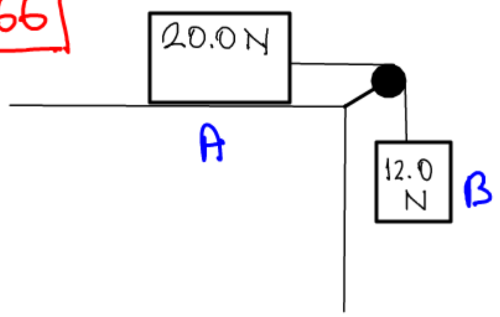
6.11.2014

Recitation 6

6.66



- The system moves 75.0 cm

a) no friction btw. table and 20.0N block

b) $\mu_s = 0.500$ and $\mu_k = 0.325$ for 20.0N block

- Total work done?

- Draw free body diagrams

- Use $\sum \vec{F} = ma$

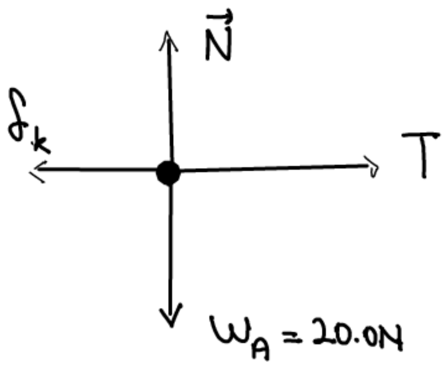
- Find tension in the string

We also need the masses of the blocks

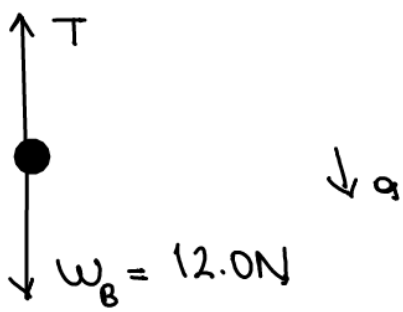
$g = 9.80665 \ m/s^2$ (1) back of the book

$m_A = \dfrac{20.0\,N}{g} = 2.04\ kg$ (2) $\quad m_B = \dfrac{12.0N}{g} = 1.22kg$ (3)

A



B



a is +

Two blocks as a single unit

$T - f_k = m_A a$ (1) $\quad T - W_B = -m_B a$ (2) $\quad W_B - f_k = (m_A + m_B) a$ (3)

a) $f_k = 0$ $\qquad a = \dfrac{w_B}{m_A + m_B}$ (4) $\qquad T = \dfrac{m_A w_B}{m_A + m_B} = w_B\left(\dfrac{w_A}{w_A + w_B}\right) = 7.50\text{N}$ (5)

(4) → (1)

20.0N block $\qquad W_{tot} = T \cdot 0.75\text{m} = 5.62\ \text{J}$

12.0N block $\qquad W_{tot} = (w_B - T)\ 0.75\text{m} = 3.38\ \text{J}$

$$W = \vec{F} \cdot \vec{s} = (T - w_B)\hat{j} \cdot (-0.75\text{m})\hat{j}$$

b) $f_k = \mu_k w_A = 0.325 \times 20 = 6.50\ \text{N}$ (6)

kinetic fric. since blocks are moving

VERIFICATION

$w_B > \mu_s w_A$

$12.0\text{N} > 0.5 \times 20.0\text{N}$

From (3)

$a = \dfrac{w_B - \mu_k w_A}{m_A + m_B}$ (7) $\qquad T = f_k + (w_B - \mu_k w_A)\left(\dfrac{m_A}{m_A + m_B}\right)$ (8)

(6) → (1)

$T = 6.50\text{N} + (5.50\text{N})(0.625) = 9.94\text{N}$ (9)

20.0N block $\qquad W_{tot} = (T - f_k)\ 0.75\text{m} = 2.58\ \text{J}$

10.0N block $\qquad W_{tot} = (w_B - T)\ 0.75\text{m} = 1.54\ \text{J}$

6.74 Spring not obeying Hooke's law

- One end is fixed
- To keep the other end still the following force is applied
  $F_x = kx - bx^2 + cx^3$

Spring exerts force $-F_x$ in the x-direction

a) Work to stretch this spring by 0.050 m = ?

b) Work to compress this spring by 0.050 m = ?

c) Is it easier to strech or compress

$k = 100 N/m \qquad b = 700 N/m^2 \qquad c = 12\,000 \ N/m^3$

Spring is unstreched at the beginning.

(a)

$$W = \int_0^{x_2} F_x \, dx = \int_0^{x_2} (kx - bx^2 + cx^3)\, dx = \frac{k}{2}x_2^2 - \frac{b}{3}x_2^3 + \frac{c}{4}x_2^4$$

$x_2 = 0.050 m$ in part a

$$W = (50.0 N/m)\, x_2^2 - (233 N/m^2)\, x_2^3 + (3000 N/m^3)\, x_2^4 \quad (1)$$

$W = 0.12 J$

b) $x_2 = -0.050 m$ this time

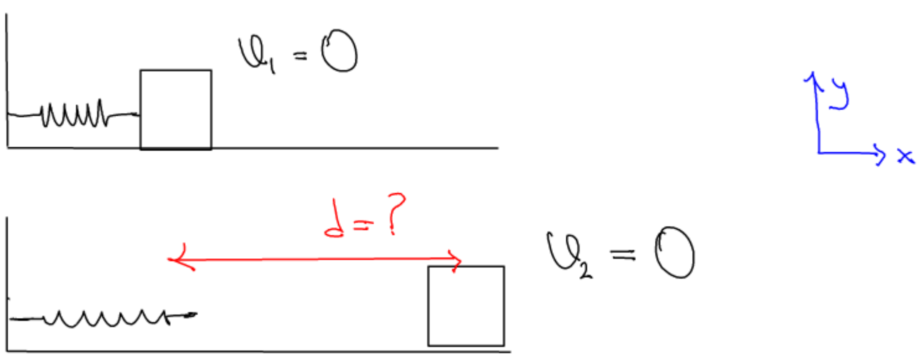
Evaluating Eq. (1) at this value yields $W = 0.17 J$

c) It is easier to strech the spring, the $-bx^2$ term in the force is always in the negative x-direction and the work needed is less when $x_2 > 0$. Since $b > 0$ and the other terms are positive, the magnitude of work is less.

6.81

- 2.50 kg textbook
- $k = 250$ N/m
- Compression of 0.250 m
- $\mu_k = 0.30$ coefficient of kinetic friction

- Use work-energy theorem to find how far the textbook moves from its initial position



Work energy theorem states that the total work done on an object is equal to its change in K.E.

$$W_{tot} = W_{spring} + W_{friction} = \Delta K \quad (1)$$

$$W_{spring} + W_{friction} = K_2 - K_1 = 0 \quad (2)$$

$$W_{spring} = \int_{-x_1}^{0} -kx\,dx = -\frac{1}{2}kx^2 \Big|_{-x_1}^{0} = \frac{1}{2}kx_1^2 \quad (3)$$

work done on the textbook

The force exerted by Spring is opposite to the dir. of comp. stre

$$W_{friction} = -\mu_k m g d \quad (4)$$

$$W_{tot} = 0 = \frac{1}{2}kx_1^2 - \mu_k m g d = 0$$

(3), (4) → (2)

Solve for d

$$d = \frac{kx_1^2}{2\mu_k m g} = \frac{(250\,N/m)(0.250m)^2}{2(0.30)(2.50kg)(9.80m/s^2)}$$

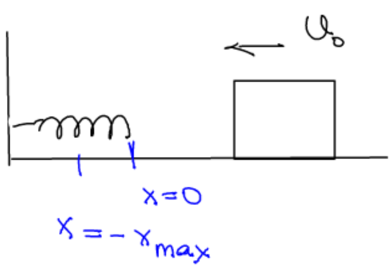
$$d = 1.1 \ m$$

The textbook slides this much before stopping

6.85 $m = 5.00\ kg$ No friction

$v_0 = 6.00\,m/s$

$k = 500\,N/m$



(a) Max. dist. spring can be compressed

$$W_{tot} = K_2 - K_1 = 0 - \frac{1}{2} m v^2 \quad (1)$$

K_2 ↑ 0

Only spring can do work

$$W_{spring} = \int_0^{-x_{max}} -kx\,dx = -\frac{1}{2} k x_{max}^2 \quad (2)$$

(1) = (2)

$$m v^2 = k x_{max}^2 \quad (3)$$

$$x_{max} = \sqrt{\frac{m v^2}{k}} = \sqrt{\frac{5.00\,kg}{500\,N/m}}\,(6.00\,m/s) = 0.600\,m$$

(b)

$$m v_{max}^2 = k x^2$$

$$v_{max} = \sqrt{\frac{k x^2}{m}} = \sqrt{\frac{500\,N/m}{5\,kg}}\,(0.150\,m) = 1.50\,m/s$$