

Closed book. No calculators are to be used for this quiz.

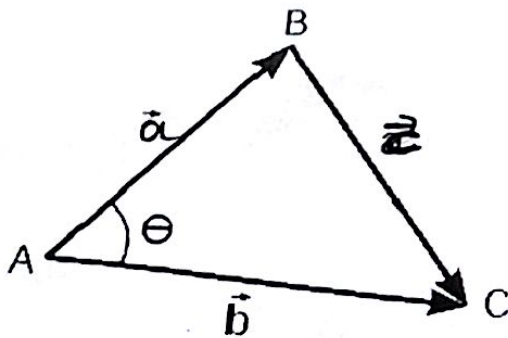
Quiz duration: 10 minutes

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Using vector techniques and considering the triangle formed by three vectors as shown above, show that $c^2 = a^2 + b^2 - 2ab \cos \theta$. (Hint: Consider the vector relation $\vec{c} = \vec{b} - \vec{a}$)

$$\vec{c} = \vec{b} - \vec{a}$$

$$(\vec{c})^2 = (\vec{b} - \vec{a})^2$$

$$|\vec{c}|^2 = |\vec{b} - \vec{a}|^2$$

$$|\vec{c}|^2 = |\vec{b}|^2 + |\vec{a}|^2 - 2|\vec{b} \cdot \vec{a}|$$

And $\vec{b} \cdot \vec{a} = |\vec{b}||\vec{a}|\cos\theta$ *

$$\Rightarrow |\vec{c}|^2 = |\vec{b}|^2 + |\vec{a}|^2 - 2|\vec{b}||\vec{a}|\cos\theta$$

$$c^2 = a^2 + b^2 - 2ab \cos\theta$$

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By direct substitution show that $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$ for the vectors $\vec{A} = \hat{i} - 2\hat{j}$, $\vec{B} = \hat{j} + \hat{k}$, and $\vec{C} = \hat{k}$.

$$\vec{B} \times \vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \hat{i}$$

$$\vec{A} \times (\vec{B} \times \vec{C}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 0 \\ 1 & 0 & 0 \end{vmatrix} = 2\hat{k}$$

$$\vec{A} \cdot \vec{C} = (\hat{i} - 2\hat{j}) \cdot (\hat{k}) = 0 \Rightarrow \vec{B}(\vec{A} \cdot \vec{C}) = 0$$

$$\vec{A} \cdot \vec{B} = (\hat{i} - 2\hat{j}) \cdot (\hat{j} + \hat{k}) = 1 \cdot 0 + (-2) \cdot 1 + 0 \cdot 1 = -2$$

$$\vec{C}(\vec{A} \cdot \vec{B}) = \hat{k}(-2) = -2\hat{k}$$

$$\rightarrow \vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$$

$$2\hat{k} = 0 - (-2\hat{k})$$

$$2\hat{k} = 2\hat{k} \quad \checkmark$$

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You are given vectors $\vec{A} = 5\hat{i} - 6.5\hat{j}$ and $\vec{B} = -3.5\hat{i} + 7\hat{j}$. A third vector \vec{C} lies in the xy-plane. Vector \vec{C} is perpendicular to vector \vec{A} , and the scalar product of \vec{C} with \vec{B} is 15. From this information, find the components of vector \vec{C} .

$$\vec{C} \perp \vec{A}, \quad \vec{C} \cdot \vec{B} = 15$$

If \vec{C} is perpendicular to \vec{A} , then the angle between them is 90°

$$\vec{C} \cdot \vec{A} = |\vec{C}| |\vec{A}| \cos \theta, \quad \theta = 90^\circ$$

$$\vec{C} \cdot \vec{A} = 0$$

$$\vec{C} \cdot \vec{A} = (c_x \hat{i} + c_y \hat{j}) \cdot (5\hat{i} - 6.5\hat{j}) = 0$$

$$5c_x - 6.5c_y = 0 \quad (1)$$

$$\vec{C} \cdot \vec{B} = (c_x \hat{i} + c_y \hat{j}) \cdot (-3.5\hat{i} + 7\hat{j}) = 15$$

$$-3.5c_x + 7c_y = 15 \quad (2)$$

Using (1) and (2):

$$3.5 / \quad 5c_x - 6.5c_y = 0$$

$$5 / \quad -3.5c_x + 7c_y = 15$$

$$12.25c_y = 75$$

$$c_y = 6.12 \approx 6$$

$$5c_x - 6.5(6.12) = 0$$

$$c_x = 7.95 \approx 8$$

$$\vec{C} = 6\hat{i} + 8\hat{j}$$

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Consider vectors $\vec{A} = -4\hat{i} + 5\hat{j}$ and $\vec{B} = 2\hat{i} + 5\hat{k}$. Find the \vec{C} which has magnitude 5 and which is parallel to $\vec{A} \times \vec{B}$.

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & 5 & 0 \\ 2 & 0 & 5 \end{vmatrix} = 25\hat{i} + 20\hat{j} - 10\hat{k}$$

$$|\vec{A} \times \vec{B}| = \sqrt{625 + 400 + 100} = \sqrt{1125} = 33.6$$

$$\frac{\vec{C}}{|\vec{C}|} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$$

$$\vec{C} = \frac{5 \cdot (\vec{A} \times \vec{B})}{33.6} = \frac{5 \cdot (25\hat{i} + 20\hat{j} - 10\hat{k})}{33.6}$$

$$\vec{C} = \frac{125}{33.6} \hat{i} + \frac{100}{33.6} \hat{j} - \frac{50}{33.6} \hat{k}$$

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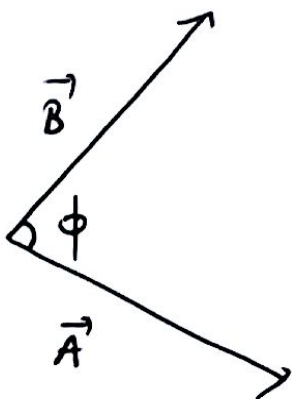
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When two vectors \vec{A} and \vec{B} are drawn from a common point, the angle between them is ϕ . Using vector techniques show that the magnitude of their vector sum ($|\vec{A} + \vec{B}|$) is given by $\sqrt{A^2 + B^2 + 2AB \cos \phi}$.



$$(|\vec{A} + \vec{B}|)^2 = (\vec{A} + \vec{B}) \cdot (\vec{A} + \vec{B})$$

$$= |\vec{A}|^2 + |\vec{B}|^2 + 2\vec{A} \cdot \vec{B}$$

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \phi$$

$$(|\vec{A} + \vec{B}|)^2 = |\vec{A}|^2 + |\vec{B}|^2 + 2|\vec{A}| |\vec{B}| \cos \phi$$

$$|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB \cos \phi}$$