

Closed book. No calculators are to be used for this quiz.

Quiz duration: 10 minutes

Name:

Student ID:

Signature:

The position vector of a particle of mass 2.0 kg is given as a function of time by $\vec{r} = (6.0\hat{i} + 5.0t\hat{j})$ (here, time t is given in seconds). Determine the angular momentum of the particle as a function of time.

Section 5

$$m = 2 \text{ kg}$$
$$\vec{r} = 6\hat{i} + 5t\hat{j} \text{ (m)}$$
$$\vec{L} = ?$$
$$\vec{L} = \vec{r} \times \vec{p}$$
$$= \vec{r} \times m\vec{v}$$
$$\vec{v} = \frac{d\vec{r}}{dt} = 5\hat{j} \text{ m/s}$$
$$= (6\hat{i} + 5t\hat{j}) \times (2 \text{ kg})(5\hat{j} \text{ m/s})$$
$$\vec{L} = \underbrace{60}_{\text{kg m}^2/\text{s}} \hat{k}$$

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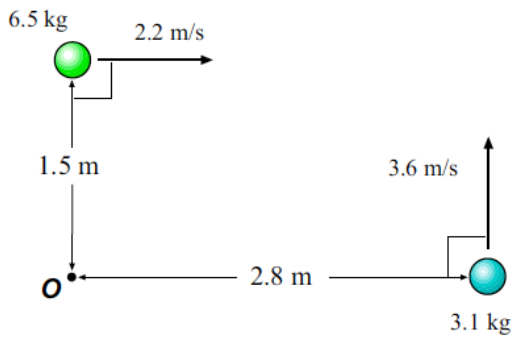
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Two objects are moving as shown in the figure. What is their total angular momentum about point O?



Section 2

$\vec{L}_{total} = ?$ $\vec{L}_{tot} = \vec{L}_1 + \vec{L}_2$

$L_1 = r_1 m_1 v_1$
 $= (2.8 m)(3.1 kg)(3.6 m/s)$

$L_2 = r_2 m_2 v_2$
 $= (1.5 m)(6.5 kg)(2.2 m/s)$

$\vec{L} = \vec{r} \times \vec{p}$
 $= \vec{r} \times m\vec{v}$

$\vec{L}_1 \text{ in } \hat{z}$
 $\vec{L}_2 \text{ in } -\hat{z}$

$\vec{L}_{total} = \vec{L}_1 + \vec{L}_2$
 $= 31.248 \hat{z} - 21.45 \hat{z}$
 $= 9.8 (kg \cdot m^2/s) \hat{z}$

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Suppose that the Sun runs out of nuclear fuel and suddenly collapses to form a white dwarf star, with a diameter equal to that of the Earth. Assuming no mass loss, what would then be the Sun's new rotation period, which currently is about 25 days? Assume that the Sun and the white dwarf are uniform, solid spheres; and the present radius of the Sun is approximately 100 times the radius of Earth. Hint: moment of inertia of a uniform solid sphere with mass M and radius R is $\frac{2MR^2}{5}$.

Section 3

$$R_{\text{sun}} = 100 R_E \quad T = 25 \text{ day} \quad I_1 \omega_1 = I_2 \omega_2 \quad \left(\omega = 2\pi f = \frac{2\pi}{T} \right)$$

$$R'_{\text{sun}} = R_E \quad T' = ?$$

$$\frac{2}{5} M (100 R_E)^2 \frac{2\pi}{(25 \text{ day})} = \frac{2}{5} M R_E^2 \frac{2\pi}{T'} \quad \left(I_{\text{solid sphere}} = \frac{2}{5} M R^2 \right)$$

$$T' = \frac{25 \text{ day}}{10000} = \frac{1 \text{ day}}{400} \cdot \frac{24 \text{ h}}{1 \text{ day}} \cdot \frac{60 \text{ min}}{1 \text{ h}}$$

$$T' = \frac{3.6 \text{ min}}{1}$$

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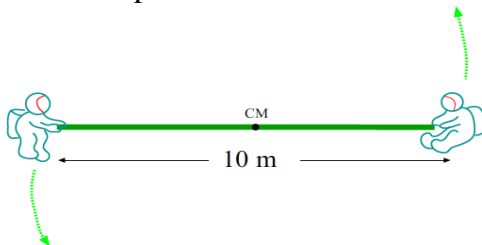
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Two astronauts each having a mass of 75 kg are connected by a 10m rope of negligible mass. They are isolated in space, orbiting their center of mass at speeds of 5.0 m/s . If, by pulling on the rope, the astronauts shorten the distance between them to 5.0m, what are their new speeds?



Section 4

Handwritten solution showing the conservation of angular momentum. It starts with a diagram of two 75 kg masses separated by 10 m, moving at $v = 5 \text{ m/s}$. The distance is then shortened to 5 m, and the new speed v' is to be found. The solution uses the equation $I_1 \omega_1 = I_2 \omega_2$ and calculates the final speed as $v' = 10 \text{ m/s}$.

$v = 5 \text{ m/s}$. The distance shortened to 5 m.
 $v' = ?$
 $L_1 = L_2$
 $I_1 \omega_1 = I_2 \omega_2$
 $\omega_2 = \frac{I_1}{I_2} \omega_1$
 $= \frac{2 \cdot 75 \cdot 25}{2 \cdot 75 \cdot (2,5)^2} = 4$
 $v' = \omega_2 \cdot r_2 = 4 \cdot 2,5 = \frac{10 \text{ m/s}}{}$
 $\omega_1 = \frac{v_1}{r_1} = \frac{5 \text{ m/s}}{5 \text{ m}} = 1$
 $I_1 = m r_1^2 = 2 \cdot 75 \cdot (5 \text{ m})^2$
 $I_2 = m r_2^2 = 2 \cdot 75 \cdot (2,5 \text{ m})^2$

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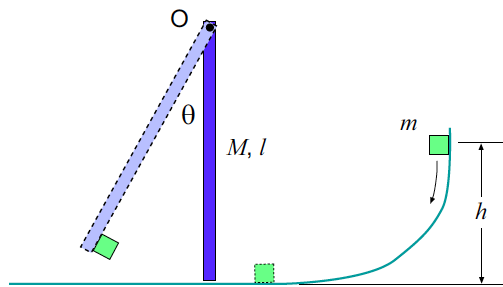
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The particle of mass m shown in the figure slides down the frictionless surface and collides with the uniform vertical rod, sticking to it. The rod pivots about O through the angle θ before momentarily coming to rest. Find θ in terms of the other parameters given in the figure.



Section 1

(I) $K_1 + U_1 = K_2 + U_2$

$v = \sqrt{2gh}$ ($v = \omega r$)

$\omega = \frac{\sqrt{2gh}}{l}$

(II) $L_1 = L_2$ since the external torque is zero, angular momentum is conserved.

$I_1 \omega_1 = I_2 \omega_2$

(III) $K_1 + U_1 = K_2 + U_2$ you can denote as K_2, U_2 and K_3, U_3

$\frac{1}{2} \left(\frac{4}{3} ml^2 \right) \omega_2^2 + M \frac{l}{2} g = mgl + Mgd \frac{1}{2}$

$\frac{1}{2} \left(\frac{4}{3} ml^2 \right) \frac{g^2}{16} \frac{2gh}{l^2} + M \frac{l}{2} g = d \frac{(2mg + Mg)}{2}$

$\frac{3}{2} mgh + Mgl = d \frac{2mg + Mg}{2}$

$d = \frac{3mgh + 2Ml}{4m + 2M}$

$d = l(1 - \cos\theta)$

$\cos\theta = 1 - d/l \rightarrow \theta = \cos^{-1}(1 - d/l)$

=(moment of inertia of the rod + moment of inertia of the particle) x ω_2