

Closed book. No calculators are to be used for this quiz.

Quiz duration: 10 minutes

First Name:

Last name:

Student ID:

Signature:

Consider the motion shown in the figure, where a ball starts moving at time $t = 0$ with speed v_0 and making an angle of $\pi/4$ radians with the horizontal axis. If the horizontal and vertical accelerations of the ball are given by $a_x = A$ and $a_y = -B$, respectively, find the maximum displacements in the vertical and horizontal directions. Here A and B are positive constants. You may ignore the effects of gravity.



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Consider the motion shown in the figure, where a ball starts moving at time $t = 0$ with speed v_0 and making an angle of 37° with the horizontal axis. If the horizontal and vertical accelerations of the ball are given by $a_x = A$ and $a_y = -Bt$, respectively, find the maximum height reached by the object during its motion. Here A and B are positive constants. You may ignore the effects of gravity. Warning: acceleration is time-dependent.



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Consider the motion shown in the figure, where a ball starts moving at time $t = 0$ with speed v_0 and making an angle of 53° with the horizontal axis. If the horizontal and vertical accelerations of the ball are given by $a_x = A t$ and $a_y = -B$, respectively, find the range of the motion in the horizontal direction. Here A and B are positive constants. You may ignore the effects of gravity. Warning: acceleration is time-dependent.



Section 1

10 October 2013

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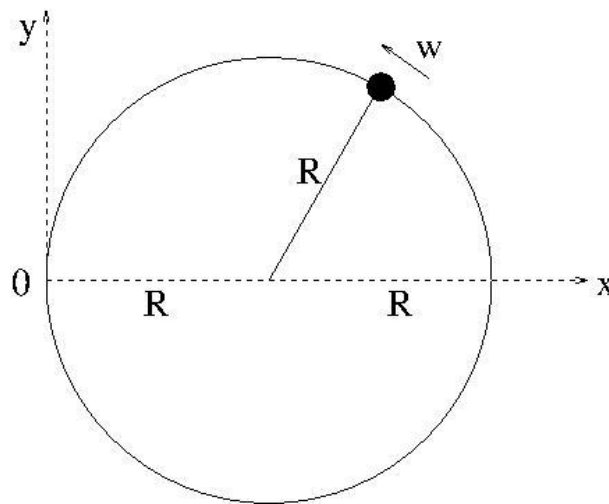
First Name:

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Consider the uniform circular motion shown in the figure, where an object is rotating along the circular track with a constant angular frequency $\omega = 2\pi/T$. Here T is the period of the motion. Using the x - y coordinate system indicated in the figure, find the position and acceleration of the object as a function of time t , assuming the object passes $(x = 0, y = 0)$ point at time $t = 0$ s.



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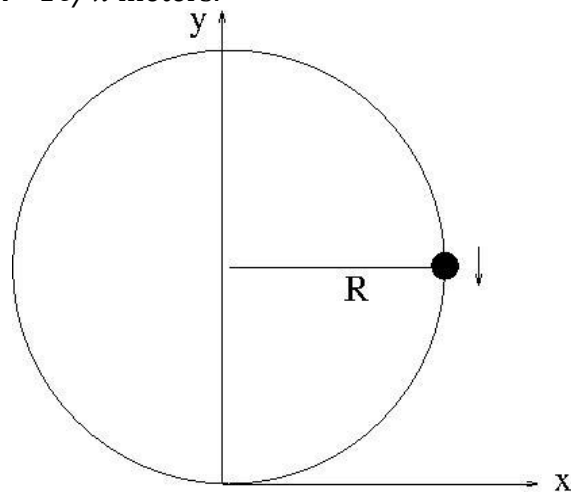
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Consider the nonuniform circular motion shown in the figure, where an object is rotating along the circular track with a time-dependent tangential acceleration given by $a_{\text{tang}} = A t$, where A is a positive constant and t is time. If the object starts at rest from the position shown in the figure, find its position in the x - y coordinate system at time $t = 2\text{s}$. Here, you may set $A = 3\text{m/s}^3$ and $R = 16/\pi$ meters.



1 Section 1

From the figure, it is clear that

$$\begin{aligned}x &= R + R\cos(\alpha) \\ y &= R\sin(\alpha)\end{aligned}$$

and the equation for the angle α at any time is equal to

$$\alpha = \omega t$$

so, equations of motion for x and y axes are

$$\begin{aligned}x &= R + R\cos(\omega t) \\ y &= R\sin(\omega t)\end{aligned}$$

To find velocities, differentiate x and y coordinates with respect to time

$$\begin{aligned}\frac{dx}{dt} &= v_x = -R\sin(\omega t) \\ \frac{dy}{dt} &= v_y = R\cos(\omega t) \\ a_x &= \frac{dv_x}{dt} = -R\omega^2\cos(\omega t) \\ a_y &= \frac{dv_y}{dt} = -R\omega^2\sin(\omega t)\end{aligned}$$

2 Section 2

Equations of motion for x and y axes are

$$y = -\frac{1}{2}Bt^2 + \frac{v_0t}{\sqrt{2}}$$

(equation of motion with cons. acceleration; $a=-B$, $v_0=v_0 \sin(45)$, $y_0=0$)

$$x = \frac{1}{2}At^2 + \frac{v_0t}{\sqrt{2}}$$

(same as y where $a=A$, $v_0=v_0\cos(45)$, $x_0=0$)

Then, in order to find maximum of y-function;

$$\begin{aligned}\frac{dy}{dt} &= 0 = -Bt + \frac{v_0}{\sqrt{2}} \\ t &= \frac{v_0}{B\sqrt{2}}\end{aligned}$$

Plug $t = \frac{v_0}{B\sqrt{2}}$ into y equation of motion to find maximum of y.

$$y_{max} = \frac{v_0^2}{4B}$$

Same procedure for the range in x-direction with $t_{flight} = 2t$

$$x_{max} = v_0^2 \left(\frac{A+1}{B} \right)$$

3 Section 3

$$a_{tang} = At$$

Since the tangential acceleration is given, this motion does not have any difference from motion along a straight line.

$$v_{tang} = \int a_{tang} dt = \int At dt = \frac{At^2}{2} \quad x_{tang} = \int v_{tang} dt = \int At^2/2 = \frac{At^3}{6}$$

In two seconds, total distance covered is

$$x(t=2) = A(2)^3/6 = 4m.$$

The circumference of the circle is

$$2\pi R = 32m.$$

Total distance covered is equal to the 1/8 of circumference. Therefore, the total angle covered is

$$\alpha = \frac{2\pi}{8} = \pi/4$$

In this problem, x and y coordinates can be found via

$$\begin{aligned} x &= R\cos(\alpha) \\ y &= R - R\sin(\alpha) \end{aligned}$$

where

$$\begin{aligned} R &= \frac{16}{\pi} \\ \alpha &= \pi/4 \end{aligned}$$

4 Section 4

Equations of motion;

$$x = \frac{At^2}{2} + v_0 \cos(\alpha)$$

since it is motion with constant acceleration, and

$$v_y = \int -Btdt + v_0 \sin(\alpha)$$
$$y = \int \left(-\frac{Bt^2}{2} + v_0 \sin(\alpha)\right) dt = \frac{Bt^3}{6} + v_0 \sin(\alpha)t$$

To find maximum of y, take the derivative with respect to time and set equal to zero

$$\frac{dy}{dt} = 0 = -\frac{Bt^2}{2} + v_0 \sin(\alpha)$$
$$t' = \sqrt{\left(\frac{2v_0 \sin(\alpha)}{B}\right)}$$

Maximum height that the particle will achieve is

$$y(t = t') = -\frac{B}{6} \left(\frac{2v_0 \sin(\alpha)}{B}\right)^{3/2} + v_0 \sin(\alpha) \sqrt{\left(\frac{2v_0 \sin(\alpha)}{B}\right)}$$

where $\alpha = 37$

5 Section 5

Equations of motion for x and y directions are

$$y = -\frac{Bt^2}{2} + v_0 \sin(\alpha)t$$
$$v_x = \int Atdt + v_0 \cos(\alpha) = \frac{At^2}{2} + v_0 \cos(\alpha)$$
$$x = \int \left(\frac{At^2}{2} + v_0 \cos(\alpha)\right) dt = \frac{At^3}{6} + v_0 \cos(\alpha)t$$

Time of flight t_{flight} can be calculated from the time required for the ball to reach its maximum height

$$\frac{dy}{dt} = 0 = -Bt + v_0 \sin(\alpha)$$
$$t' = \frac{v_0 \sin(\alpha)}{B}$$
$$t_{flight} = 2t'$$

and maximum displacement in x direction will be

$$x(t = t_{flight}) = \frac{A}{6} \left(\frac{8v_0^3 \sin^3(\alpha)}{B^3}\right) + v_0 \cos(\alpha) \left(\frac{2v_0 \sin(\alpha)}{B}\right)$$