

Closed book. No calculators are to be used for this quiz.

Quiz duration: 10 minutes

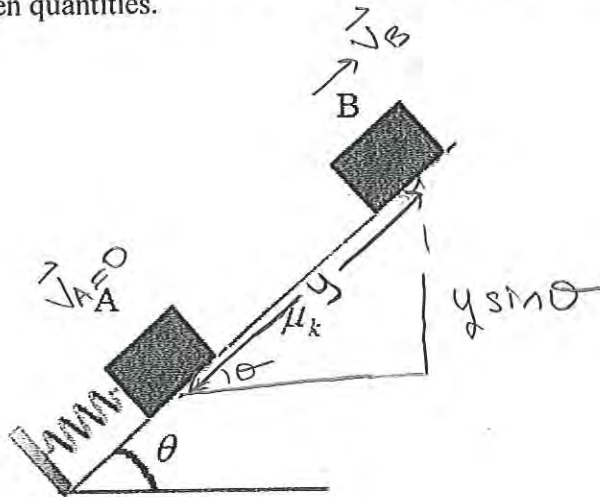
First Name:

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Student ID:

Signature:

A block with mass m is placed against a compressed spring at the bottom of an incline with angle θ at point A. When the spring is released, it projects the block up the incline. At point B, a distance of y up the incline from A, the block's velocity is v_B and is no longer in contact with the spring. The coefficient of kinetic friction between the block and the incline is μ_k . The mass of the spring is negligible. Calculate the amount of potential energy that was initially stored in the spring in terms of the given quantities.



$$* U_A + K_A + W_{oth} = U_B + K_B$$

$$U = U_{Block} + U_{el}$$

at point A;

$$U_{Block} = 0 \quad // \text{we take the point A as an origin.}$$

$$U_A = 0 + U_{el} \quad \text{Spring is compressed}$$

$$K_A = 0 \quad // \vec{v}_A = 0$$

at point B;

$$U_{Block} = mgy \sin \theta \quad // \text{spring is not compressed.}$$

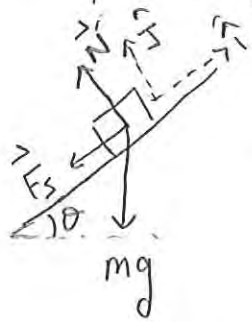
$$U_{el} = 0$$

$$U_B = mgy \sin \theta$$

$$K_B = \frac{1}{2} m v_B^2$$

$W_{oth} = W_f$ \therefore only external force applied is friction.

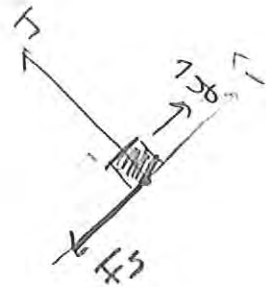
$$W_f = \vec{F}_s \cdot \vec{y}$$



$$N = mg \cos \theta \hat{j}$$

$$|F_s| = |N \mu_k|$$

$$\vec{F}_s = -mg \cos \theta \mu_k \hat{i}$$



$$W_f = -mg \cos \theta \mu_k \hat{i} \cdot y \hat{i} \quad \hat{i} \cdot \hat{i} = 1$$

$$W_f = -mg \cos \theta \mu_k y \quad //$$

$$W_{oth} = W_f = -mg \cos \theta \mu_k y$$

put all into 1st formula

$$U_A + K_A + W_{oth} = U_B + K_B$$

$$U_{el} + 0 - mg \cos \theta \mu_k y = mgy \sin \theta + \frac{1}{2} m v_B^2$$

$$\Rightarrow U_{el} = mgy \sin \theta + \frac{1}{2} m v_B^2 + \mu_k m g \cos \theta y$$

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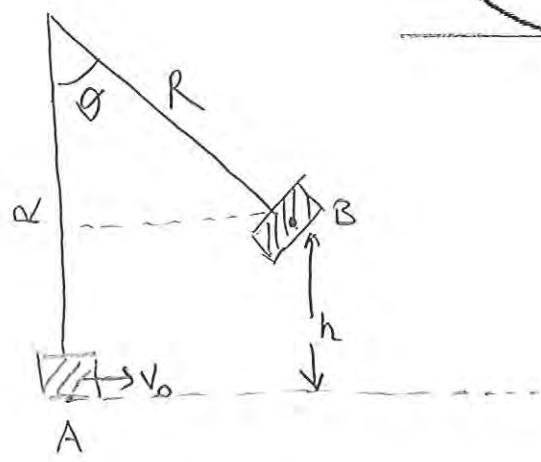
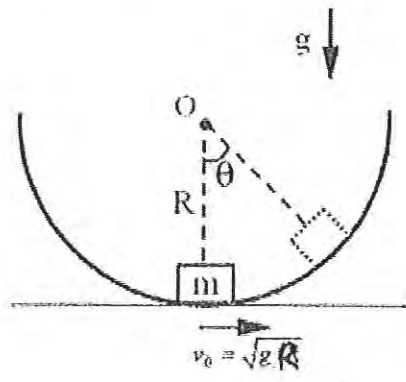
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A small particle with mass m is moving in a vertical circular track that has a radius R as shown in the figure below. The particle is initially at the lowest point of the track, and has an initial velocity \sqrt{gR} in the direction shown. (g is the magnitude of the gravitational acceleration).

Consider no friction, what is the maximum angle θ that a line from the center of the circle to the particle makes with the vertical when the particle reaches its maximum height?



$$h = R - R \cos \theta$$

$$U_A + K_A + W_{oth} = U_B + K_B$$

$W_{oth} = 0$ // no external force is applied
no friction

$$U_A = 0 \quad // \text{on the ground}$$

$$K_A = \frac{1}{2} m v_0^2 = \frac{1}{2} m (\sqrt{gR})^2 = \frac{m g R}{2}$$

$$U_B = mgh \\ = mg(R - R\cos\theta) = mgR(1 - \cos\theta)$$

$$K_B = 0 \quad // \text{ Since it is maximum height } v_B = 0$$

put all into;

$$\Rightarrow \quad " \quad U_A + K_A = U_B + K_B$$

$$0 + \frac{mgR}{2} + 0 = mgR(1 - \cos\theta) + 0$$

$$\frac{mgR}{2} = mgR(1 - \cos\theta)$$

$$\frac{1}{2} = 1 - \cos\theta \Rightarrow \cos\theta = \frac{1}{2}$$

$$\theta = \arccos\left(\frac{1}{2}\right) = \underline{\underline{60^\circ}}$$

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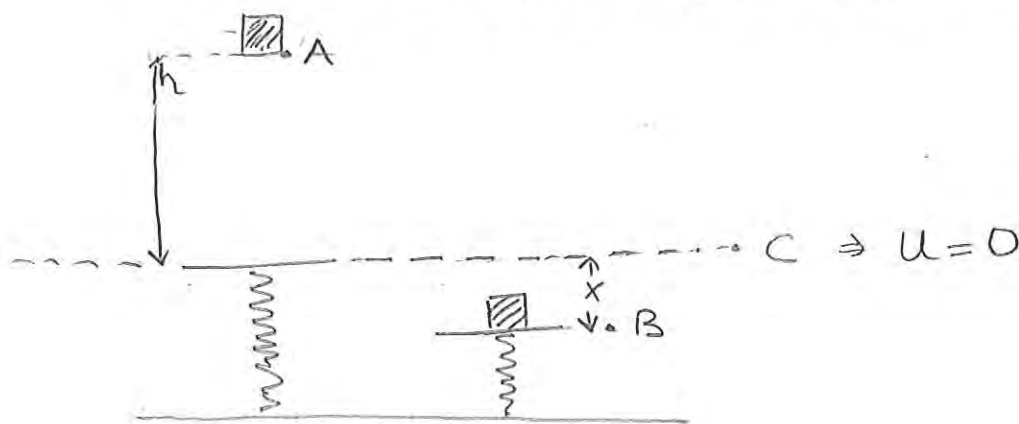
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A block of mass m is dropped from height h onto a spring of spring constant k . Find the maximum distance the string is compressed. (Express your results in terms of given quantities.)



$$U_A + K_A + W_{oth} = U_B + K_B$$

$W_{oth} = 0$ no external force is applied.

$$U = U_{\text{Block}} + U_{\text{el}}$$

at point A;

$$U_{\text{Block}} = mgh \quad // \text{ if i take the line C as ground level as shown in the figure.}$$

$$U_{\text{el}} = 0 \quad // \text{ spring is not compressed.}$$

$$K_A = 0 \quad // v_A = 0 \quad \text{dropped} \Rightarrow \text{no initial velocity.}$$

at point B;

$$U_{\text{Block}} = -mgx \quad // \text{ in negative side of the ground}$$

$$U_{\text{el}} = \frac{1}{2} kx^2$$

$$K_B = 0 \quad // v_B = 0 \quad \text{since it is maximum distance at point B.}$$

put all the equations in 1st formula

$$U_A + K_A + W_{oth} = U_B + K_B$$

$$mgh + 0 + 0 = -mgx + \frac{1}{2}kx^2 + 0$$

$$\Rightarrow \frac{1}{2}kx^2 - mgx - mgh = 0$$

Reminder;

$$ax^2 + bx + c = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow a = \frac{k}{2}, \quad b = -mg \quad c = -mgh$$

$$x_{1,2} = \frac{+mg \pm \sqrt{m^2g^2 + 4 \cdot \frac{k}{2}mgh}}{k}$$

$$= \frac{mg \pm \sqrt{m^2g^2 + 2mghk}}{k}$$

// we have \pm positive (negative) answer.

we know that x is in negative side of the ground.
so x is negative

$$x = \frac{mg - \sqrt{m^2g^2 + 2mghk}}{k}$$

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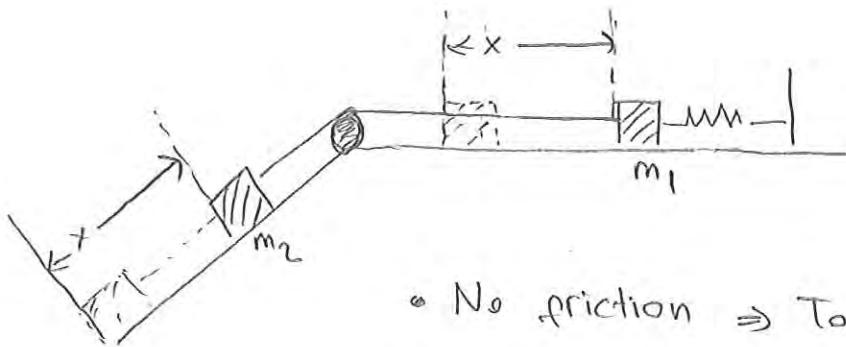
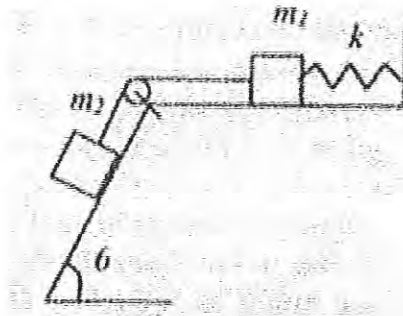
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In the figure, the block of mass m_1 is connected by a spring to a wall on the right side and to block of mass m_2 by a string through a pulley on the left side. The block m_2 is suspended on an inclined plane. The string is always tight. The gravitational acceleration is g . The whole system is frictionless. Answer the following questions. Express your answer only in terms of masses, g , k and θ .

Suppose that the system is released from the position, when the spring is unstretched/uncompressed and the masses are not moving. What will be the maximum potential energy stored in the spring during the first downward motion?



// Assume spring is stretched x , when it has max. potential energy

• No friction \Rightarrow Total energy is conserved.

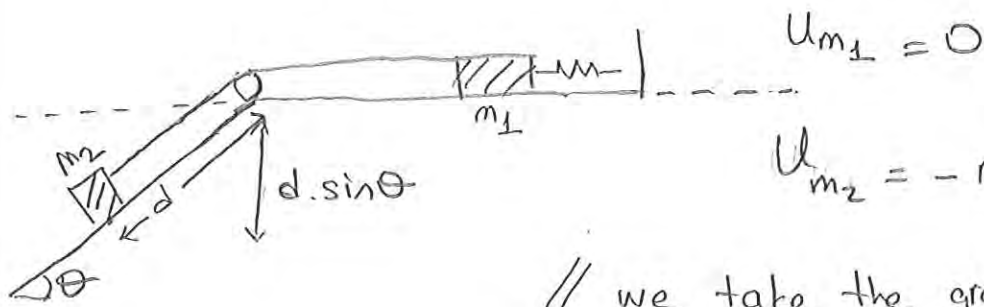
$$U_i + K_i = U_f + K_f$$

$$U = U_{m_1} + U_{m_2} + U_{el}$$

$$K = K_{m_1} + K_{m_2}$$

// U_{m_1} = potential energy of the block m_1

initial condition



$$U_{m_1} = 0$$

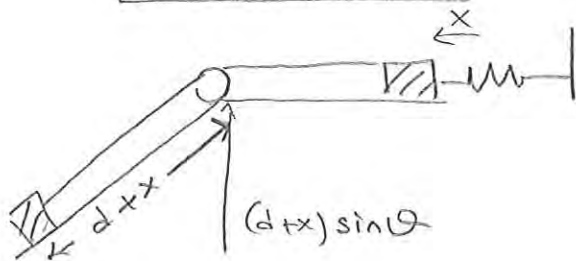
$$U_{m_2} = -m_2 g d \sin \theta$$

// we take the ground level "0" as line showed with dashed line (---)

$U_{el} = 0$, initially spring is unstretched.

$$U_i = -m_2 g d \sin \theta \quad K_i = 0 \quad ; \text{ since system is at rest.}$$

final condition



$$U_{m_1} = 0$$

$$U_{m_2} = -m_2 g (d+x) \sin \theta$$

$$U_{el} = \frac{1}{2} k x^2$$

$K_f = 0$: maximum distance that velocities are zero.

$$U_f = -m_2 g (d+x) \sin \theta + \frac{1}{2} k x^2$$

$$\Rightarrow U_i + K_i = U_f + K_f$$

$$-m_2 g d \sin \theta + 0 = -m_2 g (d+x) \sin \theta + \frac{1}{2} k x^2 + 0$$

$$= -m_2 g d \sin \theta = -m_2 g d \sin \theta - m_2 g x \sin \theta + \frac{1}{2} k x^2$$

$$\frac{1}{2} k x^2 = m_2 g x \sin \theta$$

$$x = \frac{2 m_2 g \sin \theta}{k} ; \quad U_{el} = \frac{1}{2} k x^2 = \frac{2 (m_2 g \sin \theta)^2}{k} //$$

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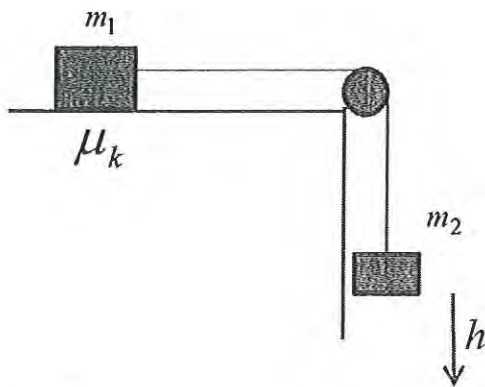
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The coefficient of kinetic friction between the block m_1 and the surface is μ_k . The system starts from rest. In terms of the given values, find the speed of m_2 when it has fallen a distance of h .

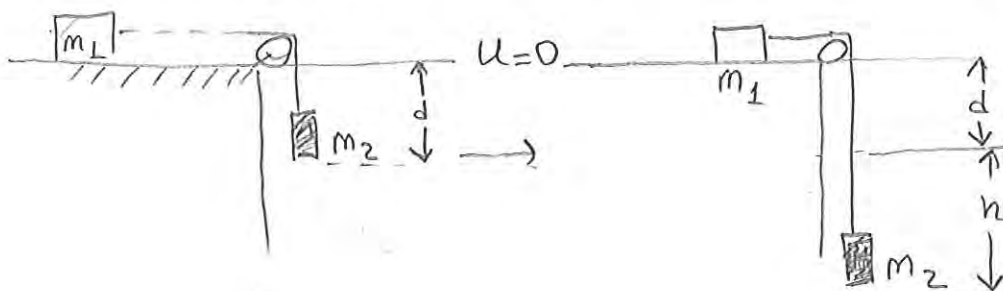


we have a friction; we need to use the formula;

$$U_i + K_i + W_{oth} = U_f + K_f$$

$$\Rightarrow U = U_{m_1} + U_{m_2}$$

$$K = K_{m_1} + K_{m_2}$$



lets take the top of the surface as ground; (zero level)

initial condition;

$$U_{m_1} = 0$$

$$U_{m_2} = -m_2 g d$$

$$U_i = -m_2 g d$$

$$K_{m_1}, K_{m_2} = 0$$

since system is at rest.

$$K_i = 0$$

final conditions

$$\left. \begin{aligned} U_{m_1} &= 0 \\ U_{m_2} &= -m_2 g (d+h) \end{aligned} \right\} U_f = -m_2 g (d+h)$$

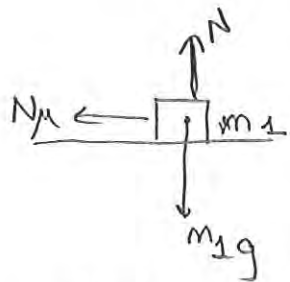
$$\left. \begin{aligned} K_{m_1} &= \frac{1}{2} m_1 v_1^2 \\ K_{m_2} &= \frac{1}{2} m_2 v_2^2 \end{aligned} \right\} |v_1| = |v_2| \text{ they're attached to each other with rope.}$$

$$K_f = \frac{1}{2} (m_1 + m_2) v^2$$

$W_{oth} = W_f$ \therefore we have a friction doing some work

$$W_f = \vec{F}_s \cdot \vec{s}$$

$$F_s = -m_1 g \mu_k \hat{i}$$



$$N = m_1 g$$

$$F_s = N \mu_k = m_1 g \mu_k$$

$\vec{s} = h \hat{i}$ \leftarrow distance that m_1 is moving

$$W_f = -m_1 g h \mu_k$$

$$\Rightarrow U_i + K_i + W_{oth} = U_f + K_f$$

$$-m_2 g d + 0 - m_1 g h \mu_k = -m_2 g (d+h) + \frac{1}{2} (m_1 + m_2) v^2$$

$$= -m_1 g h \mu_k = -m_2 g h + \frac{1}{2} (m_1 + m_2) v^2$$

$$\Rightarrow v = \sqrt{\frac{2(m_2 g h - m_1 g h \mu_k)}{m_1 + m_2}}$$