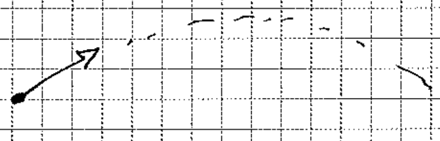
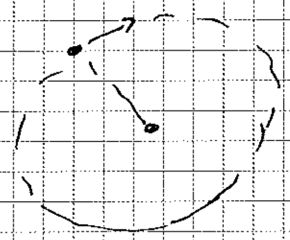
Chapter 3: Motion in 2 or 3 Dimensions

In order to analyze the motion of a particle in 2 or 3 dimensions, we have to extend the analysis developped in chapter 2.

Examples of motion in 2 or 3 dimensions are the projectile motion, or motion of a particle on a circle.

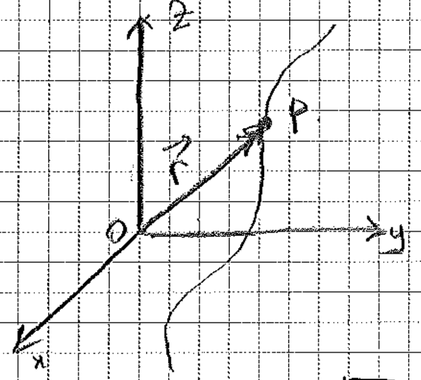




3.1. Position and Velocity Vectors:

We now develop the general formalism which will be used to analyse motion in 2 or 3D.

Consider a particle that is at a point P at a certain instant.



- You first define the coordinate sysem and its origin.

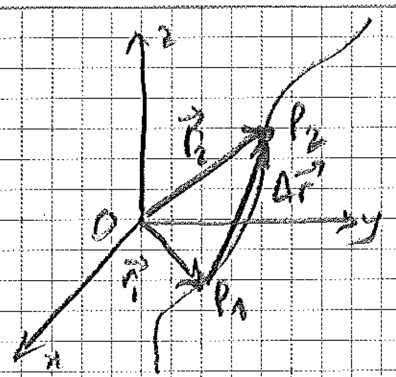
The position vector $\vec{r}$ is a vector that goes from the origin of the coordinate system to the point P.

$$\boxed{\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}}$$

x, y, z are the components of $\vec{r}$ along the x, y and z axes.

Compare this to x, (the position of the particle in one dimensional motion.

If the particle moves, it generally follows a curve path with time. Consider that it moves from a point P_1 to P_2 in a time interval Δt.



The displacement of the particle during this time interval is:

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

We can then define the average velocity:

$$\boxed{\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t}}$$ Average velocity

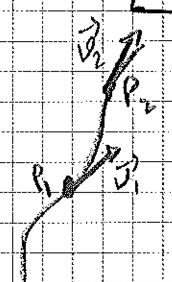
compare it with

$$v_{av-x} = \frac{\Delta x}{\Delta t}$$, in motion in 1D.

We now define the instanteneous velocity as the average velocity in the limit $\Delta t \to 0$:

$$\boxed{\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}}$$

instanteneous velocity. compare it to $v_x = \frac{dx}{dt}$, average velocity in 1D.



At every point along the path, the instanteneous velocity vector is tangent to the path at that point.

We can find the instanteneous velocity at any time by using the components of the position vector:

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \Rightarrow \boxed{\vec{v} = \frac{d\vec{r}}{dt} = \underbrace{\frac{dx}{dt}}_{v_x}\hat{i} + \underbrace{\frac{dy}{dt}}_{v_y}\hat{j} + \underbrace{\frac{dz}{dt}}_{v_z}\hat{k}}$$

$v_x = \frac{dx}{dt}$, $v_y = \frac{dy}{dt}$, $v_z = \frac{dz}{dt}$ are the components of the velocity vector.

The magnitude of $\vec{v}$ is given as: $\boxed{|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}}$ in 3D

in 2D: $\boxed{|\vec{v}| = \sqrt{v_x^2 + v_y^2}}$

→ From now on when we say "velocity" we will refer to "instantaneous velocity".

Ex. 3.1: ~~A particle has x, y components varying as~~

~~$x(t) =$~~

The x and y components of the position of a particle are given as:

$$x(t) = 2.0 - 0.25t^2$$
$$y(t) = 1.0t + 0.025t^3$$

a) What is the particle's coordinates and $|\vec{r}|$ at $t = 2.0\,s$?

$x(2) = 2 - 1 = 1m$

$y(2) = 2 + 0.025 \times 8 = 2.2m$ $\Rightarrow$ $\vec{r} = 1m\,\hat{i} + 2.2m\,\hat{j}$, $|\vec{r}| = \sqrt{(1)^2 + (2.2)^2} = 2.4m$

b) What are $\Delta\vec{r}$ and $\vec{v}_{av}$ between $t = 0.0\,s$ and $t = 2.0\,s$?

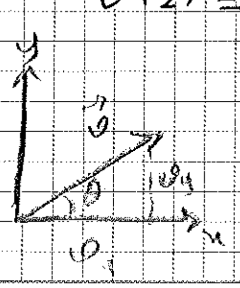
$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = x_2\hat{i} + y_2\hat{j} - (x_1\hat{i} + y_1\hat{j}) = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$$
$$= (x(2) - x(0))\hat{i} + (y(2) - y(0))\hat{j} = (1m - 2m)\hat{i} + (2.2m - 0m)\hat{j}$$
$$= -1m\,\hat{i} + 2.2m\,\hat{j} \quad //$$

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = -\frac{1}{2}\,\tfrac{m}{s}\,\hat{i} + 1.1\,\tfrac{m}{s}\,\hat{j} \quad //$$

c) Derive a general expression for the particle's instantaneous velocity, $\vec{v}$. What is $\vec{v}$ at $t = 2\,s$, express it in terms of magnitude and direction.

$$\vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = \frac{d}{dt}(2.0 - 0.25t^2)\hat{i} + \frac{d}{dt}(1.0t + 0.025t^3)\hat{j}$$
$$= (-0.5t)\hat{i} + (1.0 + 0.075t^2)\hat{j} \quad //$$

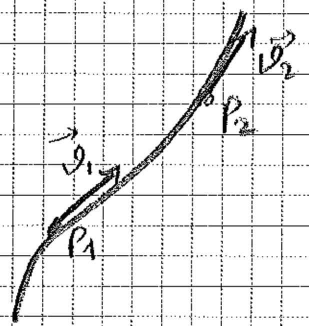
$$\vec{v}(2) = -1\,\tfrac{m}{s}\,\hat{i} + (1 + 0.3)\hat{j} = -1\,\tfrac{m}{s}\,\hat{i} + 1.3\,\tfrac{m}{s}\,\hat{j} \quad //$$



$$|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{1 + (1.3)^2} = 1.6\,\tfrac{m}{s} \quad //$$

$$\tan\theta = \frac{v_y}{v_x} = \frac{1.3\,m/s}{-1\,m/s} = -1.3 \Rightarrow \theta = 128° \quad //$$

3.2 Acceleration Vector



Definition of the acceleration vector is straight forward.

Consider a particle has instantaneous velocity $\vec{v}_1$ and $\vec{v}_2$ at times t_1 and t_2:

$$\rightarrow \boxed{\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}}$$ Average acceleration vector.

compare it with $a_{av\text{-}x} = \frac{\Delta v_x}{\Delta t} = \frac{v_{x_2} - v_{x_1}}{\Delta t}$ in motion in 1D.

Instantaneous acceleration is the average acceleration in the limit $\Delta t \rightarrow 0$.

$$\boxed{\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}}$$ Instanteneous acceleration

compare it with $a_x = \frac{dv_x}{dt}$.

~~If we know the components of the instanteneous velocity~~

We can determine the instanteneous acceleration from the components of the velocity vector.

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(v_x \hat{i} + v_y \hat{j} + v_z \hat{k}\right) = \underbrace{\frac{dv_x}{dt}}_{a_x} \hat{i} + \underbrace{\frac{dv_y}{dt}}_{a_y} \hat{j} + \underbrace{\frac{dv_z}{dt}}_{a_z} \hat{k}$$

$$= \underbrace{\frac{d^2x}{dt^2}}_{a_x} \hat{i} + \underbrace{\frac{d^2y}{dt^2}}_{a_y} \hat{j} + \underbrace{\frac{d^2z}{dt^2}}_{a_z} \hat{k}$$

$$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2} \quad , \quad a_y = \frac{dv_y}{dt} = \frac{d^2y}{dt^2} \quad , \quad a_z = \frac{dv_z}{dt} = \frac{d^2z}{dt^2}$$

$$v_x = \frac{dx}{dt} \quad , \quad v_y = \frac{dy}{dt} \quad , \quad v_z = \frac{dz}{dt}$$

$$x \quad , \quad y \quad , \quad z$$

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Ex: Consider the particle whose motion is described by:

$$\left.\begin{aligned} x(t) &= 2 - 0.25t^2 \\ y(t) &= t + 0.025t^3 \end{aligned}\right\}$$

a) What are the components of the average acceleration from t=0 to t=2s

$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j}$$

$$v_x(t) = -0.5t \Rightarrow v_x(2) = -1 \text{ m/s}, \quad v_x(0) = 0$$

$$v_y(t) = 1 + 0.075t^2 \Rightarrow v_y(2) = 1.3 \text{ m/s}, \quad v_y(0) = 1 \text{ m/s}$$

$$\Rightarrow \vec{a}_{av} = -\frac{1}{2} \text{ m/s}^2\, \hat{i} + \frac{0.3}{2} \text{ m/s}^2\, \hat{j} \quad //$$

b) What is the instantaneous acceleration at t = 2s?

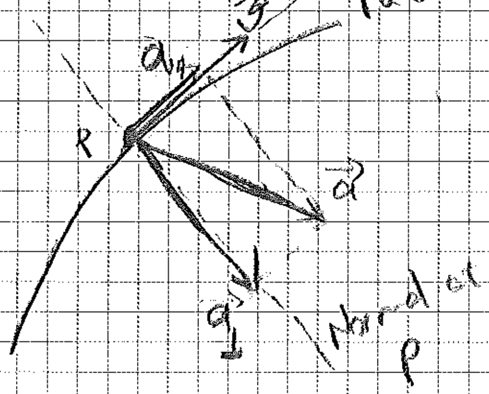
$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(-0.5t\,\hat{i} + (1 + 0.075t^2)\hat{j}\right)$$

$$= -0.5\,\hat{i} + 0.15t\,\hat{j}$$

$$\Rightarrow \boxed{\vec{a}(2) = -0.5 \text{ m/s}^2\, \hat{i} + 0.3 \text{ m/s}^2\, \hat{j}} \quad //$$

Parallel and Perpendicular Components of Acceleration:

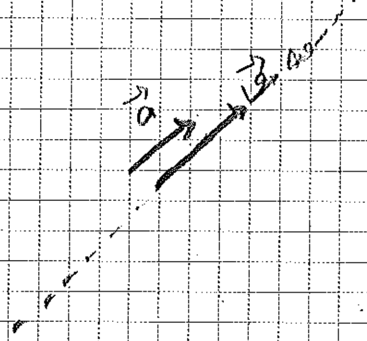
It is useful to describe the acceleration of a particle along a curved path in terms of components parallel and perpendicular to the velocity at each point.



$a_{\parallel}$ is the component of the acceleration vector parallel to velocity (along the direction of tangent at point P)

$a_{\perp}$ is the component of the acceleration vector perpendicular to the path (along the direction perpendicular to the tangent at point P) (along the normal to the path)

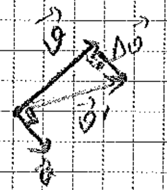
Consider the case when $a_\perp = 0$:



In this case the change in $\vec{v}$ during a small time interval Δt is a vector $\Delta\vec{v}$ in the same direction as $\vec{a}$.

Hence, the particle will follow a <u>linear</u> path. $|\vec{v}'| = |\vec{v}| + \underbrace{a\Delta t}_{1^{st}\ order}$

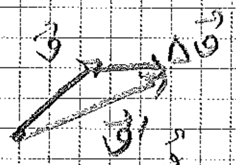
Consider the case when $a_\parallel = 0$:



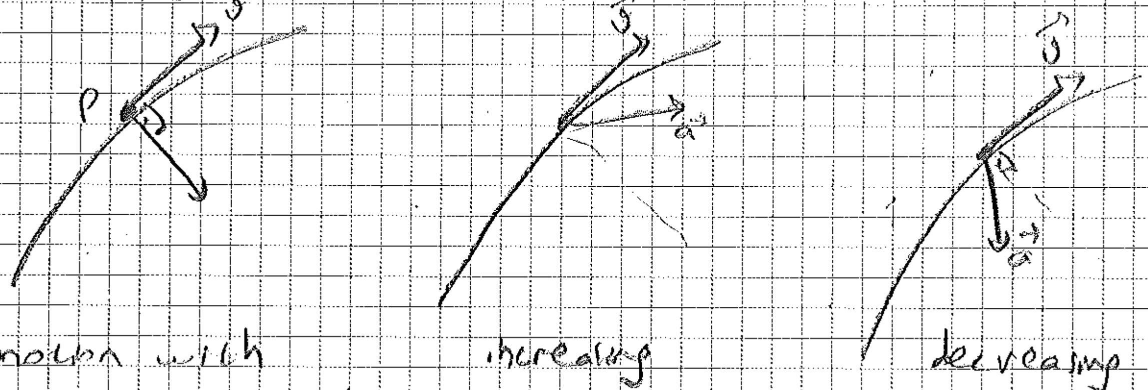
$\vec{v}' = \vec{v} + \Delta\vec{v} = \vec{v} + \vec{a}\Delta t \Rightarrow |\vec{v}'| = \sqrt{v^2 + a^2\Delta t^2} = v\sqrt{1 + \frac{a^2\Delta t^2}{v^2}} \underset{2^{nd}\ ord.}{\approx} v$

When $\vec{a}$ is perpendicular to $\vec{v}$, its effect is to change the direction of $\vec{v}$ but not its magnitude.

Consider the general case:



$\vec{v}' = \vec{v} + \Delta\vec{v} = \vec{v} + \vec{a}\Delta t = \vec{v} + \vec{a}_\parallel \Delta t + \vec{a}_\perp \Delta t$



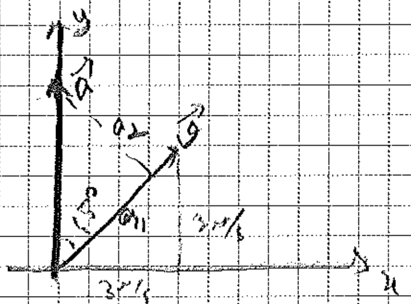
motion with constant speed | increasing speed | decreasing speed

If there is a parallel component of $\vec{a}$, speed is decreasing or increasing.

Ex: Consider that a particle's velocity and acceleration are given as:

$$\left.\begin{array}{l}\vec{v} = 3\,m/s\,\hat{i} + 3\,m/s\,\hat{j} \\ \vec{a} = 5\,m/s^2\,\hat{i}\end{array}\right\}$$ at a certain time instant.

What are $a_{\parallel}$ and $a_{\perp}$?



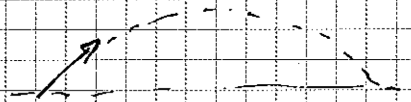
$$a_{\parallel} = |\vec{a}| \cos 45^\circ = \frac{5\sqrt{2}}{2}\ m/s^2$$

$$a_{\perp} = |\vec{a}| \sin 45^\circ = \frac{5\sqrt{2}}{2}\ m/s^2$$

~~3.3.1 A shot moves along a path shown below.~~

3.3. Projectile Motion:

Projectile motion is an ideal example to analyze motion in 2D.



A projectile is a particle that is given an initial velocity and then follows a path determined entirely by the effects of gravitational acceleration.

Trajectory is the path followed by a projectile.

In our idealized model we will
- represent the projectile as a single particle
- assume constant gravitational acceleration $\vec{g}$
- assume no air resistance and other effects.

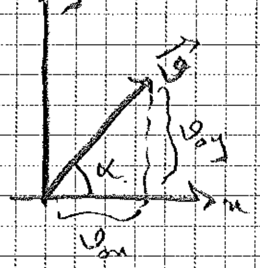
Projectile motion is a motion in 2D. We will analyze the motion as a combination of horizontal motion with constant velocity and vertical motion with constant acceleration.

$$a_x = 0 \qquad a_y = -g$$

$$v_x = v_{0x} \qquad v_y = v_{0y} - gt$$

$$x = x_0 + v_{0x} t \qquad y = y_0 + v_{0y} t - \frac{1}{2} g t^2$$

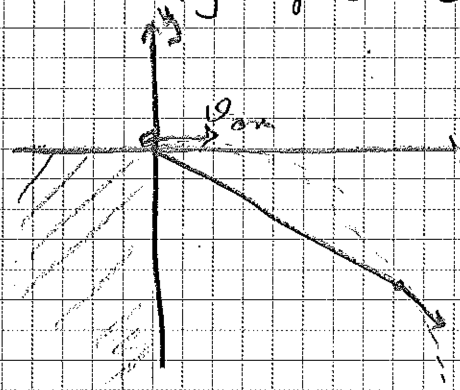
v_{0x} and v_{0y} are x and y components of the initial velocity of the projectile.



It is always convenient to choose the initial point of the projectile as the origin of the coordinate system so that $(x_0, y_0) = (0,0)$ //

These formulas are sufficient for the analysis of the projectile motion,

Ex 3.7: A motorcycle rider rides off the edge of a cliff with a horizontal velocity with magnitude $9\,m/s$.
Find the motorcycle's position, distance from the edge of the cliff and velocity after $0.5\,s$.



$v_{0x} = 9\,m/s$, $v_{0y} = 0\,m/s$

$\Rightarrow x(t) = v_{0x} t = (9\,m/s)(0.5\,s) = 4.5\,m$ //

$y(t) = -\frac{1}{2} g t^2$

$= -\frac{1}{2} \times 9.8\,m/s^2 \times \left(\frac{1}{2}s\right)^2 \approx -1.2\,m$ //

distance from the edge of the cliff.

$$r = \sqrt{x^2 + y^2} = \sqrt{(4.5\,m)^2 + (1.2\,m)^2} = 4.7\,m$$

Velocity: $v_x = 9\,m/s$, $v_y = -gt = -4.9\,m/s$

$$\Rightarrow \vec{v} = 9\,m/s\,\hat{i} - 4.9\,m/s\,\hat{j}$$

Ex 3.8; Height and range of a baseball

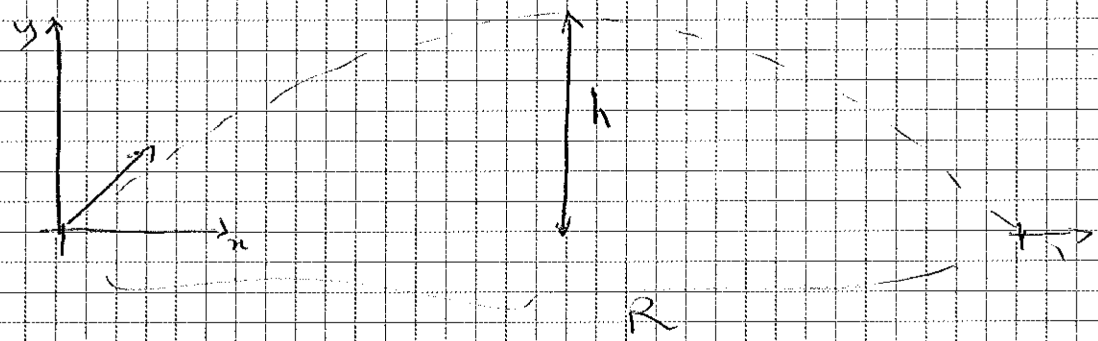
$v_0 = 37.0\,m/s$, $\alpha_0 = 53.1^\circ$

a) Pos. of the ball, magn. at $t = 2s$

$\vec{r}$ $|\vec{v}|$, $\vec{v}$

b) The time at which ball reaches the highest point of its flight and its height at this point

c) horizontal range R



$v_{0x} = v_0 \cos\alpha = 22.2\,m/s$, $v_{0y} = v_0 \sin\alpha = 29.6\,m/s$

(a) $x = v_{0x} t = 44.4\,m$

$y = v_{0y} t - \frac{1}{2} g t^2 = 39.6\,m$

$v_x = v_{0x} = 22.2\,m/s$ $\Rightarrow$ $|\vec{v}| = \sqrt{(22.2)^2 + (10)^2} = 24.3\,m/s$

$v_y = v_{0y} - g t = 10\,m/s$ $\alpha = \arctan\left(\frac{10}{22.2}\right) = 24.2^\circ$

(b) At max point $v_y = 0 = v_{y0} - g t_1$ $\Rightarrow$ $t_1 = \frac{v_{0y}}{g} = 3.02\,s$

derive

$h = v_{0y} \underset{t_1}{t} - \frac{1}{2} g t^2 = 44.7\,m$ $h = v_{0y} \frac{v_{0y}}{g} - \frac{1}{2} g \frac{v_{0y}^2}{g^2} = \boxed{\frac{1}{2} \frac{v_{0y}^2}{g}}$

(c) $y = 0 = v_{0y} t_2 - \frac{1}{2} g t^2 \Rightarrow t_2 \left(v_{0y} - \frac{1}{2} g t_2\right) = 0 \Rightarrow t_2 = 0$

$t_2 = \frac{2 v_{0y}}{g}$

Max height when $\alpha = 45^\circ$

$\Rightarrow R = v_{0x} t_2 = \boxed{134\,m}$, $R = v_{0x} 2 t_1 = \boxed{\frac{2 v_{0x} v_{0y}}{g}} = \frac{2 v_0^2 \cos\alpha \sin\alpha}{g} = \boxed{\frac{v_0^2 \sin 2\alpha}{g}}$

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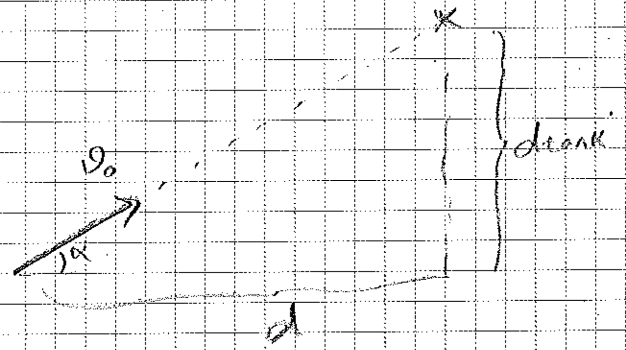
Example 3.9:

Target is released at the same time as the rifle is fired. Will the bullet hit the target?



The bullet reaches $x=d$ at time:

$$v_{0x}t = d \Rightarrow t = \frac{d}{v_{0x}} = \frac{d}{v_0\cos\alpha}$$

At this time:

$$y_{target} = d\tan\alpha - \frac{1}{2}gt^2$$

$$y_{bullet} = v_{0y}t - \frac{1}{2}gt^2 = v_0\sin\alpha\frac{d}{v_0\cos\alpha} - \frac{1}{2}gt^2 = d\tan\alpha - \frac{1}{2}gt^2$$

$\therefore$ $y_{target} = y_{bullet}$ independent from v_0. v_0 should be large that the bullet does not hit the ground before:

$$\frac{d}{v_0\cos\alpha} < \sqrt{\frac{2d\tan\alpha}{g}} \Rightarrow v_0\cos\alpha > d\sqrt{\frac{g}{2d\tan\alpha}}$$

$$\Rightarrow v_0 > \frac{1}{\cos\alpha}\sqrt{\frac{gd}{2\tan\alpha}}$$

$$v_0 > \sqrt{\frac{gd}{\sin 2\alpha}}$$

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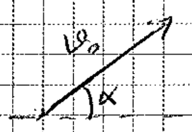
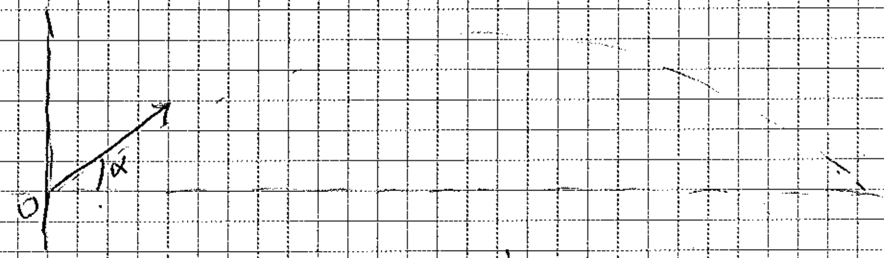
Example 3.10: Height and range of a projectile.



derive general expressions for the max. height (h) and horizontal range (R)

What value of α_0 gives max. height?

What value gives max. horizontal range?



$$v_y = 0 \Rightarrow v_0 \sin\alpha_0 - g t_1 = 0 \Rightarrow \boxed{t_1 = \frac{v_0 \sin\alpha_0}{g}}$$

$$h = v_0 \sin\alpha \left(\frac{v_0 \sin\alpha_0}{g}\right) - \frac{1}{2} g \left(\frac{v_0 \sin\alpha_0}{g}\right)^2 = \boxed{\frac{v_0^2 \sin^2\alpha_0}{2g}} = h$$

, h is minimum when $\alpha_0 = 90°$

To find the general exp. for R, first find the time t_2 at which $y = 0$:

$$(v_0 \sin\alpha_0) t_2 - \frac{1}{2} g t_2^2 = t_2 \left(v_0 \sin\alpha_0 - \frac{1}{2} g t_2\right) = 0$$

$$\Rightarrow t_2 = 0 \quad \text{or} \quad \boxed{t_2 = \frac{2 v_0 \sin\alpha_0}{g}}$$

$$\Rightarrow R = v_{0x} t_2 = v_{0x} \frac{2 v_0 \cos\alpha_0 \, v_0 \sin\alpha_0}{g} = \boxed{\frac{v_0^2 \sin 2\alpha_0}{g}} = R$$

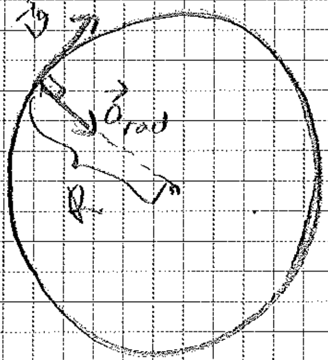
Maximum when $2\alpha_0 = 90°$

$\Rightarrow \boxed{\alpha = 45°}$

3.4. Motion in a Circle

Uniform circular motion:

When a particle moves in a circle with constant speed, the motion is called uniform circular motion.

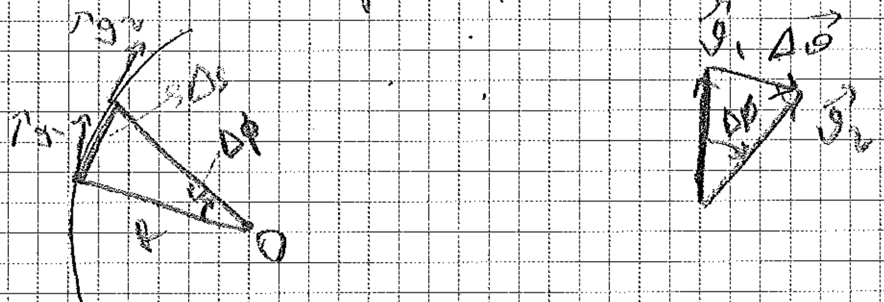


- Since the particle moves with constant speed, there is no parallel component of acceleration.

- The acceleration vector at each point in the circular path is directed toward the center point.

$$\vec{a}_{rad} \perp \vec{v}$$

Consider an infinitesimal rotation of the particle along the circle with constant speed:



$$|\vec{v}_1| = |\vec{v}_2| = v$$

We want to find an expression for $a_{rad} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$

$$\overbrace{\Delta s}^{\text{length of the arc}} = R\Delta\phi \quad , \quad \Delta s = v\Delta t$$

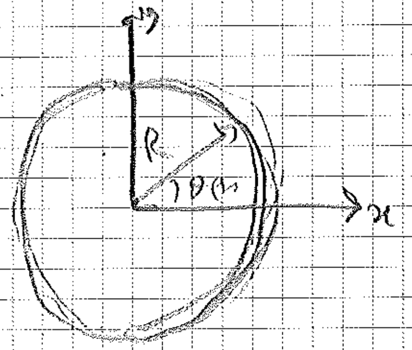
$$\Delta v = \underbrace{v\Delta\phi}_{\text{length of the arc}}$$

$$\Rightarrow v\Delta t = R\,\Delta\phi = R\,\frac{\Delta v}{v}$$

$$\Rightarrow \frac{\Delta v}{\Delta t} = \frac{v^2}{R}$$

$$\Rightarrow \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{v^2}{R} \Rightarrow \boxed{a_{rad} = \frac{v^2}{R}}$$

Uniform Circular Motion:



A particle moving in a circle with constant speed.

$x(t) = R\cos(\theta(t)),\ y(t) = R\sin(\theta(t)) \Rightarrow \vec{r} = R\cos\theta\,\hat{i} + R\sin\theta\,\hat{j}$

$\vec{v} = \frac{d\vec{r}}{dt} = -R\sin\theta \frac{d\theta}{dt}\hat{i} + R\cos\theta\frac{d\theta}{dt}\hat{j}$, $vT = 2\pi R$

$v = \frac{2\pi R}{T}$

$= -R\sin\theta\,w\,\hat{i} + R\cos\theta\,w\,\hat{j}$

$wT = 2\pi \Rightarrow w = \frac{2\pi}{T} = \frac{d\theta}{dt}$

$\vec{a} = \frac{d\vec{v}}{dt} = -Rw^2\cos\theta\,\hat{i} - Rw^2\sin\theta\,\hat{j}$

$= -w^2(\underbrace{R\cos\theta\,\hat{i} + R\sin\theta\,\hat{j}}_{\vec{r}}) \Rightarrow \vec{a} = -w^2\vec{r}$, in the radial direction

$\Rightarrow |a| = w^2 R$

$= \boxed{\frac{v^2}{R}}$

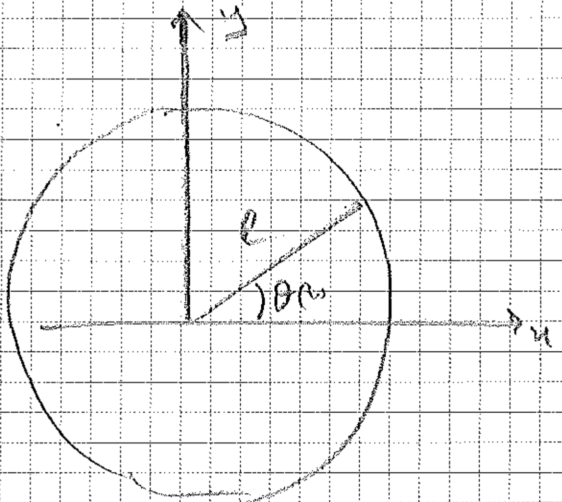
Remark: Uniform circular motion:

A particle moves in a circle with cons speed / A particle moves with a cons acceleration in radial direction

$v = \frac{2\pi R}{T} \Rightarrow |a| = \frac{4\pi^2 R}{T^2}$

//

Non-Uniform Circular Motion



A particle moving in a circle with varying speed.

$$\vec{r} = R\cos\theta\,\hat{i} + R\sin\theta\,\hat{j}$$

$$\vec{v} = -R\omega\sin\theta\,\hat{i} + R\omega\cos\theta\,\hat{j} \Rightarrow v = \omega R$$

$$\vec{a} = -R\omega^2\cos\theta\,\hat{i} - R\omega^2\sin\theta\,\hat{j} - R\sin\theta\frac{d\omega}{dt}\,\hat{i} + R\cos\theta\frac{d\omega}{dt}\,\hat{j}$$

$$= \underbrace{-\omega^2\vec{r}}_{\vec{a}_{rad}} + \underbrace{\frac{d\omega}{dt}\frac{\vec{v}}{\omega}}_{\vec{a}_{tan}}$$

$$\boxed{|\vec{a}_{rad}| = \frac{v^2}{R}} \quad , \quad |\vec{a}_{tan}| = \frac{d\omega}{dt}\frac{v}{\omega}$$

$$= \frac{d\omega}{dt}R = \boxed{\left|\frac{dv}{dt}\right|}$$

In uniform circular motion, the direction of $\vec{a}$ is perpendicular to $\vec{v}$ and inward along the radius. Because the acceleration is always directed toward the center of the circle, it is also called as "centriputal acceleration".

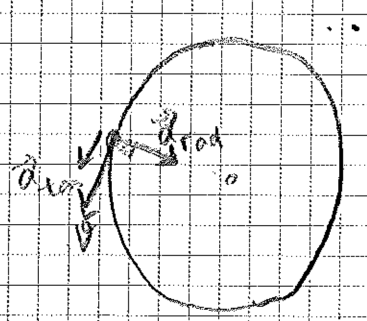
Remark: Uniform circular motion:

A particle moves in a circle with constant speed / A particle moves with a constant acceleration in radial direction

We can also express a_{rad} in terms of the period of motion:

$$v = \frac{2\pi R}{T} \Rightarrow \boxed{a_{rad} = \frac{4\pi^2 R}{T^2}}$$

Non-Uniform Circular Motion



When a particle moves in a circle with varying speed, it makes non-uniform circular motion.

In that case both the radial and tangential components of acceleration are nonzero.

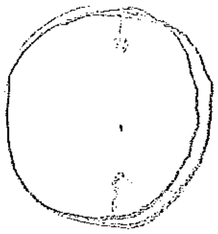
$$\vec{a}_{tan} \neq 0, \quad \vec{a}_{rad} \neq 0$$

At a given instant, these components of acceleration are given by:

$$\boxed{\vec{a}_{rad} = \frac{v^2}{R}}, \quad \boxed{a_{tan} = \frac{d|\vec{v}|}{dt}}$$

Prob [illegible] A wheel with radius 16 cm is rotating about a horizontal axis through its centre. Consider a point P on the rim [illegible]

Mag. and direction of [illegible]



a) [illegible]

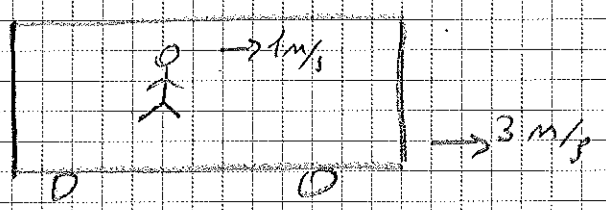
b) [illegible]

c) period of rotation [illegible] $\frac{[illegible]}{12}$

3.5. Relative Velocity:

In general, when two observers measure the velocity of a moving body, they obtain different results if one observer is moving relative to the other. The velocity seen by a particular observer is called the velocity relative to that observer.

in One Dimension:

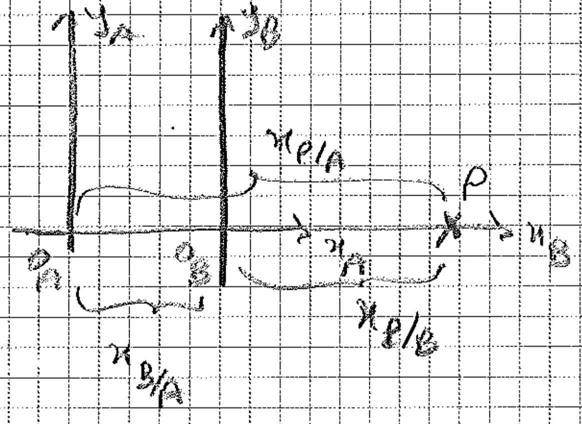


Consider a person moving with a velocity of 1 m/s in +x direction in a train moving with a velocity of 3 m/s in +x direction.

A passenger sitting in the train sees the person moving with a velocity of 1 m/s

A person standing outside the train sees " " " " " 4 m/s

Let us formulate this observation. Consider two ~~frames of reference~~ coordinate systems (frames of reference)



$$x_{P/A} = x_{B/A} + x_{P/B}$$

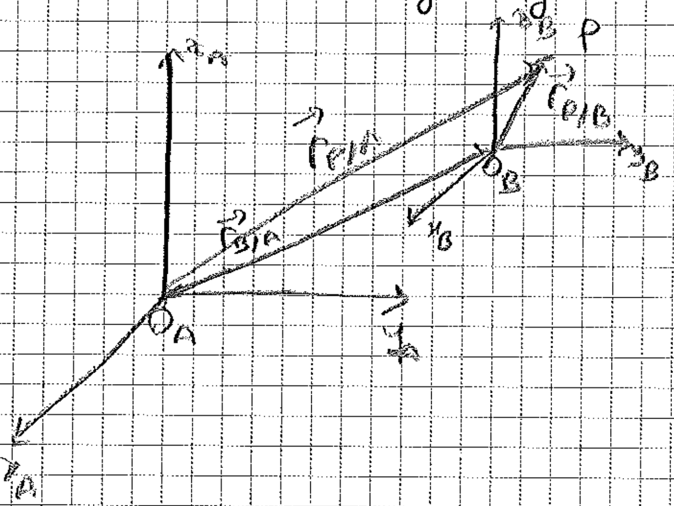
Position of point P in frame of reference A

$\Rightarrow$ Take the derivative of both sides with respect to time:

$$\frac{dx_{P/A}}{dt} = \frac{dx_{B/A}}{dt} + \frac{dx_{P/B}}{dt}$$

$$\Rightarrow \boxed{v_{P/A} = v_{B/A} + v_{P/B}}$$

This derivation is generally true in 2D or 3D



$$\vec{r}_{P/A} = \vec{r}_{B/A} + \vec{r}_{P/B}$$

$$\Rightarrow \boxed{\vec{v}_{P/A} = \vec{v}_{B/A} + \vec{v}_{P/B}}$$

Galilean transformations:

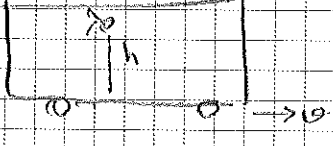
$$\vec{r}_{P/A} = \vec{r}_{B/A} + \vec{r}_{P/B}, \quad \vec{v}_{P/A} = \vec{v}_{B/A} + \vec{v}_{P/B}, \quad \vec{a}_{P/A} = \vec{a}_{B/A} + \vec{a}_{P/B}$$

If $v_{B/A}$ is very large ($\frac{v}{c} < 1$, $c = 3.0 \times 10^8$ m/s) Galileo transformations are not valid.

For const. $v_{B/A}$

$$\left\{\begin{array}{l} x_{P/A} = \gamma\left(x_{P/B} + \beta c t_B\right), \quad \gamma = \dfrac{1}{\sqrt{1 - \frac{v_{BA}^2}{c^2}}}, \quad \beta = \dfrac{v_{BA}}{c} \\ y_{P/A} = y_{P/B}, \quad z_{P/A} = z_{P/B} \\ c t_A = \gamma\left(c t_B + \beta x_{P/B}\right) \end{array}\right.$$

Lorentz Transformations



$$\Delta t = \frac{h}{c}, \quad \Delta t = \frac{\sqrt{h^2 + (v\Delta t)^2}}{c} \rightarrow \Delta t = \sqrt{1 - \frac{v^2}{c^2}}\,\Delta t$$

moving clocks run slower!
time dilation.

$$\Delta t_1 = \frac{\Delta x + v\Delta t_1}{c}, \quad \Delta t_2 = \frac{\Delta x - v\Delta t_2}{c}$$

$$\Delta t_1 = \frac{\Delta x}{c-v}, \quad \Delta t_2 = \frac{\Delta x}{c+v}$$

$$\Delta t = \Delta t_1 + \Delta t_2 = 2\frac{\Delta x}{c}\cdot\frac{1}{1-\frac{v^2}{c^2}}, \quad \Delta\tilde{t} = \sqrt{1-\frac{v^2}{c^2}}\,\Delta t$$

$$\boxed{\Delta\tilde{t} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\,\Delta x}$$

Moving objects are shortened.

Lorentz contraction