Chapter 8: Momentum, Impulse and Collisions

8.1. Momentum and Impulse

Consider a particle with a mass m moving at a velocity $\vec{v}$, the momentum of the particle is defined as:

$$\boxed{\vec{p}(t) = m\vec{v}(t)}$$

momentum is a vector quantity that has a magnitude 'mv' and the same direction as the velocity.

Newton's second law can be written as:

$$\sum \vec{F} = m\frac{d\vec{v}}{dt} = \frac{d}{dt}(\underset{\substack{\uparrow \\ \text{constant}}}{m}\vec{v}) = \frac{d}{dt}(\vec{p})$$

$$\Rightarrow \boxed{\sum \vec{F} = \frac{d\vec{p}}{dt}}$$ Newton's 2^{nd} Law in terms of momentum.

Consider a constant force $\sum \vec{F}$ is applied to a particle during a time Δt.

$$\Rightarrow \sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t} \Rightarrow \Delta \vec{p} = \sum \vec{F}(\Delta t)$$

$$\Rightarrow \boxed{\vec{p}_2 - \vec{p}_1 = \left(\sum \vec{F}\right)(t_2 - t_1)}$$

We define $\sum \vec{F}(\Delta t) = \vec{J}$, as the <u>impulse</u> of the net force.

SI units $(\vec{J}) = (Ns) = (kg\,\frac{m}{s})$

For when the net force $\sum \vec{F}$ is not constant:

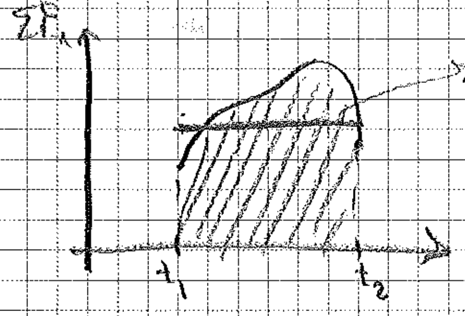
$$\boxed{\vec{J} = \int_{t_1}^{t_2} \sum \vec{F}\, dt} \Rightarrow \sum \vec{F} = \frac{d\vec{p}}{dt} \Rightarrow \int_{t_1}^{t_2} \sum \vec{F}\, dt = \int_{\vec{p}_1}^{\vec{p}_2} d\vec{p}$$

$$\Rightarrow \boxed{\vec{J} = \vec{p}_2 - \vec{p}_1}$$

Impulse-momentum theorem.

The change in momentum of a particle during a time interval equals the impulse of the net force that acts on the particle during that interval.

Consider a one dimensional net force $\sum F_x$:



$$\rightarrow \text{area} = \int \sum F_x\, dt = J_x = F_{av}\, \Delta t = F_{av}(t_2 - t_1)$$

$$\boxed{F_{av} = \frac{1}{\Delta t} J_x}$$

Average force applied to the particle.

In general for a 3 dimensional applied force:

$$\boxed{\vec{F}_{av} = \frac{1}{\Delta t} \vec{J} = \frac{1}{(t_2 - t_1)} \int_{t_1}^{t_2} \sum \vec{F}\,(dt)}$$

$\Rightarrow$ If $\sum \vec{F}$ is constant then $\vec{F}_{av} = \sum \vec{F}$.

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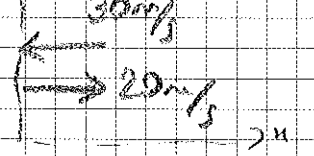
Ex 8.2: A ball with a mass of 0.4 kg is thrown against a wall. The ball hits the wall horizontally to the left with 30 m/s and rebounds horizontally to the right at 20 m/s.



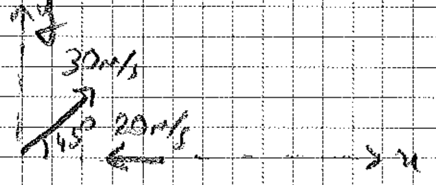
a) What is the impulse of the net force on the ball during its collision with the wall
b) If the ball is in contact with the wall for 0.01 s, what is the average force that the wall applies to the ball?

a) In 1D:

$$J_x = \Delta p_x = p_{2x} - p_{1x} = m(v_{2x} - v_{1x})$$
$$= m(20\,m/s - (-30\,m/s)) = 0.4 \cdot 50 = 20\ kg\,m/s$$

b) $$J_x = \int_{t_1}^{t_2} \Sigma F_x\, dt = F_{av_x}\Delta t \Rightarrow F_{av_x} = \frac{1}{\Delta t} J_x$$
$$= \frac{1}{0.01} \cdot 20 = \boxed{2000\ N}$$

Example 8.3: A ball with m = 0.4 kg is initially moving to the left at 20 m/s, it is given a velocity at 45° upward to the right with a magnitude of 30 m/s.
What is the impulse of the net force and the average net force for a collision time $\Delta t = 0.01 s$?



$$\vec{J} = \Delta\vec{p} = \vec{p}_2 - \vec{p}_1 \Rightarrow J_x = p_{2x} - p_{1x} = \left(30\frac{\sqrt{2}}{2} - (-20)\right)0.4\,kg$$

≈ 0.7

$$\approx (41)(0.4) \approx 16.4\ kg\,m/s$$

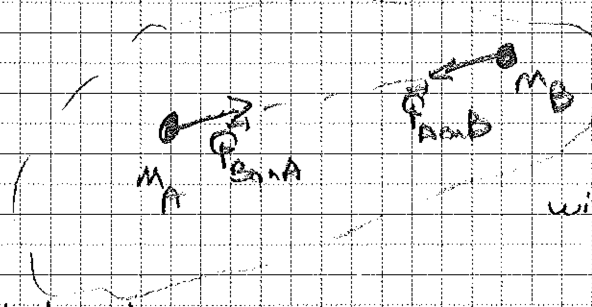
$$J_y = p_{2y} - p_{1y} = \left(30\frac{\sqrt{2}}{2} - 0\right) \cdot 0.4\,kg = 8.4\ kg\,m/s$$

Average net force: $\vec{F}_{av} \Delta t = \Delta \vec{p} = \vec{J}$

$$\Rightarrow F_{av_x} = \frac{1}{\Delta t} J_x = 1640N$$

$$F_{av_y} = \frac{1}{\Delta t} J_y = 840N$$

8.2. Conservation of Momentum:



isolated system

Consider an idealized system consisting of two bodies that interact with each other but not with anything else.

$\Downarrow$

isolated system: There are no external forces applied to the system.

Each particle applies a force to the other; according to Newton's 3rd law:

$$\vec{F}_{B on A} = -\vec{F}_{A on B} \quad \text{: Internal forces.}$$

For an isolated system:

$$\vec{F}_{B on A} = \frac{d}{dt}(\vec{p}_A) \quad , \quad \vec{F}_{A on B} = \frac{d}{dt}(\vec{p}_B) = -\vec{F}_{B on A}$$

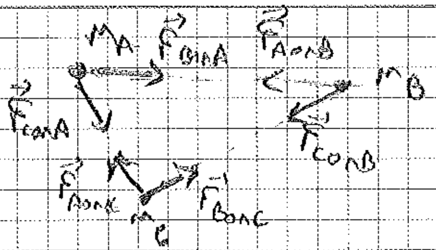
$$\Rightarrow \frac{d}{dt}\vec{p}_B = -\frac{d}{dt}\vec{p}_A \Rightarrow \frac{d}{dt}(\vec{p}_A + \vec{p}_B) = 0$$

$$\boxed{\frac{d}{dt}\vec{P} = 0} \quad , \quad \vec{P} = \vec{p}_A + \vec{p}_B \text{ : Total momentum of the system.}$$

If the vector sum of the external forces on a system is zero, the total momentum of the system is constant.

$\Rightarrow$ Principle of conservation of momentum.

Generalization:



For a system containing any number of particles: A, B, C, ...

$$\frac{d}{dt}\vec{p}_A + \frac{d}{dt}\vec{p}_B + \frac{d}{dt}\vec{p}_C = \vec{F}_{B\,on\,A} + \vec{F}_{C\,on\,A} + \vec{F}_{A\,on\,B} + \vec{F}_{C\,on\,B} + \vec{F}_{A\,on\,C} + \vec{F}_{B\,on\,C}$$

$$= 0$$

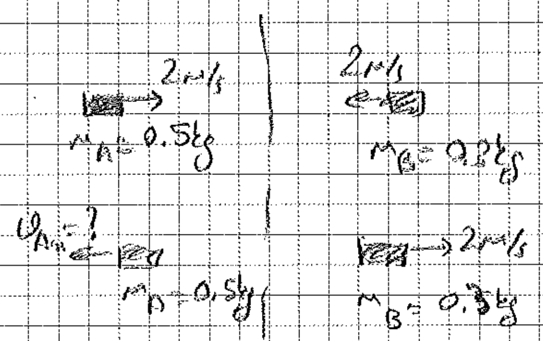
$\Rightarrow \vec{P} = \vec{p}_A + \vec{p}_B + \vec{p}_C + \dots = m\vec{v}_A + m\vec{v}_B + m\vec{v}_C + \dots$, total momentum of a system of particles

$\Rightarrow$ If there are no external forces applied to the system:

$$\boxed{\frac{d\vec{P}}{dt} = 0}$$

Total momentum of the system is conserved.

Ex 8.5: Two particles move toward each other on a frictionless surface, they both move with a speed of 2 m/s before the collision. After the collision particle B moves away with a speed of 2 m/s towards right.



What is the final velocity of particle A? How does the total kinetic energy change?

$m_A v_A + m_B v_B =$ conserved

$$\Rightarrow 0.5\,kg\,(2\,m/s) - 0.3\,kg\,2\,m/s = 0.3\,kg\,2\,m/s + m_A v_A$$

$$\Rightarrow m_A v_A = 1 - 0.6 - 0.6 = -0.2 \Rightarrow \boxed{v_A = -0.4\,m/s}$$

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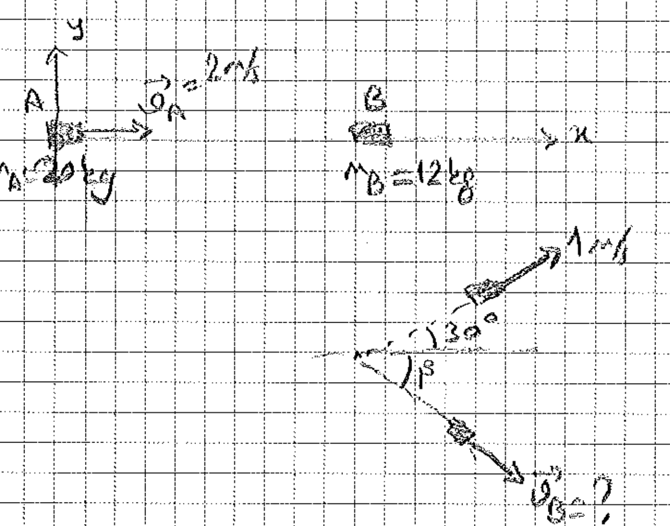
$$K_1 = K_{A_1} + K_{B_1} = \frac{1}{2} m_A (4) + \frac{1}{2} m_B (4)$$
$$K_2 = \frac{1}{2} m_A (0.16) + \frac{1}{2} m_B 4$$
$$\Rightarrow K_2 - K_1 = -\frac{1}{2} m_A (3.84)$$

Kinetic energy decreased after collision. "Inelastic collision"

Ex 8.6:



Conservation of momentum:

$$P_{Ax_1} + P_{Bx_1} = P_{Ax_2} + P_{Bx_2} \Rightarrow 20 \times 2 + 0 = 20 \times 1 \times \frac{\sqrt{3}}{2} + P_{Bx_2}$$
$$\Rightarrow P_{Bx_2} = 20\left(2 - \frac{\sqrt{3}}{2}\right) = 12\, v_B \cos\beta$$

$$P_{Ay_1} + P_{By_1} = P_{Ay_2} + P_{By_2} \Rightarrow 0 + 0 = 20 \times 1 \times \frac{1}{2} - 12 v_B \sin\beta \Rightarrow v_B \sin\beta = \frac{10}{12} = \frac{5}{6}$$

$$v_B \cos\beta = \frac{20}{12}\left(2 - \frac{\sqrt{3}}{2}\right) = \frac{5}{3}\left(2 - \frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow v_B \cos\beta = 1.89 \text{ m/s}$$
$$v_B \sin\beta = 0.83 \text{ m/s}$$
$$\Rightarrow \beta = 24°$$

$$\Rightarrow v_B^2 \sin^2\beta + v_B^2 \cos^2\beta = v_B^2 = (1.89)^2 + (0.83)^2$$
$$\Rightarrow v_B = 2.1 \text{ m/s}$$

8. 3. Inelastic Collisions:

- If no mechanic energy is lost or gained in the collision, the total kinetic energy of the system is the same after the collision as before. Such a collision is called elastic collision.

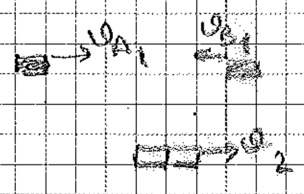
↳ Total kinetic energy of the system is conserved.

- If the total kinetic energy after the collision is less than that before the collision, the collision is called an inelastic collision.

Note that, in any collision in which external forces are 0, total momentum is conserved. Only in elastic collisions, total kinetic energy is also conserved.

Examples:

Ex 8.7: Completely inelastic collision: If the colliding bodies stick together and move as one body after the collision.



$$m_A v_{A_1} - m_B v_{B_1} = (m_A + m_B) v_2$$

Suppose $v_{B_1} = 0 \Rightarrow m_A v_{A_1} = (m_A + m_B) v_2$

$$\Rightarrow K_{A_1} + K_{B_1} = \frac{1}{2} m_A v_{A_1}^2$$

$$K_{A_2} + K_{B_2} = \frac{1}{2}(m_A + m_B) v_2^2 = \frac{1}{2}(m_A + m_B) \cdot \frac{m_A^2}{(m_A + m_B)^2} \cdot v_{A_1}^2$$

$$= \frac{1}{2} \frac{m_A^2}{(m_A + m_B)} v_{A_1}^2$$

$$= \frac{1}{2} m_A v_A^2 \underbrace{\left(\frac{m_A}{m_A + m_B}\right)}_{<1} < \frac{1}{2} m_A v_{A_1}^2$$

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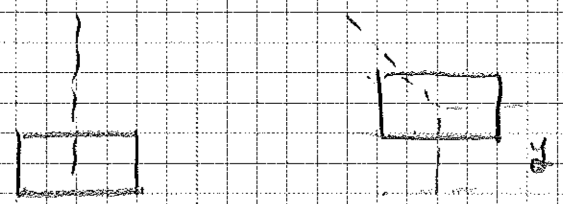
Ex 8.8: A bullet with mass m is fired into a block of wood with mass M, making a completely inelastic collision.
After the impact the block swings up to a maximum height y, given y, m and M, what is v_1?



$$m v_1 = (m+M) v_2 \Rightarrow v_1 = \frac{m+M}{m} v_2$$

$$\frac{1}{2}(m+M) v_2^2 = (m+M) g y$$

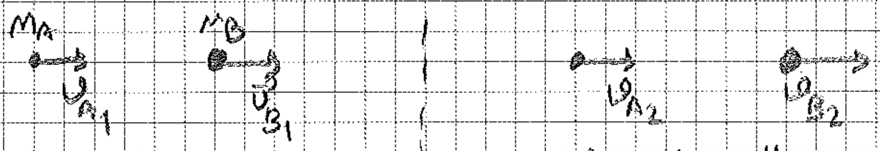
$$\Rightarrow \frac{1}{2} \cancel{(m+M)} \cdot \frac{m^2}{(m+M)^2} v_1^2 = \cancel{(m+M)} g y \Rightarrow v_1^2 = \frac{2(m+M)^2}{m^2} g \cdot y$$

$$\Rightarrow \boxed{v_1 = \left(\frac{m+M}{m}\right)\sqrt{2gy}}$$

8.4) Elastic Collisions :

Elastic collision is a collision in which kinetic energy as well as momentum is conserved.

Consider a one-dimensional collision:



Before the collision | After the collision

$$\left.\begin{aligned} m_A v_{A_1} + m_B v_{B_1} &= m_A v_{A_2} + m_B v_{B_2} \\ \frac{1}{2} m_A v_{A_1}^2 + \frac{1}{2} m_B v_{B_1}^2 &= \frac{1}{2} m_A v_{A_2}^2 + \frac{1}{2} m_B v_{B_2}^2 \end{aligned}\right\}$$

Two equations and two unknowns (v_{A_2}, v_{B_2})

If the body B is at rest before the collision:
$v_{B_1} = 0$

$$\Rightarrow \left.\begin{aligned} m_A v_{A_1} = m_A v_{A_2} + m_B v_{B_2} &\Rightarrow m_A (v_{A_1} - v_{A_2}) = m_B v_{B_2} \\ m_A v_{A_1}^2 = m_A v_{A_2}^2 + m_B v_{B_2}^2 &\Rightarrow m_A (v_{A_1}^2 - v_{A_2}^2) = m_B v_{B_2}^2 \end{aligned}\right\}$$

$$\Rightarrow \cancel{m_B} \cancel{v_{B_2}} (v_{A_1} + v_{A_2}) = \cancel{m_B} v_{B_2}^{\cancel{2}} \Rightarrow \boxed{v_{B_2} = v_{A_1} + v_{A_2}}$$

$$\Rightarrow m_A v_{A_1} = m_A v_{A_2} + m_B (v_{A_1} + v_{A_2}) \Rightarrow \boxed{v_{A_2} = \left(\frac{m_A - m_B}{m_A + m_B}\right) v_{A_1}}$$

$$v_{B_2} = v_{A_1} + v_{A_2} = \left(1 + \frac{m_A - m_B}{m_A + m_B}\right) v_{A_1} = \boxed{\left(\frac{2 m_A}{m_A + m_B}\right) v_{A_1}}$$

Interpretations: if $m_A > m_B \Rightarrow v_{A_2} > 0,\ v_{B_2} > 0$

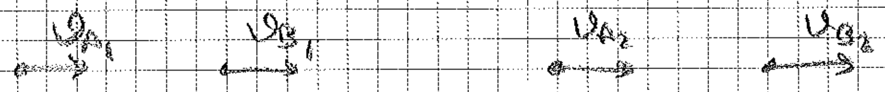
$m_A < m_B \Rightarrow v_{A_2} < 0,\ v_{B_2} > 0$

$m_A = m_B \Rightarrow v_{A_2} = 0,\ v_{B_2} > 0$

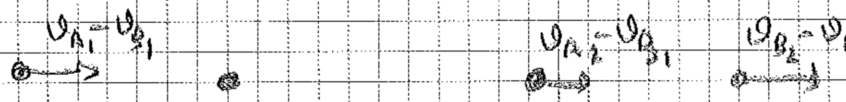
Generalizations:

$$v_{B_2} = v_{A_1} + v_{A_2} \Rightarrow v_{B_2} - v_{A_2} = v_{A_1}$$

Any collision in which the body B is not at rest can be analyzed in the frame of reference where B is stationary before the collision:



Laboratory frame



Frame of reference where B is initially stationary

use

$\Rightarrow v_{B_2} = v_{A_1} + v_{A_2}$

$$v_{B_2} - \cancel{v_{B_1}} = v_{A_1} - \cancel{v_{B_1}} + v_{A_2} - v_{B_1} \Rightarrow v_{A_1} - v_{B_1} = v_{B_2} - v_{A_2}$$

or $\boxed{v_{B_2} - v_{A_2} = -(v_{B_1} - v_{A_1})}$ "In an elastic collision the relative velocity of the two bodies has the same magnitude before and after the collision."

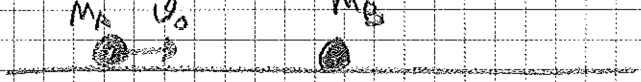
Similarly: $v_{B_2} - v_{B_1} = \left(\frac{2m_A}{m_A+m_B}\right)(v_{A_1} - v_{B_1}) \Rightarrow v_{B_2} = \left(\frac{2m_A}{m_A+m_B}\right)(v_{A_1} - v_{B_1}) + v_{B_1}$

$$v_{A_2} - v_{B_1} = \left(\frac{m_A-m_B}{m_A+m_B}\right)(v_{A_1} - v_{B_1}) \Rightarrow v_{A_2} = \left(\frac{m_A-m_B}{m_A+m_B}\right)(v_{A_1} - v_{B_1}) + v_{B_1}$$

$$= \left(\frac{m_A-m_B}{m_A+m_B}\right)v_{A_1} + \left(\frac{2m_B}{m_A+m_B}\right)v_{B_1}$$

$$v_{B_2} = \left(\frac{2m_A}{m_A+m_B}\right)v_{A_1} + \left(\frac{m_B-m_A}{m_A+m_B}\right)v_{B_1}$$

Prob 8.82:



In an elastic collision, initial velocities are v_0 and 0.

For what values of $\frac{m_A}{m_B}$ is the original kinetic energy shared equally by the two objects after the collision?

$K = \frac{1}{2} m_A v_0^2$: total kinetic energy

after the collision

$$K_A = \frac{1}{2} m_A v_A^2 = \frac{1}{2} m_A \left(\frac{m_A - m_B}{m_A + m_B}\right)^2 v_0^2$$

$$K_B = \frac{1}{2} m_B \left(\frac{2 m_A}{m_A + m_B}\right)^2 v_0^2$$

call $\frac{m_A}{m_B} = \alpha \Rightarrow$ $K_A = \frac{1}{2} m_A \left(\frac{\alpha - 1}{\alpha + 1}\right)^2 v_0^2$

$$K_B = \frac{1}{2} m_B \left(\frac{2\alpha}{\alpha + 1}\right)^2 v_0^2$$

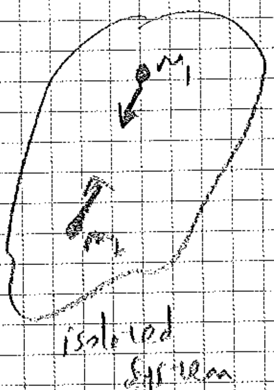
$$\Rightarrow K_A = \cancel{\frac{1}{2}} \cancel{m_A} \left(\frac{\alpha - 1}{\alpha + 1}\right)^2 \cancel{v_0^2} = \frac{1}{2} \cancel{\frac{1}{2}} \cancel{m_A} \cancel{v_0^2} \Rightarrow \left(\frac{\alpha - 1}{\alpha + 1}\right)^2 = \frac{1}{2}$$

$$\Rightarrow 2(\alpha^2 - 2\alpha + 1) = \alpha^2 + 2\alpha + 1 \Rightarrow \alpha^2 - 6\alpha + 1 = 0$$

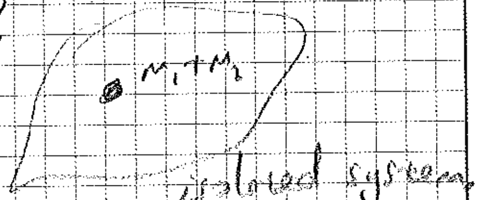
$$\Rightarrow \alpha = \frac{6 \pm \sqrt{36 - 4}}{2} = \frac{6 \pm \sqrt{32}}{2} = 3 \pm \sqrt{8}$$

$$\Rightarrow \boxed{\alpha = 5.8} \text{ or } \boxed{\alpha = 0.17}$$

8.5. Center of Mass:



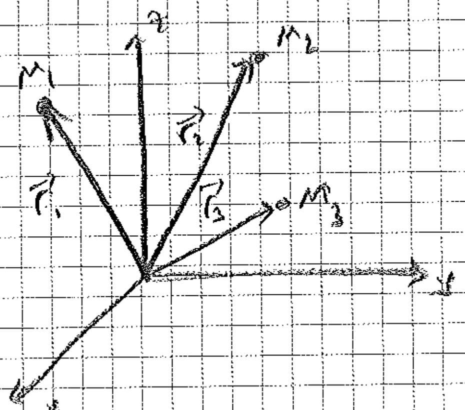
Can we represent the motion of the system of particles as the motion of a point particle with mass m_1+m_2?



$\vec{P} = m_1\vec{v}_1 + m_2\vec{v}_2$ is conserved

The answer is yes. We can analyze the motion of system of particles by analyzing the motion of the "center of mass".

Consider an isolated system of several particles:



If $\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots$ correspond to the position vectors of the particles with masses $m_1, m_2, m_3, \ldots$ the position vector of the center of mass is given as:

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \ldots}{m_1+m_2+m_3+\ldots} = \boxed{\frac{\sum_i m_i\vec{r}_i}{\sum_i m_i}}$$

//

⇒ The coordinates of the center of mass:

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \ldots}{m_1 + m_2} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

$$y_{cm} = \frac{\sum_i m_i y_i}{\sum_i m_i}, \quad z_{cm} = \frac{\sum_i m_i z_i}{\sum_i m_i}$$

In an isolated system: (no external forces)

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 + \ldots = \vec{P} = \text{constant}$$

$$\Rightarrow \vec{P} = \frac{d}{dt} \underbrace{\left(m_1 \vec{r}_1 + m_2 \vec{r}_2 + \ldots \right)}_{(m_1 + m_2 + \ldots)\vec{r}_{cm}} = (m_1 + m_2 + \ldots) \underbrace{\frac{d}{dt} \vec{r}_{cm}}_{\vec{v}_{cm}}$$

$\vec{v}_{cm}$: velocity of the center of mass

$\Rightarrow \vec{P} = M \, \vec{v}_{cm}$: Total momentum can be viewed as the momentum of a single point particle with mass M moving at a velocity $\vec{v}_{cm}$.

For an isolated system, total momentum $\vec{P}$ is constant

⇒ The velocity of the center of mass

$\vec{v}_{cm} = \dfrac{\vec{P}}{M}$ is also constant!

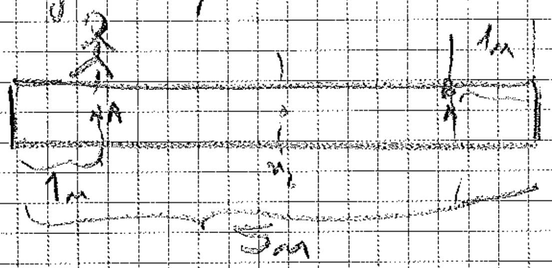
Prob. 8.44: Three boxes with masses of 0.3kg, 0.4kg and 0.2kg are at coordinates (0.2m, 0.3m), (0.1m, −0.4m), and (−0.3m, 0.6m).

Find the coordinates of the center of mass.

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1+m_2+m_3} = \frac{0.06 + 0.04 - 0.06}{0.9} = \frac{0.4}{9}\,m$$

$$y_{cm} = \frac{0.09 - 0.16 + 0.12}{0.9} = \frac{0.05}{0.9} = \frac{0.5}{9}\,m$$

Prob 8.94: A 45kg person stands up in a 60kg canoe of length 5m. The person moves from a point A to B. How far does the canoe move during this process?

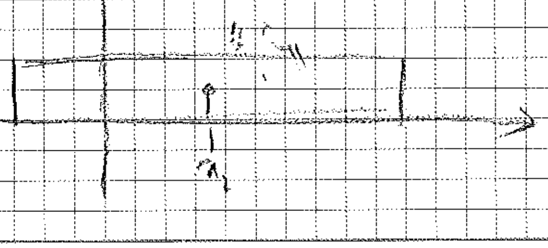


Isolated system ⇒ Center of mass should move at constant speed.

Initially center of mass is at rest so it should always stay at rest.

$$x_{cm} = \frac{45kg \times 1m + 60kg \times 2.5m}{105kg} = \frac{195}{105}\,m$$

At the end of the motion;



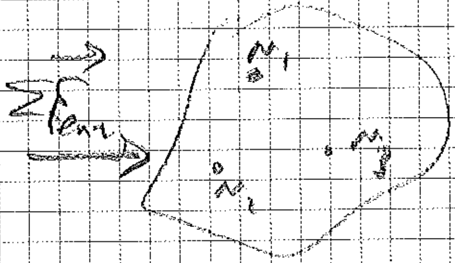
$$x_{cm} = \frac{45kg\cdot(x_2+1.5) + 60\cdot x_2}{\cancel{105}} = \frac{195}{\cancel{105}}$$

$$\Rightarrow 105x_2 = 195 - 67.5$$

$$\Rightarrow \boxed{x_2 = 1.2m}$$

External Forces:

If the net external force on a system of particles is not zero, then the total momentum is not conserved.

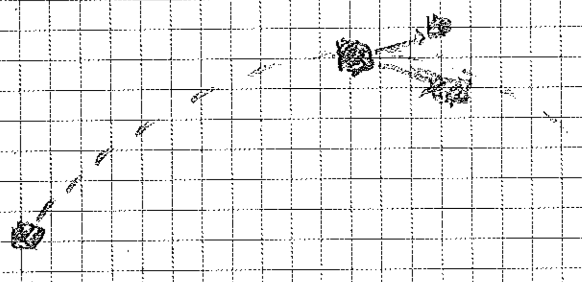


$$\Rightarrow \frac{d}{dt}\vec{p}_1 + \frac{d}{dt}\vec{p}_2 + \ldots = \underbrace{\Sigma \vec{F}_{ext}}_{\text{external forces}} + \underbrace{\Sigma \vec{F}_{int}}_{\text{internal forces}}$$

$\Sigma \vec{F}_{int} = 0$ because of Newton's 3^{rd} Law.

$$\Rightarrow \frac{d}{dt}\vec{P} = \Sigma \vec{F}_{ext} = \frac{d}{dt}(M\vec{v}_{cm}) \Rightarrow \boxed{M\vec{a}_{cm} = \Sigma \vec{F}_{ext}}$$

When a collection of particles is acted on by an external force, the center of mass moves just as if all mass were concentrated at the center of mass and were acted on by a net force equal to the sum of the external forces on the system.



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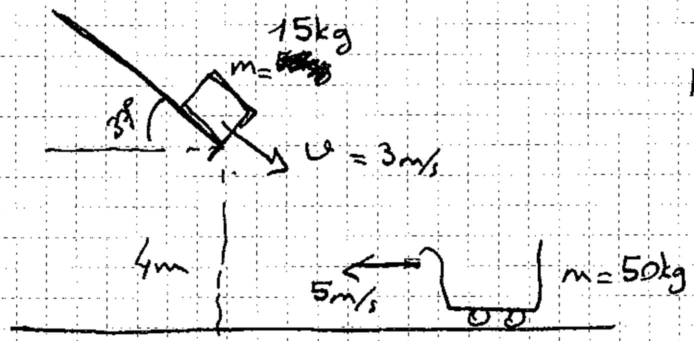
Completely inelastic collision:

Problem 8.81:



Package and cart roll off together

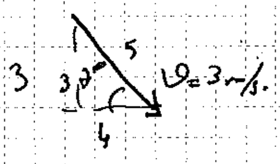
a) What is the speed of the package just before it lands in the chart?

b) What is the final speed of the cart?

a) Conservation of energy: $\cancel{m} g h + \frac{1}{2}\cancel{m} v^2 = \frac{1}{2}\cancel{m} v_2^2$

$$\Rightarrow v_2^2 = 10\,m/s^2 \times 4m + \frac{1}{2}\, 9 = \frac{89}{2} \Rightarrow v_2 \approx 6.7\,m/s$$

b) Conservation of momentum.



$$15\,kg \times 3\,m/s \times \cos 37° - 5\,m/s \times 50\,kg = (65\,kg)\, v_x$$

$$\overset{3}{\cancel{15}} \times 3 \times \frac{4}{\cancel{5}} - 250 = -214 = 65\,kg\, v_x \Rightarrow v_x = -\frac{214}{65} \approx -3.3\,m/s$$

$\cancel{-15kg \times 3 m/s \times \sin 37° = 0}$

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Problem 8.80:

(a) If the collision is elastic, what percentage of the original energy does each object have after the collision?

$$K_{A_1} = \frac{1}{2} m_A v_0^2$$

$$m_A v_0 = m_A v_{A_2} + m_B v_{B_2}$$

$$K_{A_2} = \frac{1}{2} m_A v_{A_2}^2 \quad , \quad v_{A_2} = \left(\frac{m_A - m_B}{m_A + m_B}\right) v_0 \quad \Rightarrow K_{A_2} = \frac{1}{2} m_A v_0^2 \left(\frac{m_A - m_B}{m_A + m_B}\right)^2$$

$$K_{B_2} = \frac{1}{2} m_B v_{B_2}^2 \quad , \quad v_{B_2} = \left(\frac{2 m_A}{m_A + m_B}\right) v_0 \quad \Rightarrow K_{B_2} = \frac{1}{2} m_B v_0^2 \left(\frac{2 m_A}{m_A + m_B}\right)^2$$

$$\Rightarrow \frac{K_{A_2}}{K_{A_1}} = \left(\frac{m_A - m_B}{m_A + m_B}\right)^2 \quad , \quad \frac{K_{B_2}}{K_{A_1}} = \frac{4 m_A m_B}{(m_A + m_B)^2}$$

(b) $m_A = m_B \Rightarrow K_{A_2} = 0 \quad , \quad K_{B_2} = K_{A_1}$

$$m_A = 5 m_B \Rightarrow K_{A_2} = \frac{16}{36} K_{A_1} \quad , \quad K_{B_2} = \frac{4 \times 5}{36} = \frac{20}{36} K_{A_1} = \frac{5}{9} K_{A_1}$$

$$= \frac{4}{9} K_{A_1}$$

(c) $$\left(\frac{m_A - m_B}{m_A + m_B}\right)^2 = \frac{4 m_A m_B}{(m_A + m_B)^2} \Rightarrow (m_A - m_B)^2 = 4 m_A m_B$$

$$\Rightarrow m_A^2 - 6 m_A m_B + m_B^2 = 0$$

$$\frac{m_A}{m_B} = \alpha \Rightarrow \quad \alpha^2 - 6\alpha + 1 = 0$$

$$\Rightarrow \alpha = \frac{6 \pm \sqrt{36 - 4}}{2} = \frac{6 \pm \sqrt{32}}{2}$$

$$\Rightarrow \alpha = \frac{6 + \sqrt{32}}{2} \quad \text{or} \quad \alpha = \frac{6 - \sqrt{32}}{2}$$

$$= 5.8 \qquad\qquad = 0.17$$

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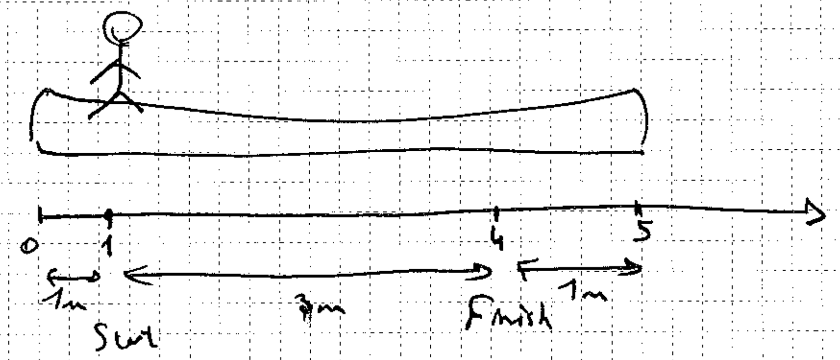
Problem 8.94 :

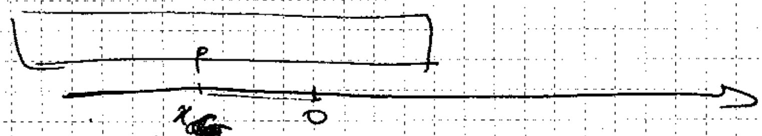


$m_A = 45kg$
$m_C = 60kg$

If you ignore resistance to motion of the canoe in the water, how far does the canoe move?

$$x_{cm} = \frac{45kg \times 1m + 60kg \times 2.5m}{105\, kg} = \frac{195\, kgm}{105\, kg}$$

after the movement: $x_{cm} = \frac{195}{\cancel{105}} = \frac{(x_c + 1.5)\, 45kg + x_c\, 60kg}{\cancel{105}}$



$\Rightarrow x_c\, 105 + 67.5kgm = 195kgm$

$$x_c = \frac{(195 - 67.5)kgm}{105kg}$$

$$\boxed{x_c = \frac{127.5\, kgm}{105\, kg}} = 1.2m$$

$$\frac{195}{105} =$$

$\Rightarrow$ Relative motion

$\Rightarrow x_{c_2} - x_{c_1} = 2.5 - 1.2 = 1.3$