Chapter 4: Newton's Laws of Motion

We have seen how to analyze the motion of a particle. This was called kinematics, we understood the quantities: displacement, velocity and acceleration.

Dynamics: Now we will describe the underlying causes of motion. We will introduce the concepts of force and mass.

4.1 Force and Interactions:

Force describes the interaction between two bodies. When you drag a box, a force is applied to the box.

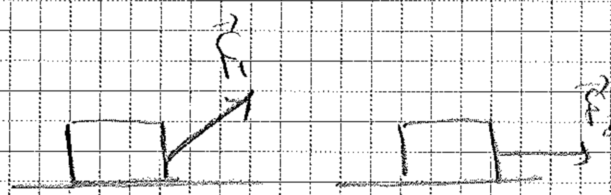
Examples of Forces:

Contact-Force: When a force involves direct contact between two bodies.

Push-pulls with your hand
Friction force
The force of a rope pulling on a block

Long-Range Force: Forces that act even when two bodies are separated by empty space.

Magnetic Force
Gravity $\longrightarrow$ The force of gravitational attraction that the earth exerts on your body is called weight.

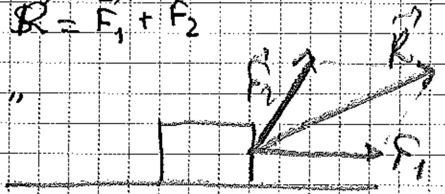


$\vec{F}_1 \neq \vec{F}_2$

Force is a vector quantity.
The SI unit of the magnitude of force is "newton" (N).

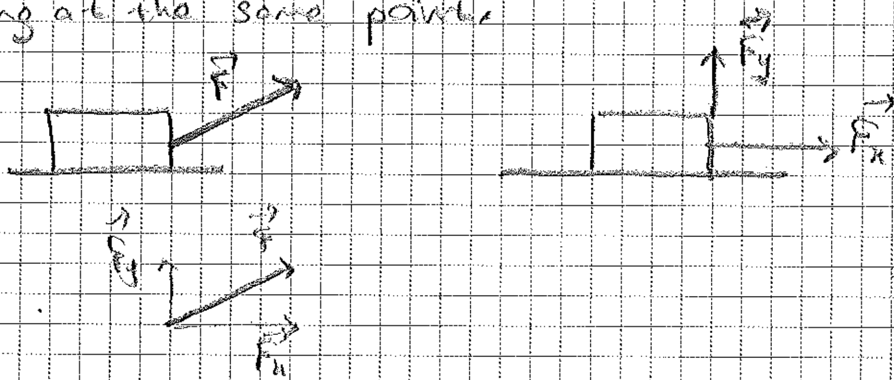
Properties: When two forces $\vec{F}_1$ and $\vec{F}_2$ act at the same time at a point A of a body, the effect on the body's motion is the same as the effect of a single force $\vec{R}$ equal to the vector sum of the original forces: $\vec{R} = \vec{F}_1 + \vec{F}_2$

"Superposition of Forces"



$\vec{R} = \vec{F}_1 + \vec{F}_2$

Conversely, any force can be replaced by its component vectors, acting at the same point.



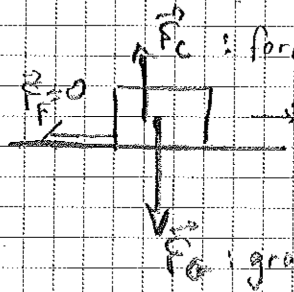
Generally, net force acting on a body is the vector sum of all the forces acting on it.

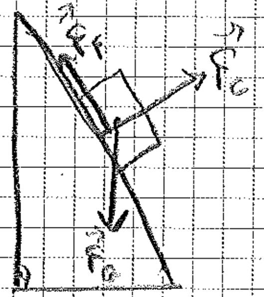
$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots = \sum \vec{F}$$
$$= (F_{1x} + F_{2x} + F_{3x} + \dots)\,\hat{i} + (F_{1y} + F_{2y} + F_{3y} + \dots)\,\hat{j}$$
$$= (\Sigma F_x)\,\hat{i} + (\Sigma F_y)\,\hat{j}$$

4.2. Newton's First Law:

"When no net force acts on a body, the body either remains at rest or moves with constant velocity in a straight line."

Consider bodies at equilibrium $\sum \vec{F} = 0$



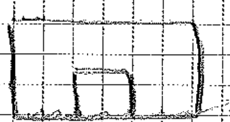


Consider friction between the body and the floor, if the body is at rest $\sum \vec{F} = 0$

$$\vec{F}_G + \vec{F}_C + \vec{F}_F = 0$$

When a body is acted by no forces, or by several forces such that their vector sum is zero, we say that the body is at equilibrium $\sum \vec{F} = 0$. [illegible] ! If a body is at rest or moving with constant velocity, it is at equilibrium.

Inertial Frame of Reference:



Consider a box in a car lying on a frictionless surface. Consider that the car moves with a constant acceleration.

According to an observer in the car, the box will start ~~moving~~ accelerating while the net force applied on the car is 0.

⇒ This is against the Newton's First Law.

Therefore an accelerating car is not a suitable frame of reference for Newton's first Law.

<u>Generally</u>: A frame of reference in which Newton's First Law is valid is called an inertial frame of reference.

– The earth is an inertial frame of reference.

– Any frame of reference which moves with a constant velocity with respect to eart is also an inertial frame of reference.

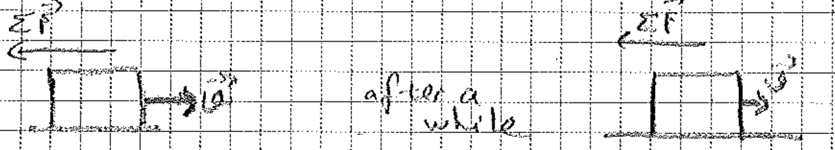
$$\text{Remember } \vec{v}_{P/A} = \vec{v}_{P/B} + \underbrace{\vec{v}_{B/A}}_{\text{constant}}$$

$$\text{if } \vec{v}_{P/A} \text{ is constant} \Rightarrow \vec{v}_{P/B} \text{ is constant.}$$

Hence, If a body is at rest or moving at a constant velocity in frame of reference A, it is also at rest or moving at a constant velocity in B.

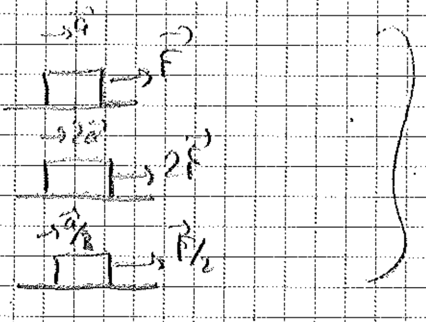
<u>4. 3. Newton's Second Law</u>:

When the net force acting on a body is not zero, the body accelerates.



velocity will decrease ⇒ so, there is a net acceleration.

Experiments show that if the magnitude of the net force applied on a body is constant, the magnitude of the acceleration of the body is also constant.



Newton's Second Law of Motion:

If a net external force is applied on a body, the body accelerates with an acceleration in the direction of the net force. The magnitude of acceleration is proportional to the magnitude of the net force.

$$\sum \vec{F} \propto \vec{a} \Rightarrow \boxed{\sum \vec{F} = \underbrace{m}_{\text{Proportionality constant}} \vec{a}}$$

Therefore mass is the proportionality constant between force and acceleration.

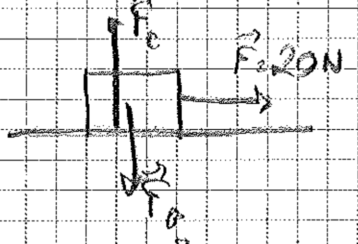
$$\sum \vec{F} = \left(\sum F_x\right)\hat{i} + \left(\sum F_y\right)\hat{j} + \left(\sum F_z\right)\hat{k} = m\left(a_x\hat{i} + a_y\hat{j} + a_z\hat{k}\right)$$

$$\Rightarrow \sum F_x = m a_x \ , \ \sum F_y = m a_y \ , \ \sum F_z = m a_z$$

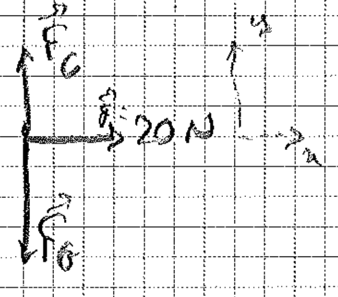
The SI unit of mass is kg.

$$(N) = (kg)\left(\frac{m}{s^2}\right)$$

Ex 4.4: A worker applies a constant horizontal force with magnitude 20N to a box with mass 40kg on a level floor with negligible friction. What is the acceleration of the box?



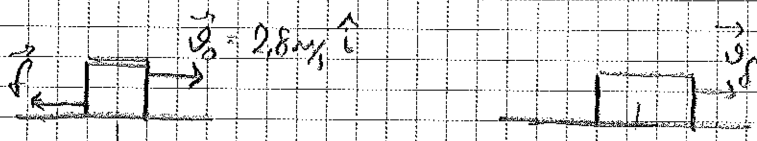
Body diagram:
Consider the body as a point and draw the force vectors applied on the point.

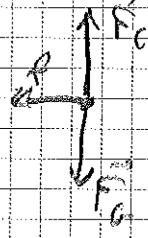


$$\sum \vec{F} = \vec{F} = (20N)\hat{i} = m\vec{a}$$

$$\Rightarrow \boxed{\vec{a} = 0.5 \frac{m}{s^2} \hat{i}}$$

Example 4.5: A body with a mass 0.45kg starts its motion on a ~~frictionless~~ horizontal surface with an initial velocity of 2.8 m/s. If the ~~surface~~ body slides for 1m before coming to rest, what is the magnitude and direction of the friction force?



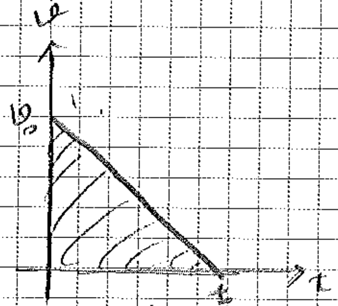


$\Delta x = 1m$

This is motion with constant acceleration.

$\vec{v}_0 = 2.8 m/s \hat{i}$

$\sum \vec{F} = m\vec{a} \Rightarrow \vec{f} = m\vec{a} \Rightarrow \vec{a} = \frac{\vec{f}}{m}$, $\boxed{a_x = \frac{f}{m}}$ in the x direction.



Area $= \Delta x = \frac{1}{2} v_0 t$

$a_x = -\frac{v_0}{t} \Rightarrow t = -\frac{v_0}{a_x}$

$\Rightarrow \Delta x = -\frac{1}{2}\frac{v_0^2}{a_x} \Rightarrow a_x = -\frac{1}{2}\frac{v_0^2}{\Delta x} = -\frac{1}{2}\frac{(2.8)^2}{(1)}$

$= -3.9 m/s^2$

$\Rightarrow \vec{f} = -a_x m \hat{i} = (-3.9 \times 0.45)\hat{i} = \boxed{-1.8 N \hat{i}}$

Prob. 4.12: A box with mass of 32.5 kg is initially at rest and it is acted by a constant net horizontal force of 140N.

a) What is the acceleration?

b) How far does the box travel in 10s?

c) What is it's speed at the end of 10 s?

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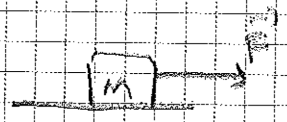


$m = 32,5\,kg, \quad \vec{F} = 140N\,\hat{i}$

(a) $\vec{a} = \frac{\vec{F}}{m} = \frac{140N}{32.5kg}\,\hat{i} \approx 4.3\ m/s\,\hat{i}$

(b) Motion with constant acceleration;

$$x(t) = \frac{1}{2}at^2 = \frac{1}{2}(4.3)\times(10)^2 = 21.5\,m$$

(c) $v(t) = v_0 + at = 0 + (4.3)\times(10) = 43\,m/s \qquad \vec{v} = (43\,m/s)\,\hat{i}$

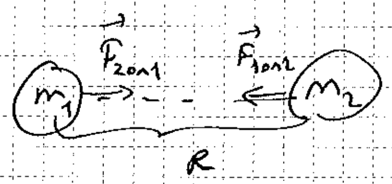
4.4. Mass and Weight

Weight: The force of the earth's gravitational attraction for a body.

The relationship between mass and weight is:

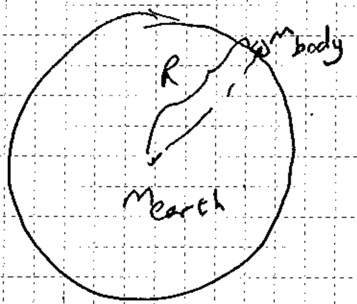
$$\underbrace{\vec{W}}_{\vec{F}} = m \underbrace{\vec{g}}_{\vec{a}}$$

Underlying cause of law is the attraction force between two masses.



$$F_{1on2} = G \frac{m_1 m_2}{R^2}$$

When we consider the earth and a small body:



$$F = G \frac{M_{earth} m_{body}}{R^2} \quad , \; R \approx R_{earth}$$

$$\Rightarrow F \approx \underbrace{\left(\frac{G M_{earth}}{R_{earth}^2}\right)}_{\text{constant} \equiv g} m_{body} = m_{body} g = \text{Weight}$$

For problems on earth: $g = 9.8 \, m/s^2$.

The value of g varies from point to point on the earth's surface, from about 9.78 to $9.82 \, m/s^2$.

because the earth is not perfectly spherical.

On the surface of the moon:

$g = 1.62 \, m/s^2 \Rightarrow$ 80 kg person will weigh;

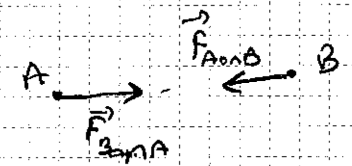
784 N on earth.

130 N on the moon.

4.5. Newton's Third Law:

Whenever two bodies interact, the two forces that they exert on each other are always equal in magnitude and opposite in direction.

$\Rightarrow$ This is Newton's third law of motion.



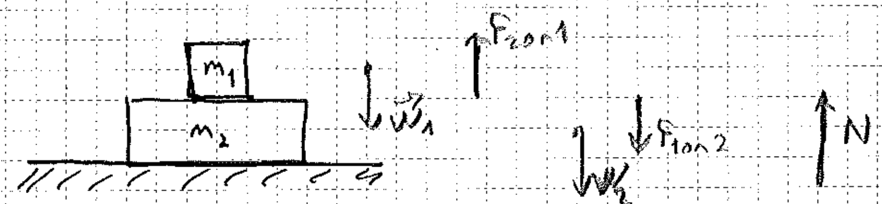
$$\boxed{\vec{F}_{AonB} = -\vec{F}_{BonA}}$$

If body A exerts a force on body B $\longrightarrow$ action

body B exerts a force on body A $\longrightarrow$ reaction

These two forces have <u>the same magnitude</u> but are <u>opposite in direction.</u>

Example 4.9: Consider two objects with masses m_1 and m_2 staying at rest. Identify the forces.



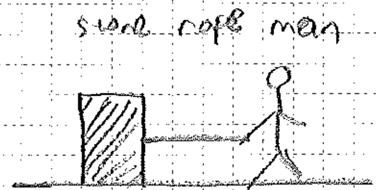
$N = W_1 + W_2$ //

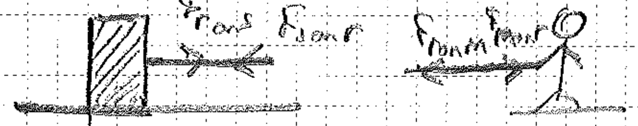
$W_1 = F_{2on1}$ //

$W_2 + F_{1on2} = N = W_1 + W_2 \Rightarrow F_{1on2} = W_1$ //

Example 4.10:

A person drags a block.
How are the various forces related?
Action reaction pairs?





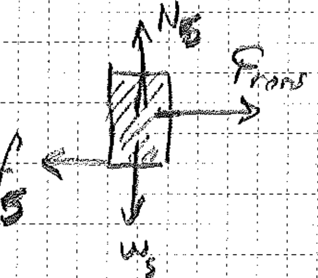
$$\boxed{\vec{F}_{rons} = -\vec{F}_{sonr} \;,\quad \vec{F}_{ronm} = -\vec{F}_{monr}}$$

$\vec{F}_{rons}$ & $\vec{F}_{sonr}$, $\vec{F}_{ronm}$ & $\vec{F}_{monr}$ form action-reaction pairs.

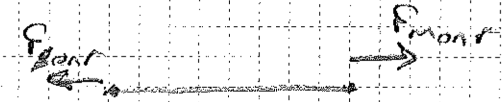
Action reaction pairs always act on different bodies.

Ex. 4.11:

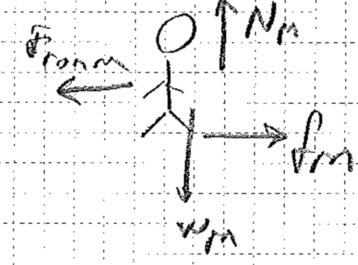
Now consider that the man starts pulling the rope and the stone starts moving, what are the forces,



$$F_{rons} - f_s = m a_s$$



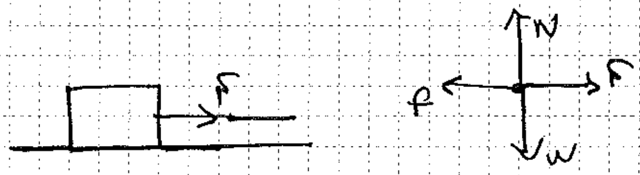
$$F_{monr} - F_{sonr} = m a_r$$

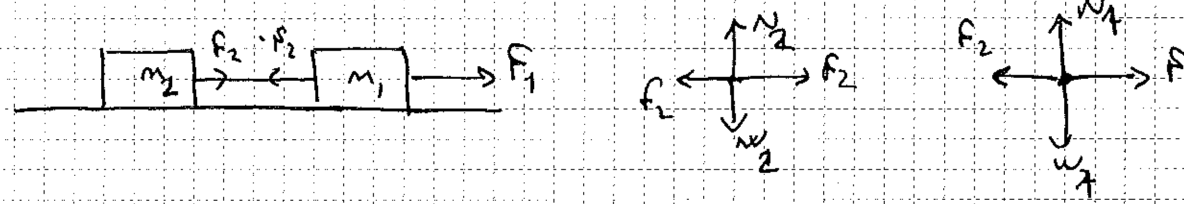


$$F_{ronm} - f_m = 0$$

4.6. Free-Body Diagram:

A free-body diagram: A diagram showing the chosen body by itself, "free" of its surroundings, with vectors drawn to show the magnitudes and directions of all the forces applied to the body.



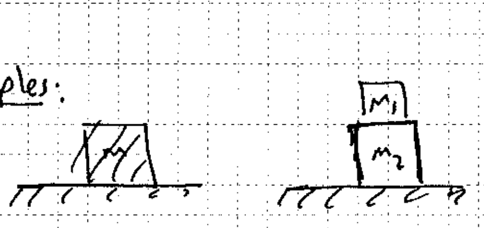


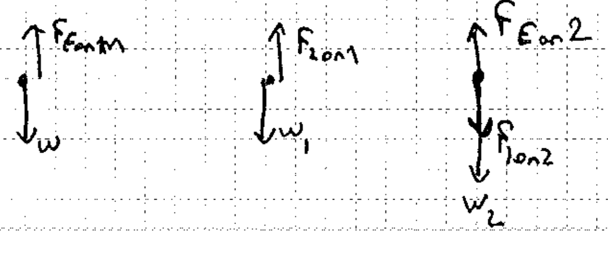
Two forces in an action-reaction pair must never appear in the same free-body diagram.

In solving problems using Newton's Laws:

- Newton's first and second Laws apply to a specific body, $(\Sigma \vec{F} = m\vec{a})$ decide on which body you are referring
- Once you have chosen the body to analyze, identify all the forces acting on it
- ~~Other~~ Draw the free-body diagram and execute.

Examples: Draw the free body diagrams





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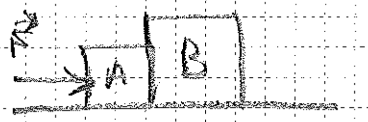
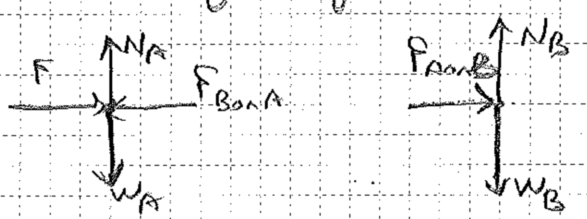
Examples:

Prob 4.24:

The boxes have masses m_A and m_B.



a) Draw free body diagrams for A and B

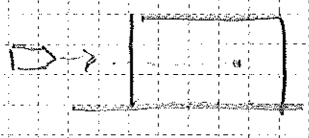


calculate $f_{AonB} = ?$

b) If the magnitude of $\vec{F}$ is less than the total weight of the two boxes, will it cause the boxes to move?

YES.

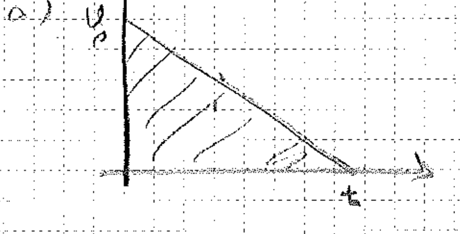
Prob 4.30: A bullet travelling at 350 m/s strikes a block of softwood and penetrates for 0.13 m. The mass of the bullet is 1.8 g. Assume a constant retarding force.



a) How much time is required for the bullet to stop?

b) What force does the wood exert on the bullet.

a)



$\frac{1}{2} v_0 t = 0.13\,m = \Delta x$

$\frac{1}{2}\, 350\,\frac{m}{s}\, t = 0.13\,m \Rightarrow t = \frac{0.26}{350}\,s$

$= \frac{26}{35} \times 10^{-3}\,s$

$\cong \boxed{2.4\,ms}$

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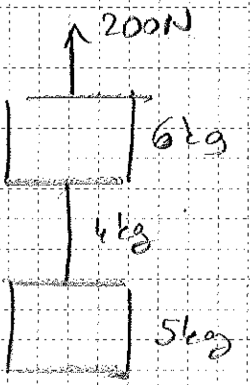
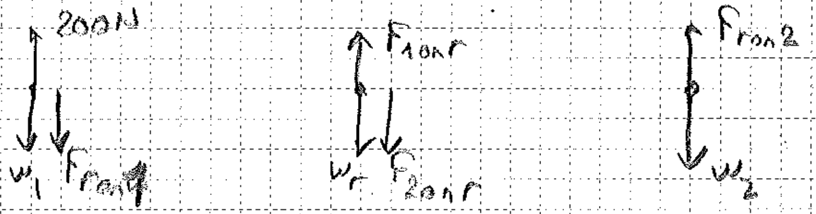
b) $F = ma = 1.8\times\cancel{10^{3}}\,kg \times \frac{350}{7.4\times\cancel{10^{-3}}} = \frac{18}{7.4}\times 350 = \boxed{85\,N}$

Prob 4.4.9.



First solve for $m_{rope}=0$

a) Draw 3 free body diagrams



b) What is a?

$200 = 15\,kg \times a \Rightarrow a = \frac{200}{15} = 13.3\, m/s^2$

c) Tension at the top of the rope:

$F_{ron1} = ?$ $\quad 200N - 6\times 9.8 - F_{ron1} = 6\times 14.3$

$\Rightarrow F_{ron1} = 200 - 6(24.1) = 55.4\,N$

d) Tension at the bottom of the rope

$F_{ron2} - w_2 = m_2 a \Rightarrow F_{ron2} - 5\times 9.8 = 5\times 14.3$

$F_{ron2} = 5\times 24.1 = 120.5\,N$

[illegible] 42!

[illegible] boxes [illegible] [illegible]

A force applied to them gives an acceleration of 2 m/s² to box B.

a) What is acc. of box A?

b) Draw a free-body diagram for box A.
Find the tension T in the rope?

c) Draw a free-body diagram for box B.
Direction of the net force on B

d) Calculate the magnitude of F