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Chapter 5: Applying Newton's Laws

We will apply Newton's laws in various circumstances.

- Particles in equilibrium
- Particles that are not in equilibrium
- Friction forces
- Circular motion.

5.1. Particles in Equilibrium

When a particle is at rest or moving with constant velocity in an inertial frame of reference, it is in equilibrium.

According to Newton's 1st Law the net force applied on a particle in equilibrium is 0.

$$\boxed{\begin{gathered}\sum \vec{F} = 0 \\ \sum F_x = \sum F_y = \sum F_z = 0\end{gathered}}$$

In ~~Re~~solving problems:

- Draw a sketch of the physical situation
- Choose a set of coordinate axes and draw the free body diagrams for each body
- Identify all the forces applied on the body, and include them in the free-body diagram. In the free-body diagram do not show forces applied by the body ~~to~~ other bodies.
- Find the components of each force and set the algebraic sum of all x-components, y-components and z-components equal to zero.
- Repeat for each body, make sure to have as many independent equations as unknown quantities.

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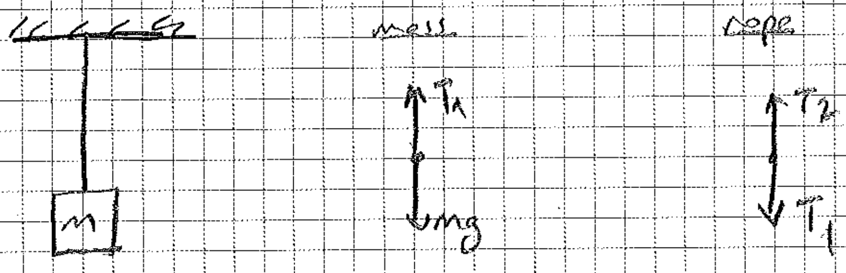
Example 5.1: A mass is ~~suspended~~ hanged from the end of a verticle rope. What force does the rope apply to the mass?
What is the tension at the top of the rope? (Assume the rope is massless)

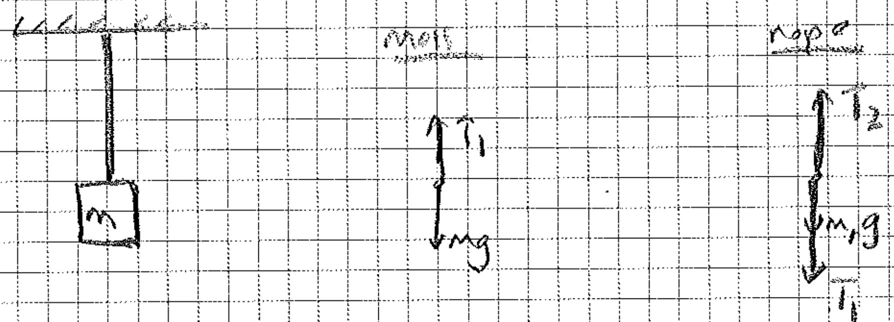


T_1: force applied by the rope to the mass
T_2: force applied by the ceiling to the rope. (Tension at the top of the rope)

$$mg = T_1 \quad , \quad T_1 = T_2$$

$$\Rightarrow \boxed{T_1 = mg} \qquad \boxed{T_2 = T_1 = mg}$$

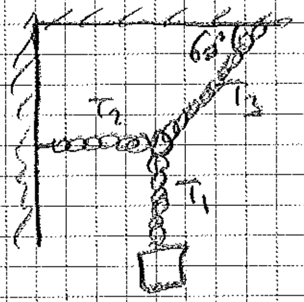
Now if the rope has a certain mass m_R:



$$\Rightarrow \boxed{T_1 = mg}$$

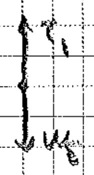
$$T_1 + m_r g = T_2 \Rightarrow \boxed{T_2 = T_1 + m_r g}$$

Ex 5.3: A car engine with weight w hangs from a chain that is linked at ring O to two other chains. What are the tensions?
Consider the weights of the ring and chains to be negligible.

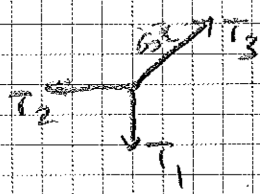


Free-body Diagrams

Engine



Ring



$\Rightarrow \boxed{T_1 = W_E}$

$T_1 = T_3 \sin 60°$

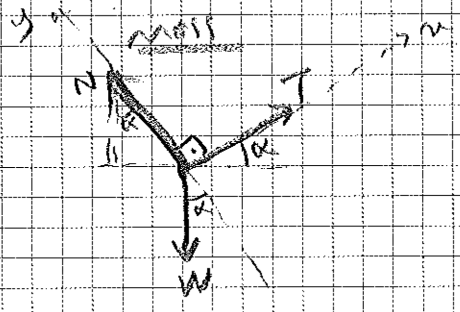
$T_2 = T_3 \cos 60°$

$\Rightarrow T_1 = T_3 \frac{\sqrt{3}}{2} \Rightarrow \boxed{T_3 = \frac{2}{\sqrt{3}} W_E}$

$T_2 = T_3 \cos 60° = \frac{2}{\sqrt{3}} \cdot \frac{1}{2} W_E = \boxed{\frac{1}{\sqrt{3}} W_E}$

Example 5.4. An inclined plane.

A mass rests on an inclined plane attached with a chain which prevents it from sliding backward. If the weight of the particle is w, what is the tension in the cable? (Ignore friction)

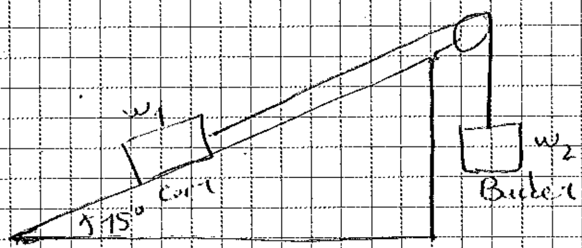


$W \cos\alpha = N$ $\boxed{N = W \cos\alpha}$

$W \sin\alpha = T \Rightarrow \boxed{T = W \sin\alpha}$

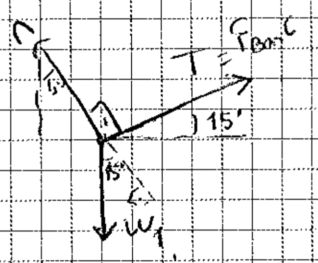
Example 5.5:

Ignoring friction in the pulley and wheels and the weight of the cable, how the weights w_1 and w_2 must be related in order for the system below to move with const. speed.



Free-body diagrams

Cart



Buckett

$w_1 \cos 15^\circ = n$

$w_1 \sin 15^\circ = T$

$T = w_2$

$\Rightarrow$ $\boxed{w_2 = w_1 \sin 15^\circ = 0.26\, w_1}$

5.2. Dynamics of Particles:

In the problems that involve particles which are not in equilibrium we apply Newton's Second Law.

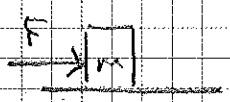
$$\sum \vec{F} = m\vec{a}$$

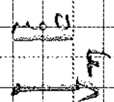
$$\Rightarrow \sum F_x = ma_x \,, \quad \sum F_y = ma_y \,, \quad \sum F_z = ma_z$$

Strategy in solving dynamics problems:

- Draw a sketch of the physical situation
- Choose a set of coordinate axes and draw the free-body diagrams for each body.
- Identify all the forces applied on the body, and include them in the free-body diagram.
- Write a separate equation for each component of Newton's 2^{nd} Law.
- Repeat for each body, make sure to have as many independent equations as unknown quantities.

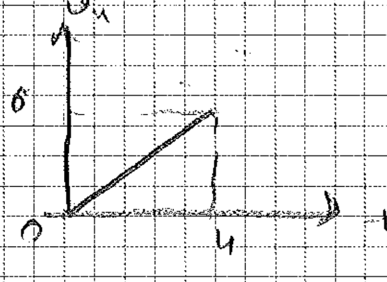
Ex. 5.6. A constant horizontal force is applied to a mass moving on a frictionless surface. 4 s after the application of the force the particle attains a velocity of 6 m/s. What is the force applied? (m = 200 kg)





$$\sum F = F = ma_x$$

$$\Rightarrow a_x = \frac{F}{m}$$



$$a_x = \frac{6\,m/s}{4\,s} = 1.5\,m/s^2$$

$$\Rightarrow F = ma_x = 200\,kg \times 1.5\,m/s^2 = \boxed{300\,N}$$

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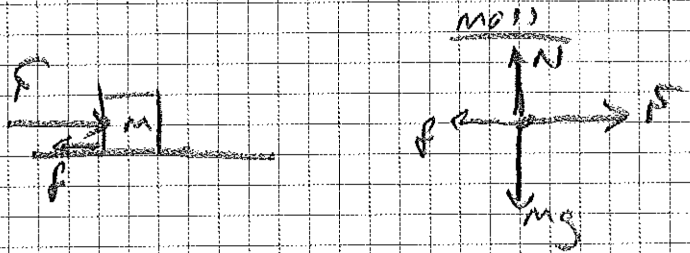
Ex. 5.8: Suppose the mass is moving with an acceleration of $a_x = 1.5\ m/s^2$, under a constant friction force of 100N. What should be the force applied to the mass? ($m = 200\,kg$)



$$\sum F = ma_x = F - f = ma_x$$

$$\Rightarrow F - 100N = 200 \times 1.5 = 300N \Rightarrow \boxed{F = 400N}$$

Ex 5.7: Consider the position of a particle as a function of time is given as:

$$x = 1.2t^2 - 0.2t^3$$

What is the force applied to the particle as a function of time?

What is the force at $t = 3s$?

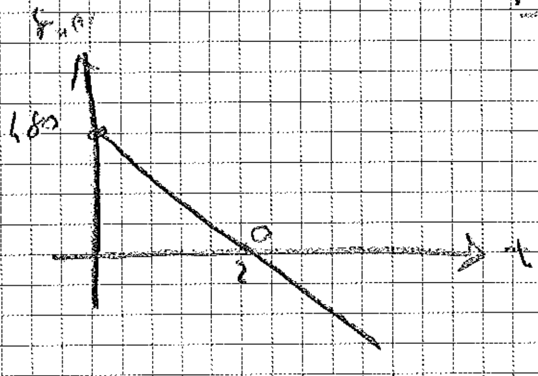
(ignore friction, $m = 200\,kg$)

$$x(t) = 1.2t^2 - 0.2t^3$$

$$\Rightarrow F_x = ma_x = m(2.4 - 1.2t)$$

$$\boxed{F_x(t) = 480 - 240t}$$

$$F_x(3) = 480 - 720 = \boxed{-240N}$$



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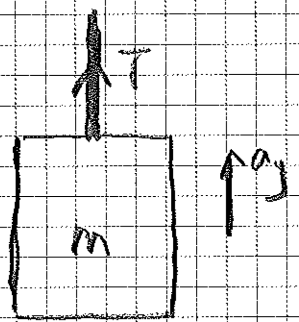
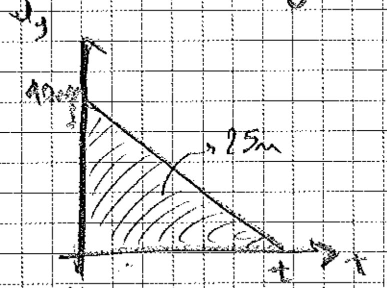
Ex 5.9: Tension in an elevator cable

Consider an elevator moving downward at 10 m/s slows to a stop with constant acceleration in a distance of 25 m. If the elevator has a total mass of 800 kg, what is the tension in the elevator cable, when the elevator is brought to rest?



free body diagram

$$\Sigma F = T - mg = ma_y$$



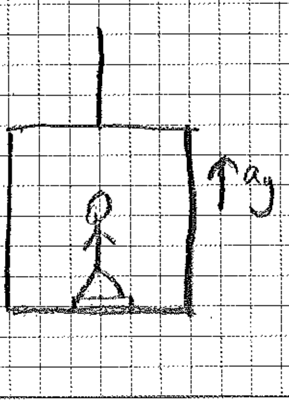
$$25\,m = 10\,m/s \times \frac{t}{2} \Rightarrow t = 5\,s$$

$$\Rightarrow a_y = \frac{10\,m/s}{5\,s} = 2\,m/s^2$$

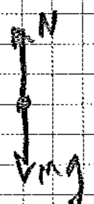
$$\Rightarrow \quad T = m(g + a_y) = 800\,kg\,(9.8 + 2) = 800 \times 11.8$$

$$= \boxed{9440\,N}$$

Ex 5.10: A 50 kg person stands on a scale while riding in the elevator. What is the reading on the scale?



Person



The reading in the scale will be given by the normal force.

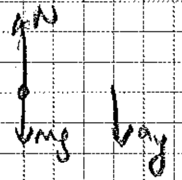
$$N - mg = ma_y$$

$$\Rightarrow N = m(g + a_y)$$

$$= 50\,kg\,(9.8 + 2) = 50 \times 11.8$$

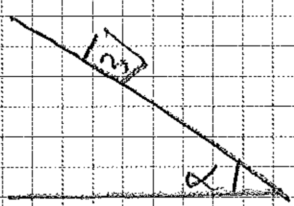
$$\boxed{N = 590\,N}$$

If the elevator was accelerating downwards:

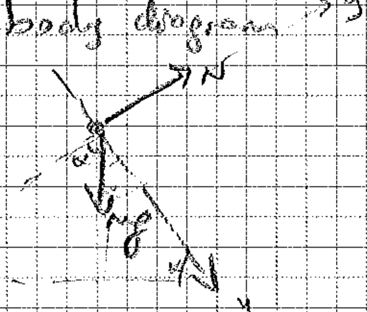


$\Rightarrow mg - N = ma_y \rightarrow \boxed{N = m(g - a_y)}$ The person will weigh less.

Ex 5.11: Consider a particle with mass m slides down an inclined plane. If the friction is negligible what is the acceleration?



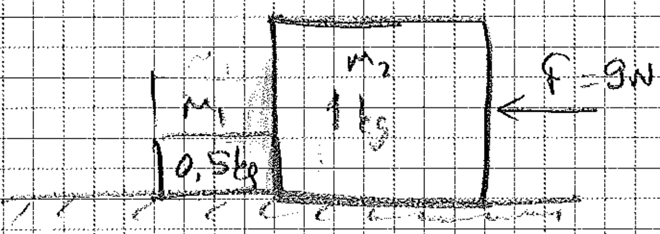
Free body diagram



$mg\cos\alpha = N$

$mg\sin\alpha = ma_x \Rightarrow \boxed{a_x = g\sin\alpha}$

Ex 5.12. Consider a system of two bodies. A constant horizontal force of 9 N is applied to the system.



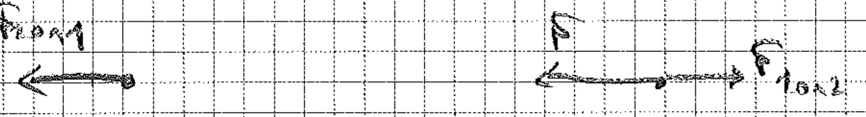
If we ignore the friction force, what is the acceleration of the system? What is the force that m_2 applies to m_1?

Since the two masses move together:

$F = (m_1 + m_2)a_x \Rightarrow 9N = 1.5\,kg\ a_x$

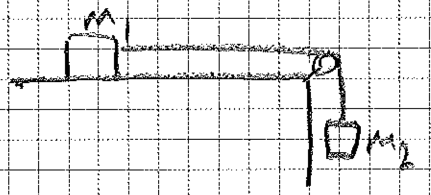
$\Rightarrow \boxed{a_x = 6\ m/s^2}$

Free body diagrams



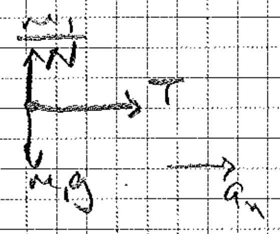
$$F_{2on1} = m_1 a = 0.5kg \times 6 \, m/s^2 = \boxed{3N}$$

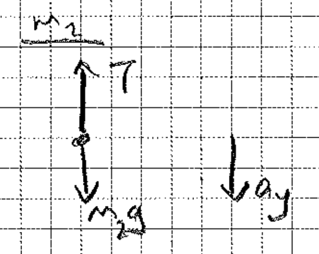
Ex 5.13: Consider a mass-pulley system. ~~Ref.~~



If friction ~~is ignored, and the~~ on the horizontal surface and on the pulley is ignored what is the acceleration of each body and tension in the string?

Free body diagrams:





$\Rightarrow T = m_1 a$ $\qquad$ $m_2 g - T = m_2 a$

$\Rightarrow$ 2 equations 2 unknowns.

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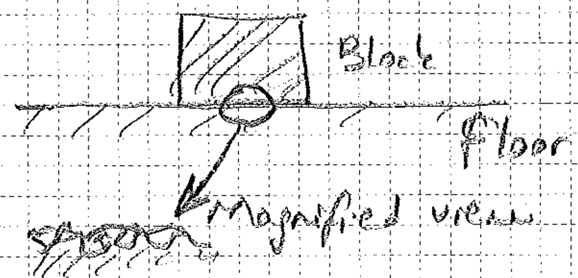
$m_2 g - m_1 a = m_2 a \Rightarrow \boxed{a = \frac{m_2}{m_1 + m_2} g}$

$T = m_1 a \Rightarrow \boxed{T = \frac{m_1 m_2}{m_1 + m_2} g}$

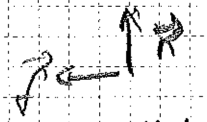
5.3 Frictional Forces:

Whenever two bodies interact by direct contact of their surfaces, ~~...~~ interaction forces are applied from one body to the other. Normal forces and friction forces are both such interaction forces.

Consider a block lying on a floor.



Normal force is the net force applied by the floor in the vertical direction.



The interaction force parallel to the surface is the friction force.

Kinetic and Static Friction

Friction that acts when a body slides over a surface is called as the kinetic friction force, f_k

Kinetic ⟺ objects moving relative to each other.

Magnitude of the ~~Kinetic~~ kinetic friction force can be approximated as:

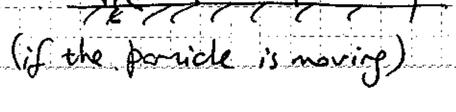
$$\boxed{f_k = \mu_k n}$$

μ_k: coefficient of kinetic friction

n: magnitude of the normal force.

Note that

~~ø~~ This equation is only an approximate representation of a complex phenomenon.



"Direction of the kinetic friction is "generally" opposite to the direction of motion"

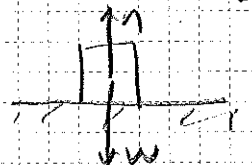
Static Friction Force, $\vec{f}_s$; is the friction force between two surfaces when there is no relative motion.

Magnitude of the static friction force is given as: $\boxed{f_s \le \mu_s n}$

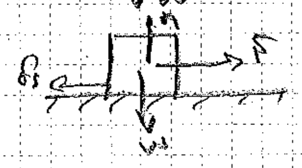
μ_s: coefficient of static friction

n: magnitude of the normal force

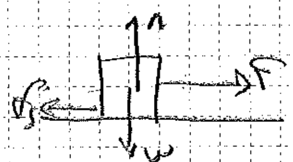
Consider a box lying on a floor:



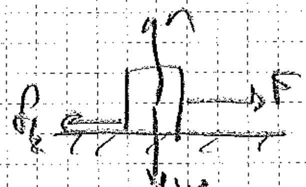
No applied force, box at rest $f = 0$



Weak applied force
Box remains at rest $f = F < \mu_s n$



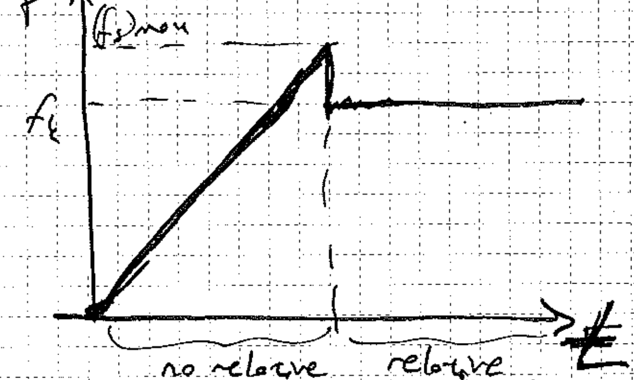
Stronger applied force,
box just about to slide:
$f_s = F = \mu_s n$



Box sliding
$f_k = \mu_k n$

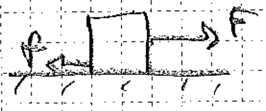
As soon as sliding starts friction force generally decreases.

$\mu_k < \mu_s$



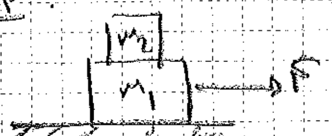
Direction of the Friction Force

Generally against the direction of motion, or if the particle is at rest against the net force applied on the particle.



There are scenarios where direction of the friction force can be along the direction of motion.

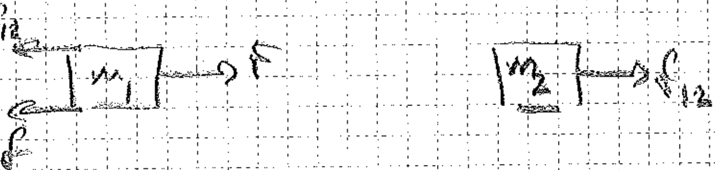
For Example:



Consider a system of two masses $m_1 = 2m_2$ moving at a constant acceleration a. How are the friction forces related.

$(F = m_1 g \quad \mu_s = 0.5, \mu_k = 0.4)$

Free body diagrams



$$\Rightarrow F - f - f_{12} = m_1 a \qquad f_{12} = m_2 a$$

$$m_1 g - (m_1+m_2)\mu_k g - f_{12} = m_1 a$$

$$2m_2 g - 1.2 m_2 g - f_{12} = 2m_2 a$$

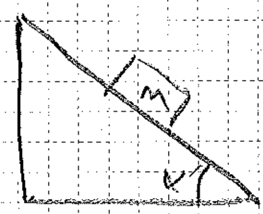
$$2m_2 g - 1.2 m_2 g - m_2 a = 2m_2 a \Rightarrow 0.8 m_2 g = 3 m_2 a \Rightarrow \boxed{a = \frac{0.8}{3} g}$$

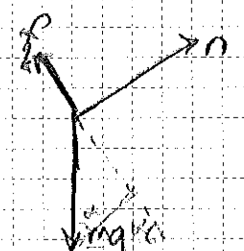
$$\Rightarrow f_{12} = m_2 \cdot \frac{0.8}{3} g = \left(\frac{0.8}{3}\right) m_2 g < 0.5\, m_2 g \quad \checkmark$$

m_2 will not move above m_1.

Ex 5.17:

Consider a box with a mass m is moving at a constant speed on an inclined plane with a kinetic friction coefficient μ_k. What should be α?





$$mg\cos\alpha = n$$

$$\cancel{m}g\sin\alpha = f_k = \mu_k n = \mu_k \cancel{m}g\cos\alpha$$

$$\Rightarrow \boxed{\tan\alpha = \mu_k} \Rightarrow \alpha = \arctan(\mu_k) = \tan^{-1}(\mu_k)$$

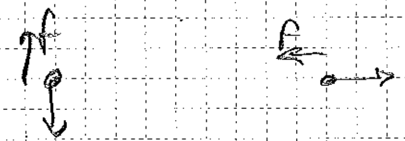
Ex 5.18: If the box is now moving downward with an acceleration, what is a?

$$mg\sin\alpha - f_k = ma \Rightarrow mg\sin\alpha - \mu_k mg\cos\alpha = ma$$

$$\Rightarrow \boxed{a = g\sin\alpha - \mu_k g\cos\alpha}$$

Fluid Resistance and Terminal Speed:

A body moving in a fluid is subject to a fluid resistance force which is directed opposite to its direction of motion ~~of the particle~~.

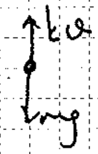


Magnitude of the fluid resistance is approximated as:

$f = kv$, at low speeds

or $f = Dv^2$, at large speeds

A particle falling under the influence of gravity will ~~reaches~~ obtain a terminal speed in ~~the~~ a fluid.

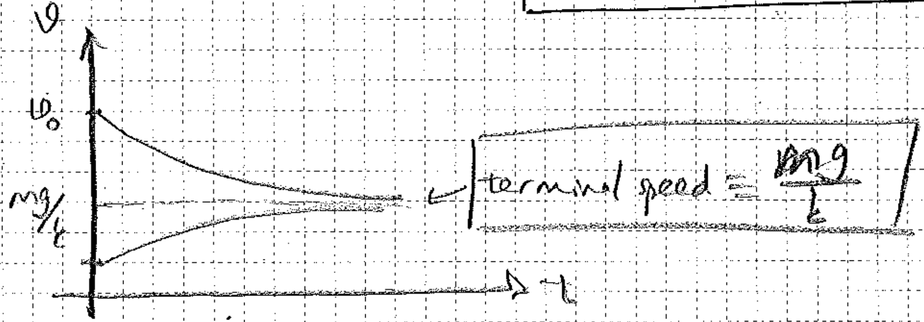


$$\Sigma F = mg - kv = ma = m\frac{dv}{dt}$$

$$\Rightarrow \int_0^t dt = \int_{v_0}^{v(t)} \frac{dv}{mg - kv}$$

$$\Rightarrow t = -\frac{1}{k}\ln(mg - kv)\Big|_{v_0}^{v(t)} \Rightarrow -kt = \ln\left(\frac{mg - kv(t)}{mg - kv_0}\right)$$

$$\Rightarrow mg - kv(t) = (mg - kv_0)e^{-kt} \Rightarrow \boxed{v(t) = \frac{mg}{k} - \left(\frac{mg}{k} - v_0\right)e^{-kt}}$$



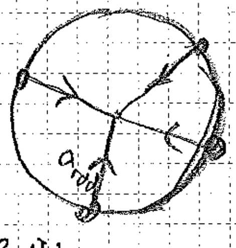
5.4. Dynamics of Circular Motion

A particle which moves along a circular path at a constant speed makes uniform circular motion.

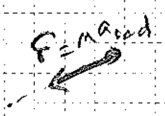
The acceleration of a particle making uniform circular direction is:

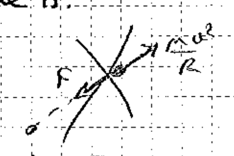
$$a_{rad} = \frac{v^2}{R}$$

denotes the radial direction



Free body diagram of the particle is:





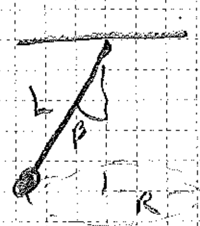
$F_{net} = m a_{rad} = \frac{mv^2}{R}$ "Incorrect interpretation"

Correct interpretation:

A net force constant in magnitude, $\frac{mv^2}{R}$, and radial in direction is applied to the particle making uniform circular motion.

uniform circular motion $\Longleftrightarrow$ Radial force with constant magnitude $\frac{mv^2}{R}$

Ex 5.22:

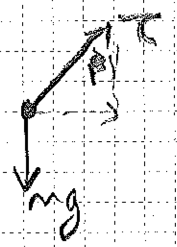


The mass at the end of a pendulum makes circular motion with constant angle β, and constant speed v.

What is the tension in the rope?

What is the period of oscillation?

Free body diagram:



$$F_{rad} = T\sin\beta$$

$$T\cos\beta = mg \Rightarrow \boxed{T = \frac{mg}{\cos\beta}}$$

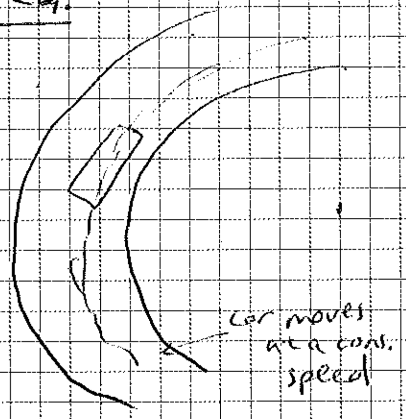
$$F_{rad} = \frac{mv^2}{R} = \frac{mv^2}{L\sin\beta} \qquad vT = 2\pi R$$

$$= \frac{m4\pi^2 R}{T^2} = \frac{4\pi^2 \cancel{m}}{T^2} \cdot L\sin\beta = T\sin\beta$$

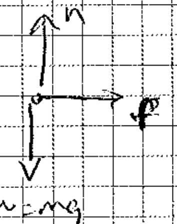
$$= \frac{\cancel{m}g\sin\beta}{\cos\beta}$$

$$\Rightarrow \frac{4\pi^2}{T^2} L = \frac{g}{\cos\beta} \Rightarrow \boxed{T = 2\pi\sqrt{\frac{L\cos\beta}{g}}}$$

Ex 5.23:



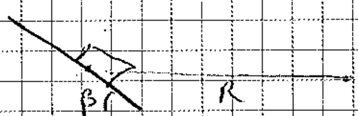
Curve with radius R,
If the coefficient of friction between tires and road is μ_s, what is the maximum speed v_{max} at which the driver can take the curve without sliding?

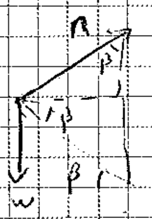


$$f = \frac{mv^2}{R} = \mu_s mg \Rightarrow \boxed{v = \sqrt{\mu_s g R}}$$

Ex 5.24:

Imagine now a curved path.
What is the angle β that will provide safe turn at velocity v?





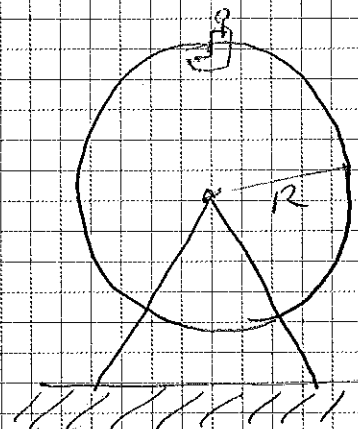
$$n\cos\beta = w = mg$$

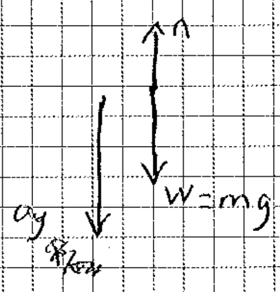
$$n\sin\beta = m\,a_{rad} = m\frac{v^2}{R}$$

$$\Rightarrow \frac{mg}{\cos\beta}\sin\beta = m\frac{v^2}{R} \Rightarrow \boxed{\tan\beta = \frac{v^2}{gR}}$$

Ex 5.25:

A passenger in a carneval wheel moves in a vertical circle with radius R at a constant speed. If the seat remains upright during the motion, derive the expressions for the force ~~the~~ seat exerts on the passenger at the top of the circle and the bottom.

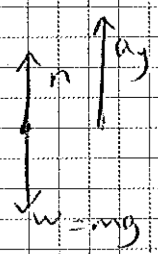




At the top:

$$w - n = ma_y$$

$$mg - n = \frac{mv^2}{R} \Rightarrow \boxed{n = m\left(g - \frac{v^2}{R}\right)}$$



At the bottom

$$n - w = ma_y \Rightarrow \boxed{n = m\left(g + \frac{v^2}{R}\right)}$$