PHYS 101: General Physics KOÇ UNIVERSITY Fall Semester 2014

College of Arts and Sciences

Quiz 1-1 25 Sept 2014

Closed book. No calculators are to be used for this quiz.

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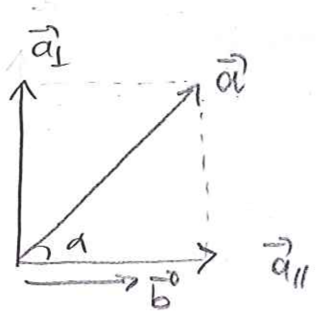
A mathematical procedure known as the "Gram-Schmidt process" involves calculating the component of a vector $\vec{a}$ perpendicular to another vector $\vec{b}$. Let

$$\vec{a} = \vec{a}_{\parallel} + \vec{a}_{\perp}$$

where $\vec{a}_{\parallel}$ lies along the direction of $\vec{b}$, and $\vec{a}_{\perp}$ is perpendicular to $\vec{b}$. Express $\vec{a}_{\perp}$ in terms of the vectors $\vec{a}$, $\vec{b}$, and their respective magnitudes a and b, by using the vector operations you have learned in the lectures.



unit vector in the direction of $\vec{b}$: $\hat{n} = \frac{\vec{b}}{b}$

Since $\vec{b} \parallel \vec{a}_{\parallel}$: $\hat{n} = \frac{\vec{b}}{b} = \frac{\vec{a}_{\parallel}}{a_{\parallel}}$

$$\vec{a}_{\parallel} = \frac{a_{\parallel}}{b}\vec{b}$$

$\vec{a} = \vec{a}_{\parallel} + \vec{a}_{\perp} \rightarrow \vec{a}_{\perp} = \vec{a} - \vec{a}_{\parallel}$

$$\vec{a}_{\perp} = \vec{a} - \frac{a_{\parallel}}{b}\vec{b}$$

For $a_{\parallel}$: $a_{\parallel} = a\cos\alpha$

$$\vec{a}\cdot\vec{b} = ab\cos\alpha$$

$$\Rightarrow a_{\parallel} = \frac{\cancel{a}}{\cancel{a}b}\,\vec{a}\cdot\vec{b} = \frac{\vec{a}\cdot\vec{b}}{b}$$

$$\Rightarrow \boxed{\vec{a}_{\perp} = \vec{a} - \left(\frac{\vec{a}\cdot\vec{b}}{b^2}\right)\vec{b}}$$

PHYS 101: General Physics KOÇ UNIVERSITY Fall Semester 2014
College of Arts and Sciences
Quiz 1-4 25 Sept 2014

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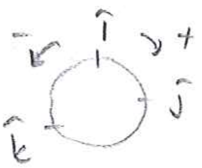
Show by substitution that,

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = \vec{b}\cdot(\vec{c}\times\vec{a})$$

for the special case

$$\begin{aligned}\vec{a} &= a_x\hat{i} + a_y\hat{j}\\ \vec{b} &= b_x\hat{i} + b_z\hat{k}\\ \vec{c} &= c_y\hat{j} + c_z\hat{k}\,.\end{aligned}$$



$$\vec{b}\times\vec{c} = (b_x\hat{i} + b_z\hat{k})\times(c_y\hat{j} + c_z\hat{k})$$

$$= b_x c_y\hat{k} - b_x c_z\hat{j} - b_z c_y\hat{i}$$

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = (a_x\hat{i} + a_y\hat{j})\cdot(-b_z c_y\hat{i} - b_x c_z\hat{j} + b_x c_y\hat{k})$$

$$= -a_x b_z c_y - a_y b_x c_z \qquad (1)$$

$$\vec{c}\times\vec{a} = (c_y\hat{j} + c_z\hat{k})\times(a_x\hat{i} + a_y\hat{j})$$

$$= -a_x c_y\hat{k} + a_x c_z\hat{j} - a_y c_z\hat{i}$$

$$\vec{b}\cdot(\vec{c}\times\vec{a}) = (b_x\hat{i} + b_z\hat{k})\cdot(-a_y c_z\hat{i} + a_x c_z\hat{j} - a_x c_y\hat{k})$$

$$= -a_y b_x c_z - a_x b_z c_y \qquad (2)$$

$(1) = (2)$ ✓

PHYS 101: General Physics KOÇ UNIVERSITY Fall Semester 2014

College of Arts and Sciences

Quiz 1-3 25 Sept 2014

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The following identity holds for all vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ in three dimensions:

$$(\vec{a}\times\vec{b})\cdot(\vec{a}\times\vec{c}) = a^2\,(\vec{b}\cdot\vec{c}) - (\vec{a}\cdot\vec{b})(\vec{a}\cdot\vec{c})\ .$$

Demonstrate the identity by using the vectors below.

$$\begin{aligned}\vec{a} &= 2\hat{\imath}+\hat{\jmath}\\ \vec{b} &= \hat{\imath}-\hat{k}\\ \vec{c} &= \hat{\jmath}+\hat{k}\ .\end{aligned}$$

For the LHS:

$\vec{a}\times\vec{b} = (2\hat{\imath}+\hat{\jmath})\times(\hat{\imath}-\hat{k}) = +2\hat{\jmath}-\hat{k}-\hat{\imath}$

$\vec{a}\times\vec{c} = (2\hat{\imath}+\hat{\jmath})\times(\hat{\jmath}+\hat{k}) = 2\hat{k}-2\hat{\jmath}+\hat{\imath}$

$(\vec{a}\times\vec{b})\cdot(\vec{a}\times\vec{c}) = (2\hat{\jmath}-\hat{k}-\hat{\imath})\cdot(2\hat{k}-2\hat{\jmath}+\hat{\imath})$

$= -4-2-1 = -7 \quad (1)$

RHS : $a^2 = \left(\sqrt{2^2+1^2}\right)^2 = 5$

$(\vec{b}\cdot\vec{c}) = (\hat{\imath}-\hat{k})\cdot(\hat{\jmath}+\hat{k}) = -1$

$(\vec{a}\cdot\vec{b}) = (2\hat{\imath}+\hat{\jmath})\cdot(\hat{\imath}-\hat{k}) = 2$

$(\vec{a}\cdot\vec{c}) = (2\hat{\imath}+\hat{\jmath})\cdot(\hat{\jmath}+\hat{k}) = 1$

$a^2(\vec{b}\cdot\vec{c}) - (\vec{a}\cdot\vec{b})(\vec{a}\cdot\vec{c}) = 5(-1) - 2\cdot 1 = -7 \quad (2)$

$(1) = (2)$ ✓

PHYS 101: General Physics KOÇ UNIVERSITY Fall Semester 2014
College of Arts and Sciences
Quiz 1-2 25 Sept 2014

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Given

$$\vec{a} = 2\hat{i} + \hat{j}$$
$$\vec{b} = \hat{i} - \hat{k}$$
$$\vec{c} = \hat{j} + \hat{k}$$

find the unit vector $\hat{n}$ perpendicular to $\vec{c}$ and parallel to the plane defined by $\vec{a}$ and $\vec{b}$ (such vectors have the form $\hat{n} = \alpha\vec{a} + \beta\vec{b}$, where α, β are real numbers).

$$\hat{n} = \alpha\vec{a} + \beta\vec{b} = \alpha(2\hat{i} + \hat{j}) + \beta(\hat{i} - \hat{k})$$
$$\hat{n} = (2\alpha + \beta)\hat{i} + \alpha\hat{j} - \beta\hat{k}$$

$$\hat{n} \perp \vec{c} \Rightarrow \quad \hat{n} \cdot \vec{c} = 0$$

$$((2\alpha + \beta)\hat{i} + \alpha\hat{j} - \beta\hat{k}) \cdot (\hat{j} + \hat{k}) = 0$$
$$\alpha - \beta = 0$$
$$\boxed{\alpha = \beta}$$

$$\hat{n} = 3\alpha\hat{i} + \alpha\hat{j} - \alpha\hat{k}$$

Since $\hat{n}$ is a unit vector $|\hat{n}| = 1$

$$\sqrt{(3\alpha)^2 + (\alpha)^2 + (-\alpha)^2} = 1$$
$$\sqrt{11\alpha^2} = 1$$
$$\alpha = \frac{1}{\sqrt{11}}$$

$$\boxed{\hat{n} = \frac{3}{\sqrt{11}}\hat{i} + \frac{1}{\sqrt{11}}\hat{j} - \frac{1}{\sqrt{11}}\hat{k}}$$

PHYS 101: General Physics KOÇ UNIVERSITY Fall Semester 2014
College of Arts and Sciences
Quiz 1-5 25 Sept 2014

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Find the unit vector perpendicular to the vectors $\vec{a}$ and $\vec{b}$, where

$$\vec{a} = 2\hat{i} + \hat{j}$$
$$\vec{b} = \hat{i} - \hat{k}$$

Cross product of two vectors $\vec{a}$ and $\vec{b}$ will give the vector perpendicular to both,

$$\vec{c} = \vec{a} \times \vec{b} = (2\hat{i} + \hat{j}) \times (\hat{i} - \hat{k})$$

$$\vec{c} = 2\hat{j} - \hat{k} - \hat{i}$$

For to unit vector; $\hat{n} = \dfrac{\vec{c}}{|\vec{c}|} = \dfrac{-\hat{i} + 2\hat{j} - \hat{k}}{\sqrt{(-1)^2 + (2)^2 + (-1)^2}}$

$$\boxed{\hat{n} = -\frac{1}{\sqrt{6}}\hat{i} + \frac{2}{\sqrt{6}}\hat{j} - \frac{1}{\sqrt{6}}\hat{k}}$$