

Closed book. No calculators are to be used for this quiz.

Quiz duration: 10 minutes

Name:

Student ID:

Signature:

A turntable rotates with a constant  $2.25 \text{ rad/s}^2$  angular acceleration. After 4 s it has rotated through an angle of 60 rad. Find the angular velocities of the wheel (i) at the beginning and (ii) at the end of the 4 s interval.

$$\alpha = 2.25 \text{ rad/s}^2 = \text{constant.}$$

$$\omega = \int \alpha dt = \alpha t + \omega_0$$

$$\theta = \int \omega dt = \int (\alpha t + \omega_0) dt = \frac{\alpha t^2}{2} + \omega_0 t + \theta_0$$

$$\text{Take } \theta_0 = 0 \rightarrow \theta = \frac{\alpha t^2}{2} + \omega_0 t.$$

$$t=4 \rightarrow \theta=60 = \frac{\alpha}{2} (4)^2 + \omega_0 \cdot 4.$$

$$\omega_0 = \frac{60 - 8 \cdot (2.25)}{4} = 10 \text{ rad/s}$$

$$\omega(t=4) = 2.25(4) + 10 = 19 \text{ rad/s}$$

## Section 2

## Quiz 10

05 December 2013

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Quiz duration: 10 minutes

Name:

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A flywheel of radius  $R$  starts from rest and accelerates with a constant angular acceleration  $\alpha$ . Give expressions for the tangential acceleration, the radial acceleration and the resultant acceleration of a point on its rim (i) at the start and (ii) after it has turned through  $\Delta\theta$ .

$$\alpha = \alpha_0 = \text{constant}$$

$$\omega = \int \alpha_0 dt = \alpha_0 t + \omega_0 \quad \omega_0 = 0 \rightarrow \omega = \alpha_0 t$$

$$a_{\text{tan.}} = r \alpha_0$$

$$a_{\text{rad.}} = \omega^2 r \quad \text{at } t=0,$$

$$a_{\text{tan}} = r \alpha_0 \quad a_{\text{rad}} = \omega^2 r = 0 \rightarrow a = r \alpha_0 = \sqrt{a_{\text{tan}}^2 + a_{\text{rad}}^2}$$

$$\theta = \int \omega dt = \frac{\alpha_0 t^2}{2} + \theta_0$$

$$\theta - \theta_0 = \Delta\theta = \frac{\alpha_0 t^2}{2}$$

$$t = \sqrt{\frac{2 \Delta\theta}{\alpha_0}}$$

$$\omega \text{ at } t = \frac{2 \Delta\theta}{\alpha_0} = \alpha_0 \sqrt{\frac{2 \Delta\theta}{\alpha_0}}$$

$$a_{\text{tan}} = r \alpha_0$$

$$a_{\text{rad}} = \omega^2 r = 2 \Delta\theta \alpha_0 r$$

$$a = \sqrt{a_{\text{rad}}^2 + a_{\text{tan}}^2} = \sqrt{r^2 \alpha_0^2 + 4 \Delta\theta^2 \alpha_0^2 r^2}$$

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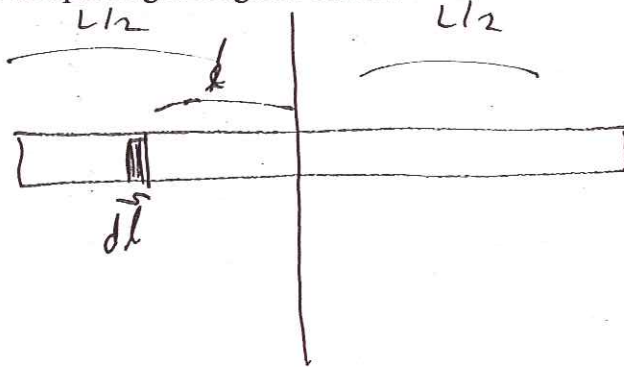
Quiz duration: 10 minutes

Name:

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Calculate the moment of inertia of a thin uniform rod of length  $L$ , about an axis perpendicular to it and passing through its center.



$$M/L = \lambda$$

$$dI = dm l^2 = \lambda dl \cdot l^2$$

$$I = \lambda \int_{-L/2}^{L/2} l^2 dl = \lambda \left. \frac{l^3}{3} \right|_0^{L/2} = 2 \lambda \frac{L^3}{24} = \frac{2M}{L} \frac{L^3}{24} = \frac{ML^2}{12}$$

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State and prove the parallel-axis theorem.

The relationship between the moment of inertia  $I_{cm}$  of a body of mass  $M$  about an axis through its center of mass and the moment of inertia  $I_p$  about any other axis parallel to the original one but displaced by a distance  $d$  is

$$I_p = I_{cm} + Md^2$$

Take  $x_{cm} = y_{cm} = z_{cm} = 0$

$$I_{cm} = \sum m_i (x_i^2 + y_i^2) \quad (\text{about center of mass})$$

about any other point  $(a, b)$

$$I_p = \sum m_i [(x_i - a)^2 + (y_i - b)^2]$$

$$= \underbrace{\sum m_i (x_i^2 + y_i^2)}_{I_{cm}} - 2a \sum m_i x_i - 2b \sum m_i y_i + \underbrace{(a^2 + b^2)}_d \sum m_i$$

since  $x_{cm} = y_{cm} = 0$

$$= I_{cm} + Md^2$$

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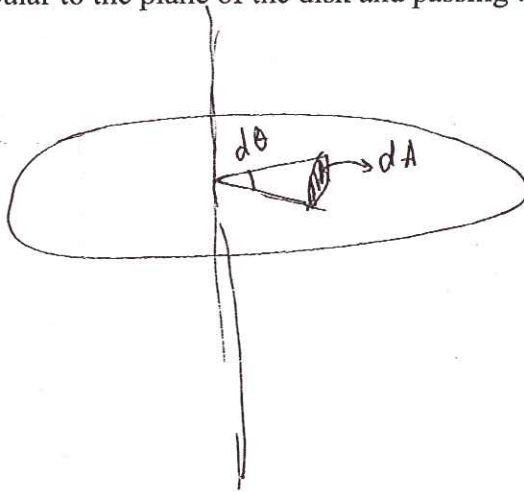
Quiz duration: 10 minutes

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Calculate the moment of inertia of a uniform, solid disk with mass  $M$  and radius  $R$  for an axis perpendicular to the plane of the disk and passing through its center.



$$\lambda = \frac{M}{\pi R^2}$$

Area element in polar coordinates

$$dA = r dr d\theta$$

$$dI = dm r^2 = \lambda r^2 dA = \lambda r^3 dr d\theta$$

$$I = \int_0^{2\pi} \int_0^R \lambda r^3 dr d\theta = \frac{\lambda R^4}{4} \cdot 2\pi = \frac{MR^2}{2}$$