation:

$$\boxed{x(t) = A e^{-b/2m\, t} \cos(\omega' t + \phi)} \quad , \omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

This solution can be verified by direct substitution.

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$$x(t) = A e^{-\frac{b}{2m}t}\left(\frac{e^{i(\omega' t+\phi)} + e^{-i(\omega' t+\phi)}}{2}\right)$$

$$= A e^{i\phi} e^{-\frac{b}{2m}t} e^{i\omega' t}$$

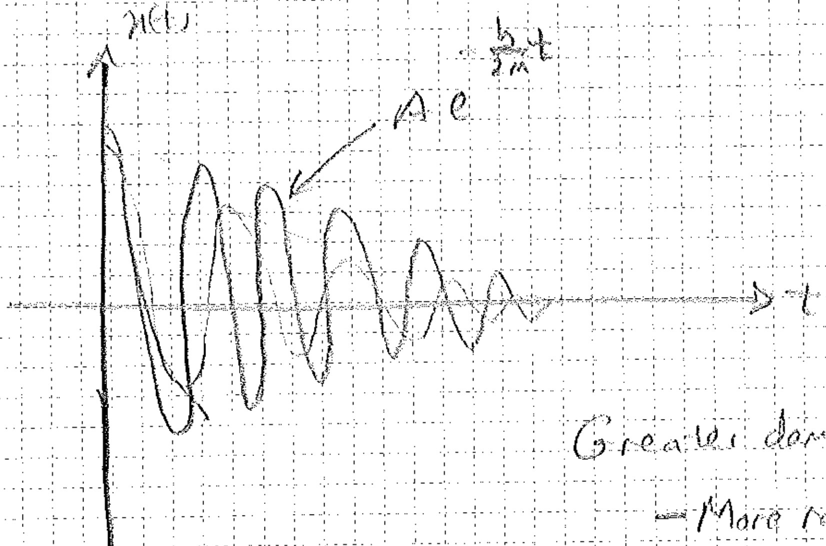
$$-k - b\left(i\omega' - \frac{b}{2m}\right) = m\left(i\omega' - \frac{b}{2m}\right)^2$$

$$\Rightarrow -k = \left(i\omega' - \frac{b}{2m}\right)\left[m\left(i\omega' - \frac{b}{2m}\right) + b\right]$$

$$\left(i\omega' - \frac{b}{2m}\right)\left[mi\omega' + \frac{b}{2}\right] = m\left(i\omega' - \frac{b}{2m}\right)\left(i\omega' + \frac{b}{2m}\right)$$

$$-k = m\left(-\omega'^2 - \frac{b^2}{4m^2}\right) \Rightarrow -\omega'^2 - \frac{b^2}{4m^2} = -\frac{k}{m}$$

$$\Rightarrow \boxed{\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}}$$



Greater damping ($b\uparrow$):

- More rapidly decreasing amplitude
- Larger T

There are indeed 3 regimes of oscillations depending on the value of the damping b:

i) $\frac{k}{m} - \frac{b^2}{4m^2} > 0 \Rightarrow b^2 < 4mk \Rightarrow \boxed{b < 2\sqrt{mk}}$

$\Rightarrow$ The oscillations are called underdamped.

The system oscillates with decreasing amplitude

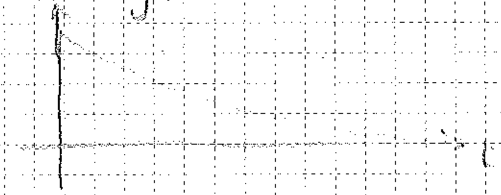
$$x(t) = A e^{-\frac{b}{2m}t}\cos(\omega' t + \phi)$$

(ii) $\frac{k}{m} - \frac{b^2}{4m^2} = 0 \Rightarrow \boxed{b = 2\sqrt{mk}}$

$\Rightarrow$ The oscillations are called critically damped.

$$x(t) = A e^{-\frac{b}{2m}t}$$

System returns to its equilibrium position without oscillating.



(iii) $\frac{k}{m} - \frac{b^2}{4m^2} < 0 \Rightarrow \boxed{b > 2\sqrt{mk}}$

$\Rightarrow$ The oscillations are called overdamped.

System returns to its equilibrium position without oscillating within a characteristic time different from the critically damped case.

$$x(t) = A e^{-\frac{b}{2m}t} e^{\sqrt{\frac{b^2}{4m^2} - \frac{k}{m}}\, t} + B e^{-\frac{b}{2m}t} e^{-\sqrt{\frac{b^2}{4m^2} - \frac{k}{m}}\, t}$$

Energy Conservation

In damped oscillations, energy does not remain constant. But decreases with time. Damping force is not a conservative force.

At a certain time t: $E = \frac{1}{2} m v_x^2 + \frac{1}{2} k x^2$

$$\frac{dE}{dt} = m v_x \frac{dv_x}{dt} + kx \frac{dx}{dt} = v_x \left(kx + m \frac{dv_x}{dt} \right)$$

$$m\frac{d^2x}{dt^2} + kx = -b\frac{dx}{dt}$$

$$\Rightarrow \frac{dE}{dt} = v_x\left(-b\frac{dx}{dt}\right) = \boxed{-bv_x^2}$$

$-bv_x^2 < 0$ always.
∴ Hence, E continuously decreases, with a nonuniform rate.

13.8. Forced Oscillations and Resonance:

If a periodic force is applied to a damped oscillator, the oscillator can maintain a constant amplitude oscillation.

The applied periodic force is called → Driving Force.

Forced Oscillations: Oscillation resulting when a periodically varying driving force is applied to a damped oscillator.

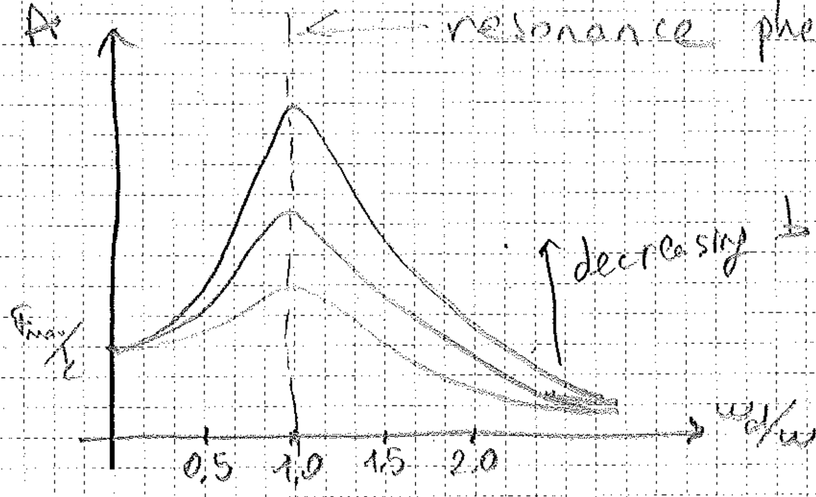
$$\sum F_x = F_{max}\cos(\omega_d t) - kx - bv_x = m\frac{d^2x}{dt^2}$$

↑ driving force frequency

Intuitively, one expects that driving will cause the biggest amplitude oscillations when the **driving frequency** matches the angular frequency ω' of the harmonic oscillator.

The solution to this differential equation reveals oscillations at frequency ω, with amplitude:

$$\boxed{A = \frac{F_{max}}{\sqrt{(k - m\omega_d^2)^2 + b^2\omega_d^2}}}$$



When the driving force is at the natural frequency of the oscillator, amplitude of the oscillation is maximum

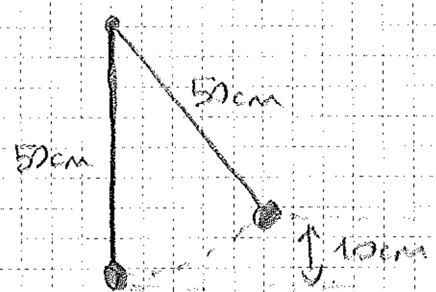
→ Resonance phenomenon,

Prob. 13.86:

The upper ball is released from rest, collides with the stationary lower ball and stick to it.

$m_{upper} = 2kg$

$m_{lower} = 3kg$



frequency, maximum angular disp. after the collision?

Frequency ; $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$, independent of mass.

$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2 \Rightarrow v = \sqrt{2gh}$ before the collision

After the collision $2kg\sqrt{2gh} = 5kg\, v_2 \Rightarrow v_2 = \frac{2}{5}\sqrt{2gh}$

At maximum angular disp: $\frac{1}{2}(\cancel{5kg})v_2^2 = \cancel{m}gh_2 \Rightarrow h_2 = \frac{1}{2g}\frac{2}{5}2gh = \frac{2}{5}h$

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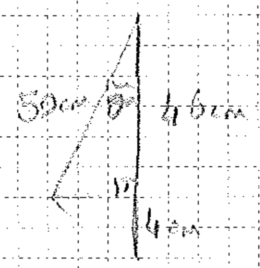
16



Maximum ang disp.

$\theta_{max} = \arccos\left(\frac{46}{50}\right)$