$\boxed{3.25}$ Earth's radius = 6380 km = $6,38 \times 10^6$ m.

$T = 24\,h = (24h \cdot 3600 s/h) = 86400$ s.

a) radial acc. of an object at equator?

$$a_{rad} = \frac{4\pi^2 R}{T^2} = \frac{4\pi^2 (6.38 \times 10^6 m.)}{(86400 s)^2} = 0.034\, m/s^2$$

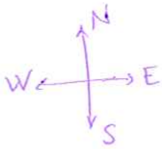
→ as a fraction of g: $a_{rad} = 0.034\, m/s^2 \cdot \frac{g}{9{,}8 m/s^2} = 3.4 \times 10^{-3} g$.

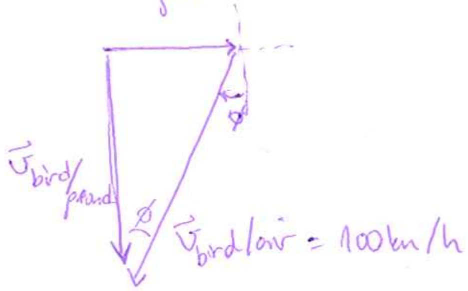
b) If $a_{rad} > g$ at the equator, objects will fly off the surface. what would be the period of earth's rotation?

$$T^2 = \frac{4\pi^2 R}{a_{rad}} \rightarrow T = \sqrt{\frac{4\pi^2 R}{a_{rad}}} \qquad a_{rad} = g$$

$$T = \sqrt{\frac{4\pi^2 (6.38 \times 10^6 m)}{9{,}8\, m/s^2}} = 5070 s. = 1.4 h.$$

$\boxed{3.39}$ $\vec{v}_{air/ground} = 40$ km/h





a) $\sin\phi = \frac{\vec{v}_{air/ground}}{\vec{v}_{bird/air}} = \frac{40 km/h}{100 km/h} \Rightarrow \phi = 24°$ west of south.

b) $\vec{v}_{bird/ground} = \sqrt{v_{bird/air}^2 - v_{air/ground}^2}$

$= 91.7$ km/h

time to take a ground distance of 500 km from N → S:

$$t = \frac{d}{v_{bird/ground}} = \frac{500 km}{91.7 km/h} = 5.5 h$$

3.40

circular track 100 m. in diameter

athlete runs at constant speed of 6 m/s.

x- and y- components of $v_{average}$ and $a_{average}$ between points:

a) A-B

b) A-C

c) C-D

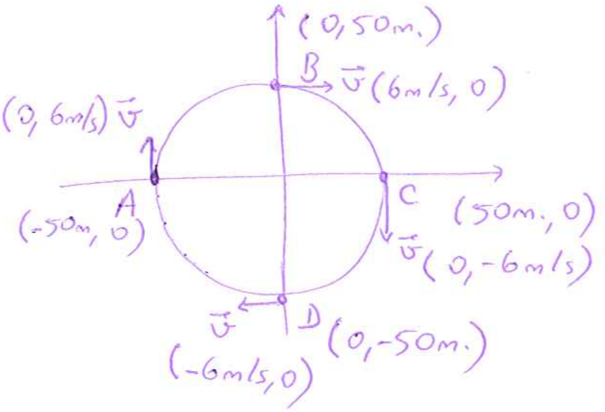
d) A-A

e) average velocity btw A-B

time for full lap:

$$t = \frac{2\pi r}{v} = \frac{2\pi(50m.)}{6m/s}$$

$$t = 52.4 s.$$



a) 1/4 lap $\rightarrow$ $t = \frac{52.4s}{4} = 13.1 s.$

$$(v_x)_{av} = \frac{\Delta x}{\Delta t} = \frac{0-(-50m)}{13.1s} = 3.8 m/s$$

$$(v_y)_{av} = \frac{\Delta y}{\Delta t} = \frac{50m - 0}{13.1s} = 3.8 m/s.$$

$$(a_x)_{av} = \frac{\Delta v_x}{\Delta t} = \frac{6m/s - 0}{13.1s} = 0,46 m/s^2$$

$$(a_y)_{av} = \frac{\Delta v_y}{\Delta t} = \frac{0 - 6m/s}{13.1s} = -0,46 m/s^2$$

b) 1/2 lap $\rightarrow t = \frac{52.4s}{2} = 26.2 s.$

$$(v_x)_{av} = \frac{50m - (-50m)}{26.2s} = 3.8 m/s$$

$$(v_y)_{av} = \frac{\Delta y}{\Delta t} = 0$$

$$(a_x)_{av} = \frac{\Delta v_x}{\Delta t} = 0$$

$$(a_y)_{av} = \frac{\Delta v_y}{\Delta t} = -0.46 m/s^2$$

c) C $\rightarrow$ D 1/4 lap $t = 13.1s$

$$(v_x)_{av} = \frac{\Delta x}{\Delta t} = \frac{0-50m}{13.1s} = -3.8 m/s$$

$$(v_y)_{av} = \frac{-50m - 0}{13.1s} = -3.8 m/s$$

$$(a_x)_{av} = \frac{\Delta v_x}{\Delta t} = \frac{-6m/s - 0}{13.1s} = -0,46 m/s^2$$

$$(a_y)_{av} = \frac{\Delta v_y}{\Delta t} = \frac{0-(6m/s)}{13.1s} = 0,46 m/s^2$$

d) A $\rightarrow$ A, $\Delta x = \Delta y = 0$ $(v_x)_{av} = (v_y)_{av} = 0$

$\Delta v_x = \Delta v_y = 0 \Rightarrow (a_x)_{av} = (a_y)_{av} = 0$

e) $v_{av} = \sqrt{v_{av_x}^2 + v_{av_y}^2}$

$= \sqrt{(3,8m/s)^2 + (3,8m/s)^2}$

$= 5.4 m/s$

speed is const. $\Rightarrow$ average speed = 6 m/s

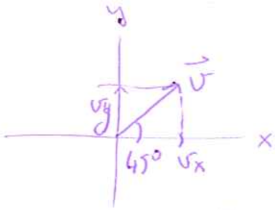
average speed > average velocity because distance traveled > magnitude of displacement

f) velocity is a vector, magnitude of velocity is constant, but the direction is changing.

$\boxed{3.43}$ $\vec{r} = bt^2\hat{i} + ct^3\hat{j}$; b, c: positive constants

when does the velocity vector make an angle θ of $45°$ with x-y axes?

$\vec{v} = \dfrac{d\vec{r}}{dt}$



$\vec{v} = 2bt\hat{i} + 3ct^2\hat{j}$ $\qquad v_x = v_y$ if $\theta = 45°$

$v_x = v_y$

$\hookrightarrow 2bt = 3ct^2 \Rightarrow t = 2b/3c$ //.

$\boxed{3.46}$ $\vec{v} = (\alpha - \beta t^2)\hat{i} + \gamma t\hat{j}$, $\alpha = 2.4\,m/s$, $\beta = 1.6\,m/s^3$, $\gamma = 4.0\,m/s^2$

Take $\uparrow +y$.

At $t=0$, bird is at origin.

a) $\vec{r} = ?$ $\vec{a} = ?$

$\vec{r} = \vec{r}_0 + \int_0^t \vec{v}(t)\,dt$ $\qquad \vec{a} = \dfrac{d\vec{v}}{dt}$

$\vec{r} = \vec{r}_0 + \left(\alpha t - \dfrac{\beta t^3}{3}\right)\hat{i} + \dfrac{\gamma t^2}{2}\hat{j}$ $\qquad \vec{a} = (-2\beta t)\hat{i} + \gamma\hat{j}$

at $t=0 \rightarrow x_0 \overset{\text{origin}}{=} y_0 = 0 \rightarrow \vec{r}_0 = 0$

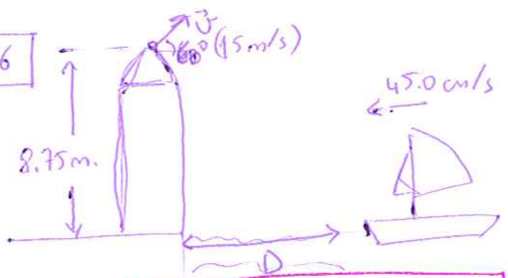
$\vec{r} = \left(\alpha t - \dfrac{\beta t^3}{3}\right)\hat{i} + \dfrac{\gamma t^2}{2}\hat{j}$

b) what is the (y-coordinate) altitude over $x = 0$ for the 1st time after $t = 0$?

$x = 0 \rightarrow \alpha t - \dfrac{\beta t^3}{3} = 0 \rightarrow t^2 = \dfrac{3\alpha}{\beta}$

$y = \dfrac{\gamma t^2}{2} = \dfrac{\gamma}{2}\left(\dfrac{3\alpha}{\beta}\right) = \dfrac{3\,(2.4\,m/s)\,(4.0\,m/s^2)}{2\cdot(1.6\,m/s^3)} = 9.0\,m.$

$\boxed{3.56}$



$\boxed{D = 24.1 + 1.44 = 25.5\,m.}$

Equipment will land at front of the ship. $D = ?$

$a_x = 0$

$a_y = -9.8\,m/s^2$

$v_{0x} = v_0\cos\alpha_0 = 7.5\,m/s$

$v_{0y} = v_0\sin\alpha_0 = 13\,m/s$

$y - y_0 = -8.75\,m.$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$

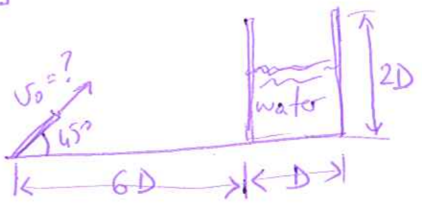
$t = \dfrac{1}{9.8}\left(13 \pm \sqrt{(-13)^2 + 4(4.9)(8.75)}\right)$

$t = 3.21\,s$ (time in air.)

* equip. horizontal range: $x - x_0 = v_{0x}t + \frac{1}{2}\cancel{a_x}t^2 = (7.5\,m/s)(3.21\,s) = 24.1\,m$

* ship moves $\Rightarrow (0.450\,m/s)t = 1.44\,m.$

$\boxed{3.60}$



let $x_0 = y_0 = 0$ $\quad \uparrow +y \quad$ $a_x = 0$, $a_y = -g$

$$y = (v_0 \sin\alpha)t - \frac{1}{2}gt^2$$

$$x = (v_0 \cos\alpha)t$$

range of launch speeds = ?

water enters tank for $\underline{\text{min.}}$ vel. : $y = 2D$, $x = 6D$

water goes in tank for $\underline{\text{max}}$ vel: $y = 2D$, $x = 7D$

$\sin\alpha = \cos\alpha = \frac{\sqrt{2}}{2}$; for min distance: $6D = \frac{\sqrt{2}}{2} v_0 t \rightarrow t = \frac{6\sqrt{2}D}{v_0}$

$$2D = \frac{\sqrt{2}}{2} v_0 t - \frac{1}{2} g t^2$$ substitute

$$2D = 6D - \frac{1}{2} g \left(\frac{6\sqrt{2}D}{v_0}\right)^2$$

$$\Rightarrow v_0 = 3\sqrt{gD}$$

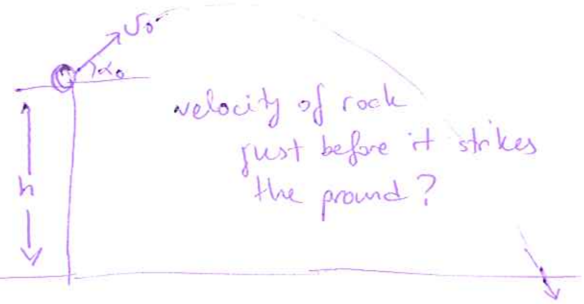
for max. distance: $7D = \frac{\sqrt{2}}{2} v_0 t$

$$t = \frac{7\sqrt{2}D}{v_0}$$ substitute

$$2D = \frac{\sqrt{2}}{2} v_0 t - \frac{1}{2} g t^2$$

$$\Rightarrow v_0 = 3.13\sqrt{gD}$$

$\boxed{3.68}$



velocity of rock just before it strikes the pround?

$$v = \sqrt{v_x^2 + v_y^2}$$

$$v^2 = v_0^2 \cos^2\alpha_0 + (v_0 \sin\alpha_0 - gt)^2 = v_0^2(\sin^2\alpha_0 + \cos^2\alpha_0) - 2v_0 \sin\alpha_0 gt + (gt)^2$$

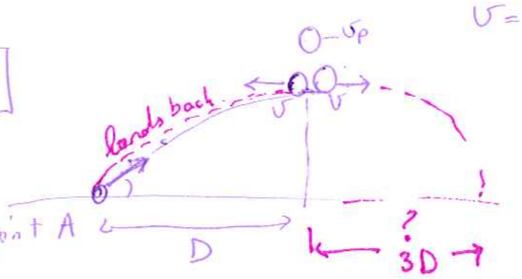
$$v^2 = v_0^2 - 2g\left(v_0 \sin\alpha_0 t - \frac{1}{2}gt^2\right)$$

$$v^2 = v_0^2 - 2gy$$

$$y = -h \rightarrow v^2 = v_0^2 + 2gh$$

$$v = \sqrt{v_0^2 + 2gh} \quad \text{(independent of } \alpha_0\text{)}$$

$\boxed{3.80}$



fragment = F
projectile = P
earth = E

$$\vec{v}_{F/E} = \vec{v}_{F/P} + \vec{v}_{P/E}$$

▲ let speeds of fragments rel. to earth v_1, v_2

▲ speeds " " rel. to projectile v

$$\begin{cases} v_1 = v + v_p \\ -v_2 = -v + v_p \end{cases}$$

$$v = v_p + v_2$$

$$vt = v_p t + v_2 t = 2D$$

$$v_1 t = v_p t + vt = D + 2D = \underline{3D}$$

▲ zero vertical comp. of vel rel. to Earth

time to fall

$v_p t = D \quad v_2 t = D$ (same time for fragments and projectile)