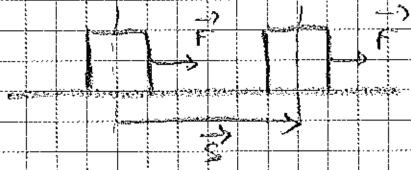
Chapter 6: Work and Kinetic Energy

6.1. Work:

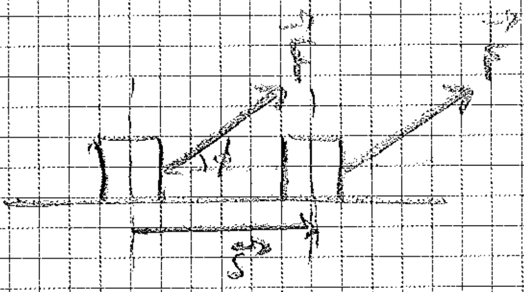


Consider a body that undergoes a displacement, s, along a straight line. While the particle moves, a constant force $\vec{F}$ acts on it in the same direction as the displacement $\vec{s}$.

$\boxed{W = Fs}$ Work done by the force $\vec{F}$ is defined as:

$$\boxed{W = Fs}$$

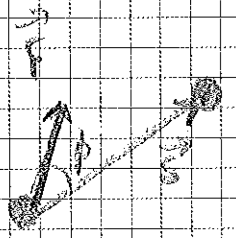
In SI units $(N)(m) = (J)$ the unit of work is "Joule".



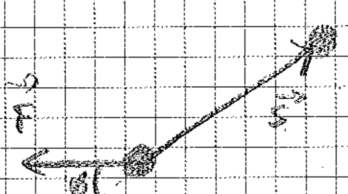
In general, considering the case when the applied force is not in the same direction as displacement, work is defined as:

$$\boxed{W = \vec{F} \cdot \vec{s}} = F\cos\phi\, s$$

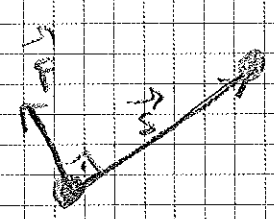
Note that work can be negative or zero.



Force has component in direction of displacement ⇒ Work done is positive



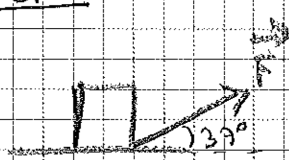
Force has component opposite to displacement ⇒ Work done is negative



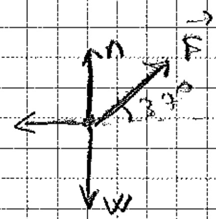
Force is perpendicular to displacement ⇒ Work done is zero

KOÇ ÜNİVERSİTESİ

<u>Ex 6.2:</u> A farmer pulls a load a distance of 20m. The applied force is 5000N at an angle of 37° above the horizontal. There is a 3500N friction force opposing the load's motion.



Find the work done by each force acting on the load and the total work done by all forces.



$\vec{s} = 20m$ in x direction

$W_w = 0$

$W_n = 0$

$W_{\vec{F}} = (5000N)(20m)(\cos 37°) = (5000N)(20m)(\frac{4}{5}) = 80000Nm = 80kJ$

$W_f = (3500N)(20m)(\cos 180°) = -70000Nm = -70kJ$

Total work done by all forces:

$$W_{tot} = W_w + W_n + W_{\vec{F}} + W_f$$
$$= \vec{w}\cdot\vec{s} + \vec{n}\cdot\vec{s} + \vec{F}\cdot\vec{s} + \vec{f}\cdot\vec{s} = (\vec{w} + \vec{n} + \vec{F} + \vec{f})\cdot\vec{s}$$
$$= \boxed{(\sum\vec{F})\cdot\vec{s}}$$

$$W_{tot} = 80kJ - 70kJ = 10kJ$$

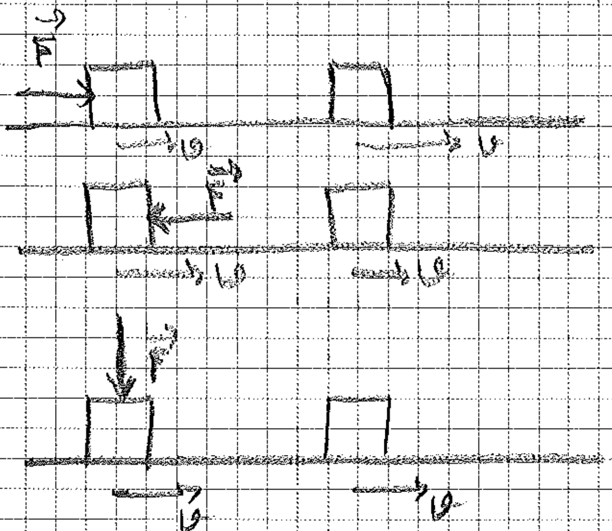
<u>Ex 6.3:</u> An electron moves in a straight line with a constant speed of 8×10^7 m/s. What is the total work done on the electron during a 1m displacement by the electric, magnetic and gravitational forces.

If a particle moves at a constant velocity:

$\sum\vec{F} = 0$, net applied force is 0. $\Rightarrow W_{tot} = (\sum\vec{F})\cdot\vec{s}$
$= 0$

6.2. Work and Kinetic Energy:

Total work is also related to changes in the speed of a body.



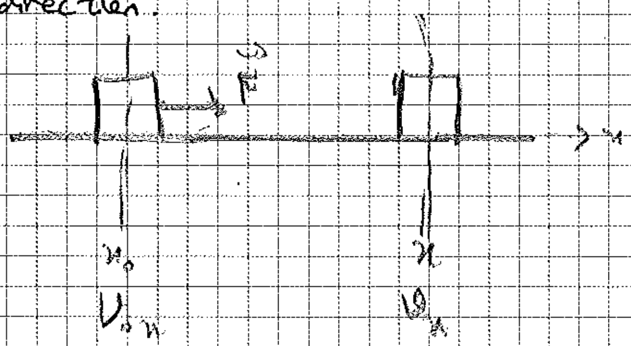
v increases

v decreases

v does not change

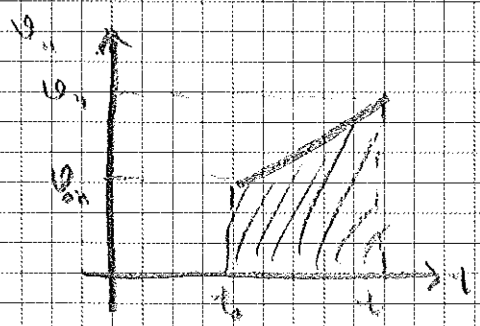
The particle's speed changes by the component of the net force along the direction of motion.

Quantitatively: Consider a particle with mass m moving along x-axis under a constant net force with magnitude F along the positive x direction.



$\sum F_x = F = ma$

$\Rightarrow a = \frac{F}{m}$, motion with constant acceleration.



$$x - x_0 = v_{0x}(t-t_0) + \frac{(t-t_0)}{2}(v_x - v_{0x})$$

$$\frac{v_x - v_{0x}}{t-t_0} = a$$

$$\Rightarrow (x-x_0) = v_{0x}\frac{(v_x - v_{0x})}{a} + \frac{(v_x - v_{0x})}{2} \cdot \frac{(v_x - v_{0x})}{a}$$

$$\Rightarrow 2a(x-x_0) = 2v_{0x}(v_x - v_{0x}) + (v_x - v_{0x})(v_x - v_{0x})$$

$$2a(x-x_0) = v_x^2 - v_{0x}^2$$

$\Rightarrow v_x^2 - v_{0x}^2 = 2a(x - x_0)$

$$= 2\frac{F}{m}(x-x_0) \Rightarrow F(x-x_0) = \left(\frac{mv_x^2}{2} - \frac{mv_{0x}^2}{2}\right)$$

$$\Rightarrow \boxed{Fs = \frac{1}{2}mv_x^2 - \frac{1}{2}mv_{0x}^2}$$

We define kinetic energy as $\boxed{K = \frac{1}{2}mv^2}$

Note that: – Kinetic energy is a scalar quantity like work.
– Kinetic energy can never be negative, it is equal to zero when the particle is at rest.

We have derived:

$F(x_2 - x_1) = Fs = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

$$\Rightarrow \boxed{W_{tot} = K_2 - K_1 = \Delta K}$$

$\Rightarrow$ The total work done on a particle equals the change in the particle's kinetic energy. This is the "Work-Energy Theorem."

Therefore when W_{tot} is $\begin{cases} \text{positive} & \text{kinetic energy increases} \\ \text{negative} & \text{kinetic energy decreases} \\ 0 & \text{kinetic energy remains constant} \end{cases}$

SI units of kinetic energy is (J). $(J) = (N)(m) = kg\frac{m}{s^2}m = (kg)\left(\frac{m^2}{s^2}\right)$

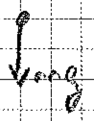
Examples:

Ex. 6.24: A 4.8 kg particle is dropped from the roof of a 25m tall building

a) Work done by gravity on the particle during its displacement from the roof to the ground

b) What is the kinetic energy of the particle just before it hits the ground?

KOÇ ÜNİVERSİTESİ



a) $W_w = mg \cdot \Delta x$

$= 4.8kg \times g \times 25m$

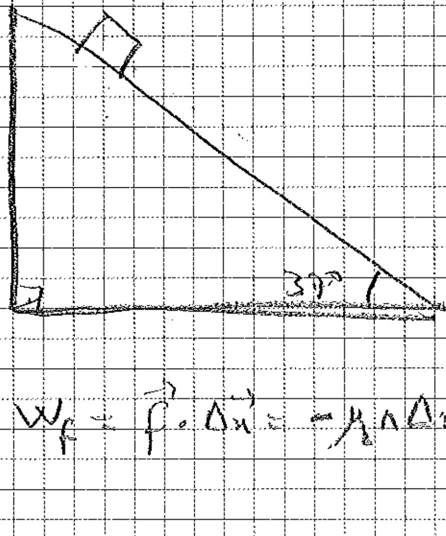
$\approx 48 \times 25 = 1200\ J$ //

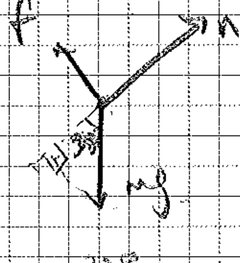
b) $\Delta K = W_w \rightarrow K_2 - \cancel{K_1}^{0} = W_w$

$\rightarrow \boxed{K_2 = 1200\ J}$ //

Prob 6.62: A 5kg package slides 1.5m down a long ramp inclined at 37°. Coefficient of kinetic friction between the package and the ramp is $\mu_k = 0.31$

a) The work done on the package by friction
b) " " " " gravity
c) " " " " by the normal force
d) Total work done on the package
e) If the package has a speed of 2.2 m/s at the top of the ramp, what is its speed after sliding for 1.5m.





$mg\cos 37° = n$

$f = \mu_k n$

a) $W_f = \vec{f} \cdot \vec{\Delta x} = -\mu_k n \Delta x = -0.31 \times mg\cos 37° \times 1.5m = -0.31 \times \overset{60}{\overline{50 \times \frac{4}{5}}} \times 1.5$

$\approx -18\ J$ //

KOÇ ÜNİVERSİTESİ

66

b) $W_w = m\vec{g} \cdot \Delta\vec{x} = mg\Delta x \cos 53° = 50 \times 1.5 \times \frac{3}{5} = 45 J$

c) $W_n = 0$

d) $W_{total} = 45 - 18 = 27 J$

e) $W_{total} = \Delta K = 27 J = K_2 - \frac{1}{2} m v_1^2 = K_2 - \frac{1}{2} 5 (2.2)^2$

$\Rightarrow K_2 \approx 15 J = \frac{1}{2} 5 \, v_2^2 \Rightarrow v_2^2 = 6 \Rightarrow \boxed{v_2 \approx \sqrt{6} \; m/s}$

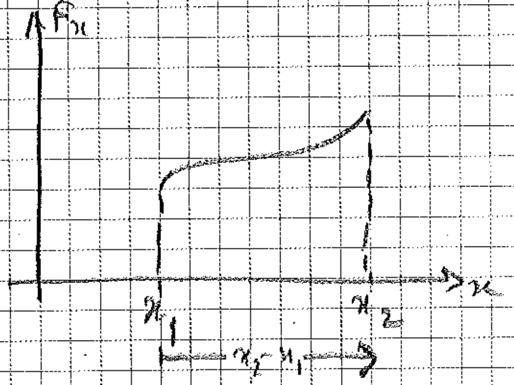
6.3. Work and Energy with Varying Forces:

If a constant force is applied to a particle during a displacement $\vec{s}$; work done by the constant force is:

$$W = \vec{F} \cdot \vec{s}$$

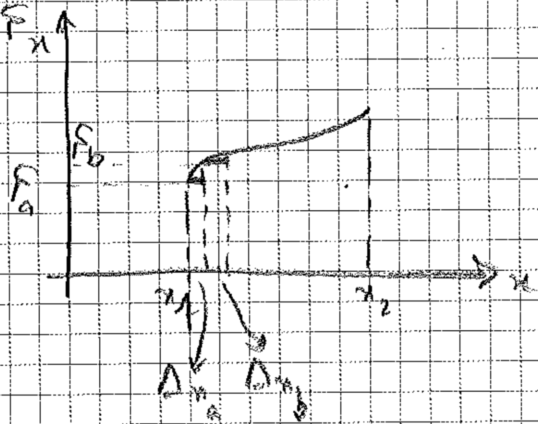
and work-energy theorem states that: $W = \Delta K = K_2 - K_1$

Now suppose a particle moves along the x-axis from point x_1 to x_2 under an applied varying force. x-component of the applied force as a function of the particle's coordinate is:



We want to find out the total work done during this travel.

This is given by "integration". We divide the total displacement into small segments; Δx_a, Δx_b, ...



Because of the "small segments assumption", ~~approximation~~ the total work done by the force in the total displacement from x_1 to x_2 is approximated as:

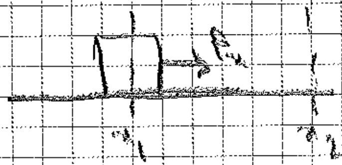
$$\boxed{W \approx F_a \Delta x_a + F_b \Delta x_b + \ldots}$$

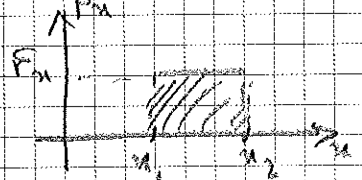
As the number of segments becomes very large and the width of each becomes very small, the approximation on W becomes an exact equality.

$$\boxed{W = \int_{x_1}^{x_2} F_x \, dx}$$

"On a graph of force as a function of position, total work done by the force is represented by the area under the curve".

Examples:



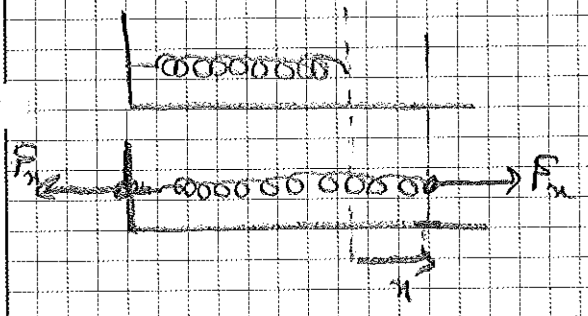


If F_x is constant:

$$W = \int_{x_1}^{x_2} F_x \, dx = F_x (x_2 - x_1)$$
$$= F_x s$$

Stretching Spring:

Consider a spring which has a certain unstretched lenght:



To keep the string stretched beyond its unstretched length by an amount x, a force F_x has to be applied to the spring.

If the spring is <u>not too much stretched</u> F_x is given as:

Hooke's Law. $\boxed{F_x = kx}$

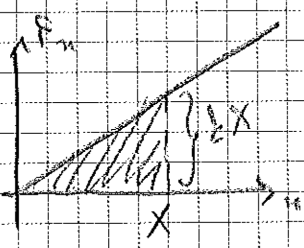
force constant (N/m)

k is larger for a stiffer spring.

Suppose the spring is initially unstretched and we slowly stretch it to an amount of X. The work done by the stretching force is found by:

$$W = \int_0^X F_x \, dx = \int_0^X kx \, dx = \frac{1}{2} k x^2 \Big|_0^X = \frac{1}{2} k X^2$$

Graphically:



Area $= \frac{1}{2} k X^2$

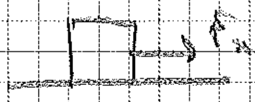
If the spring was originally stretched:

$$W = \int_{x_1}^{x_2} kx \, dx = \frac{1}{2} k x_2^2 - \frac{1}{2} k x_1^2$$

Work-Energy Theorem for Straight-Line Motion, Varying Forces:

It turns out that the work-energy theorem ($W_{tot} = K_2 - K_1$) holds true even when the force varies with position.

Proof:



Consider a particle and a varying applied force F_x along a straight line, between x_1 and x_2.

We divide the total displacement into small segments: $\Delta x_a, \Delta x_b, \ldots$

Work-energy theorem applied to each segment reveals:

$$W_a = K_a - K_1$$
$$W_b = K_b - K_a$$
$$\vdots \qquad \vdots$$
$$W_{last} = K_2 - K_{last-1}$$

$\Rightarrow$ Total work done is $W_{total} = W_a + W_b + \ldots + W_{last}$

$$= \cancel{K_a} - K_1 + \cancel{K_b} - \cancel{K_a} + \cancel{\ldots} + K_2 - \cancel{K_{last-1}}$$

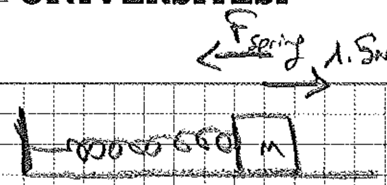
$$\Rightarrow \boxed{W_{total} = K_2 - K_1}$$

Example 6.8: A particle with a mass 0.1 kg is attached to the end of a spring with force constant 20 N/m. Initially the spring is unstretched and the particle is moving at 1.5 m/s to the right.

Find the maximum distance that the particle moves to the right

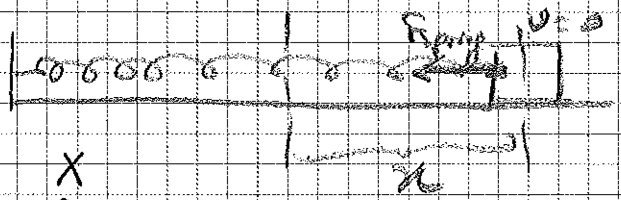
a) if there is no friction

b) if $\mu_k = 0.47$



Apply the work energy theorem:

a) At maximum displacement the particle will be at rest.



$$W_{total} = \int_0^X -kx\,dx = -\frac{1}{2}kX^2 = K_2 - K_1 = \frac{1}{2}\cancel{mv_2^2}_{0} - \frac{1}{2}mv_1^2$$

$$\Rightarrow \frac{1}{\cancel{2}}kX^2 = \frac{1}{\cancel{2}}mv_1^2 \Rightarrow X = \sqrt{\frac{m}{k}}\,v_1 = \sqrt{\frac{0.1}{20}} \times 1.5\,m/s$$

$$= 10.6\,cm$$

b) $$W_{total} = \int_0^X -kx\,dx + \int_0^X -f\,dx$$

$$= -\frac{1}{2}kX^2 - \mu_k mgX = -\frac{1}{2}mv_1^2$$

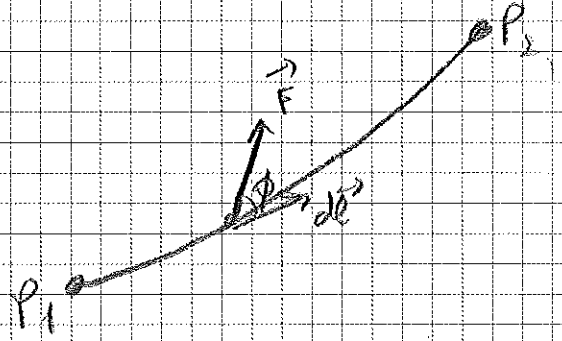
$$\Rightarrow \frac{1}{2}kX^2 = \frac{1}{2}mv_1^2 + \mu_k mgX$$

$$10X^2 - (0.47)(0.1)(10)X - \frac{1}{2}(0.1)(1.5)^2 = 0$$

$$\Rightarrow X = 0.086\,m \quad \text{or} \quad -0.\cancel{13}\,m$$

$$\Rightarrow \boxed{X = 8.6\,cm}$$

Work-Energy Theorem for Motion Along a Curve



Suppose a particle moves from point P_1 to P_2 along a curve. A varying force $\vec{F}$ is applied to the particle during its motion.

Work done on the particle by the applied force can be found by integration. Again we divide the portion of the curve between P_1 and P_2 into many infinitesimal vector displacements, $d\vec{l}$. Each $d\vec{l}$ is tangent to the curve at its position.

The element of work done on the particle during the displacement $d\vec{l}$ is:

$$dW = \vec{F} \cdot d\vec{l} = F \cos\phi \, dl$$

⇒ The total work done by $\vec{F}$ is then:

$$\boxed{W = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{l} = \int_{P_1}^{P_2} F \cos\phi \, dl}$$

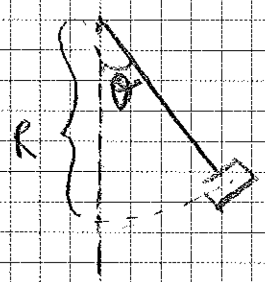
Line integral

Work-energy theorem holds true even with varying forces and a displacement along a curved path.

$$\boxed{W_{tot} = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{l} = \Delta K = K_2 - K_1}$$

is true no matter what the path and no matter what the character of the forces.

Example 6.9.:



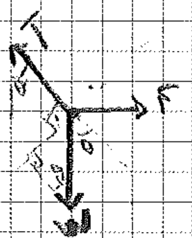
A particle with weight w is moving in a swing. A varying horizontal force $\vec{F}$ is applied to the particle such that the particle is kept in equilibrium and the angle is changed from 0 to θ_0.

(a) Total work done on the particle by all forces?
(b) Total work done by the tension in the chains?
(c) Work done by the varying horizontal force?

(a) $W_{tot} = \Delta K = 0 \Rightarrow W_{tot} = 0$

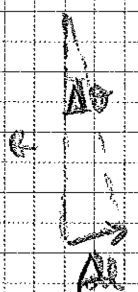
(b) $W_{Tension} = 0$, Because tension force is always perpendicular to the curve.

(c)



$$W_F + W_w = 0$$

$$W_w = -\int_0^{\theta_0} w \sin\theta \, dl$$



$$\left.\begin{array}{l} \Delta l \approx R\Delta\theta \\ dl \approx R d\theta \end{array}\right) \Rightarrow W_w = -\int_0^{\theta_0} w \sin\theta \, R d\theta = Rw\left(-\cos\theta\Big|_0^{\theta_0}\right)$$

$$= -Rw(1-\cos\theta_0)$$

$$\Rightarrow \boxed{W_F = Rw(1-\cos\theta_0)}$$

KOÇ ÜNİVERSİTESİ

6.4. Power:

Power is the time rate at which work is done.

~~[illegible]~~ so:

$$\boxed{P_{av} = \frac{\Delta W}{\Delta t} = \frac{W_2 - W_1}{t_2 - t_1}}$$ average power

$$\boxed{P_{ins} = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}}$$ instanteneous power

SI unit of power is Watt. (W)

$$1W = 1(J/s)$$

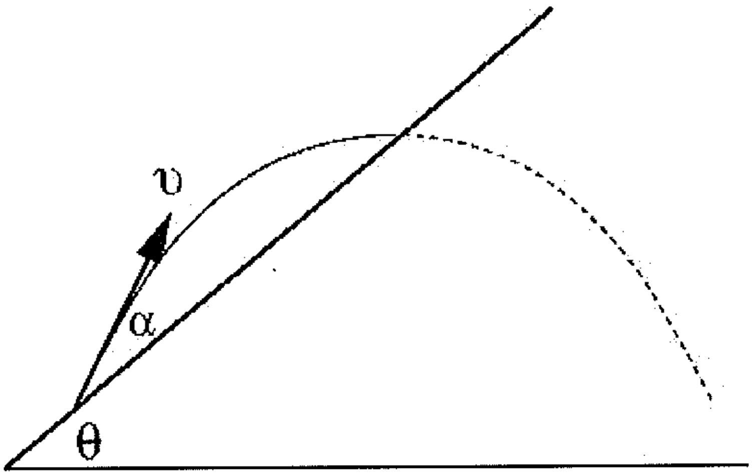
If a constant force is applied to a body

$$W = \vec{F} \cdot \vec{s} \Rightarrow P_{ins} = \vec{F} \cdot \frac{d\vec{s}}{dt} = \boxed{\vec{F} \cdot \vec{v}}$$ //

74,2

MT-I. A projectile is thrown on at an angle α and initial speed υ from an inclined plane with an inclination angle θ. The projectile touches the inclined surface at the point where it reaches its maximum height.

(a) (5 pts) Find the time of flight, **t**, in terms of given variables, using the condition that the vertical speed is zero at the moment of contact.
(b) (5 pts) Express the horizontal distance, **x**, traveled in the air as a function of given variables.
(c) (5 pts) Express the vertical distance, **y**, traveled in the air as a function of given variables.
(d) (10 pts) Using the condition that the point of maximum height is also the point of contact, show that, independent of υ, there is only one angle α that satisfies the condition.



SOLUTION:

(a) $v\sin(\theta+\alpha)=gt \Rightarrow t=v\sin(\theta+\alpha)/g$

(b) $x=v\cos(\theta+\alpha)t=v^2\sin(\theta+\alpha)\cos(\theta+\alpha)/g$

(c) $y=gt^2/2=v^2\sin^2(\theta+\alpha)/2g$

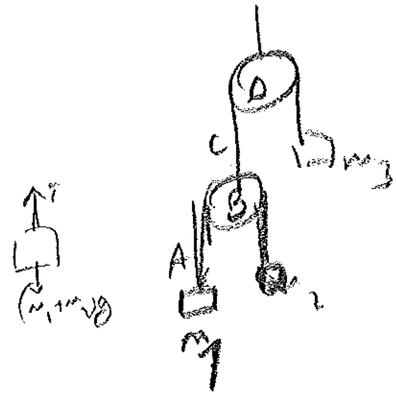
(d) $\tan(\theta)=\dfrac{y}{x}=\tan(\theta+\alpha)/2=\dfrac{1}{2}\dfrac{\tan\theta+\tan\alpha}{1-\tan\theta\tan\alpha} \Rightarrow \tan\alpha=\dfrac{\tan\theta}{1+2\tan^2\theta}$

MT-II.

-5.127

-5.125

(74.1)



a) a_{m_3}

a_B

a_{m_1}

a_{m_2}

T_A T_C

what happens if $m_1 = m_2$ and $m_3 = m_1 + m_2$?

$$\boxed{\begin{aligned} m_1 g - T_A &= m_1 a_1 \\ m_2 g - T_A &= m_2 a_2 \\ m_3 g - T_C &= m_3 a_3 \\ 2T_A - T_C &= 0 \end{aligned} \qquad \begin{aligned} a_B &= -a_3 \\ a_1 + a_2 &= 2a_B = -2a_3 \end{aligned}}$$

$$a_3 = g\,\frac{-4m_1m_2 + m_2m_3 + m_1m_3}{4m_1m_2 + m_2m_3 + m_1m_3}$$

$a_B = -a_3$

$$a_1 = g - \frac{m_3}{2m_1}(g - a_3)$$

I x (x) a_B

$y \downarrow a_1 \longrightarrow \boxed{x+y}$

$\boxed{x-y}$

$a_1 = a_B - a'$

$a_2 = a_B + a'$

$\Rightarrow a_1 + a_2 = 2a_B$