

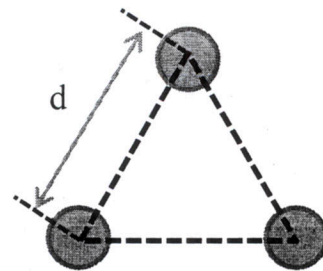
Closed book. No calculators are to be used for this quiz.
 Quiz duration: 10 minutes

Name:

Student ID:

Signature:

Three identical hard spheres each of radius R and mass m are placed on a plane such that their centers form the corners of an equilateral triangle with vertices of length $d = 5R$. Assume that, when the spheres are released from rest each sphere moves along the line from its corner towards the geometric center of the triangle. Calculate the speed of a sphere just before the spheres collide. (Gravitational constant: G)



Solution: $U_1 + K_1 = U_2 + K_2$ $K_1 = 0$

$$U_1 = 3 \cdot \frac{-Gm^2}{5R} \quad U_2 = 3 \cdot \frac{-Gm^2}{2R}$$

$$\frac{-3}{5} \frac{Gm^2}{R} = \frac{-3}{2} \frac{Gm^2}{R} + K_2 \Rightarrow K_2 = \frac{9}{10} \frac{Gm^2}{R}$$

$$K_2 = 3 \cdot \frac{1}{2} m v^2 \Rightarrow \frac{3}{10} \frac{Gm^2}{R} = \frac{1}{2} m v^2$$

$$v = \sqrt{\frac{3Gm}{5R}}$$

Closed book. No calculators are to be used for this quiz.

Quiz duration: 10 minutes

Name:

Student ID:

Signature:

A satellite of mass m travelling on a circular orbit of radius R_1 around Earth is hit by a meteorite and loses half of its kinetic energy. Does the satellite move away from Earth into space or fall on Earth? If it moves into space, what is its speed at very far away from Earth? If it falls on Earth what is its speed just before it hits Earth's surface? (Gravitational constant: G , Earth's radius R_E , mass M_E)

Solution:

$$F = \frac{GMm}{R_1^2} \quad m \frac{v^2}{R_1} = \frac{GMm}{R_1^2} \Rightarrow \frac{1}{2} m v^2 = \frac{1}{2} \frac{GMm}{R_1}$$

$$U = \frac{-GMm}{R_1}$$

$$K = \frac{1}{2} \frac{GMm}{R_1}$$

After the collision: $K = \frac{1}{4} \frac{GMm}{R_1} \Rightarrow v' = \sqrt{\frac{GM}{2R_1}}$

$$F_{\text{grav}} = \frac{GMm}{R_1^2} \quad F_{\text{cent}} = m \frac{v'^2}{R_1} = \frac{GMm}{2R_1^2}$$

$F_{\text{grav}} > F_{\text{cent}}$, so it will fall.

$$U_{\text{surface}} = -\frac{GMm}{R_E}$$

$$U_1 + K_1 = U_2 + K_2$$

$$-\frac{GMm}{R_1} + \frac{1}{4} \frac{GMm}{R_1} = -\frac{GMm}{R_E} + K_2 \Rightarrow K_2 = GMm \left(\frac{-3}{4R_1} + \frac{1}{R_E} \right)$$

$$\frac{1}{2} m v^2 = GMm \left(\frac{-3}{4R_1} + \frac{1}{R_E} \right) \Rightarrow v = \sqrt{2GM \left(\frac{-3}{4R_1} + \frac{1}{R_E} \right)}$$

Closed book. No calculators are to be used for this quiz.

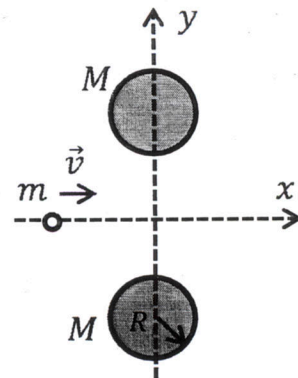
Quiz duration: 10 minutes

Name:

Student ID:

Signature:

Two identical spheres each of mass M , radius R are located with their centers on the y -axis at $y = d$ and $y = -d$, respectively ($d > R$). A small object of mass m is launched from the point $x = -d$ in the $+\hat{x}$ direction with a speed v such that it can just travel infinitely far away from the spheres (i.e. it has the escape speed for the location $x = -d$.) What is the maximum speed the object can have along its trajectory? (Gravitational constant: G)



Solution:

Escape velocity indicates $E = 0$

$E = K + U$ K_{max} implies U_{min} :

$$U_{min} = -2 \frac{GMm}{d} \quad K + U = 0 \Rightarrow K = \frac{2GMm}{d}$$

$$\frac{1}{2} m v^2 = \frac{2GMm}{d}$$

$$v = \sqrt{\frac{4GM}{d}}$$

Closed book. No calculators are to be used for this quiz.

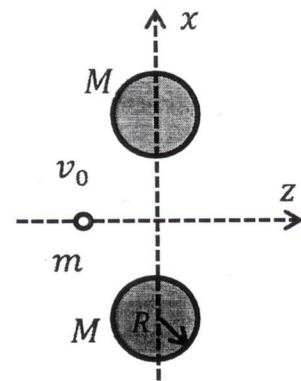
Quiz duration: 10 minutes

Name:

Student ID:

Signature:

Two identical spheres each of mass M radius R are fixed at $x = d$ and $x = -d$, respectively ($d > R$). Determine the minimum distance of a point on the z -axis from the origin such that when a small object of mass m is launched from that point along the z -axis with speed v_0 , it can escape to infinity. (Gravitational constant: G)



Solution:

$$U(z) = \frac{-2GMm}{\sqrt{d^2 + z^2}}$$

$$E = K + U = 0 \Rightarrow K = \frac{2GMm}{\sqrt{d^2 + z^2}} = \frac{1}{2} m v_0^2$$

$$\frac{1}{d^2 + z^2} = \left(\frac{v_0^2}{4GM} \right)^2$$

$$z^2 = \left(\frac{4GM}{v_0^2} \right)^2 - d^2$$

$$z = \pm \sqrt{\left(\frac{4GM}{v_0^2} \right)^2 - d^2}$$

Section

Quiz 9-5

November 2014

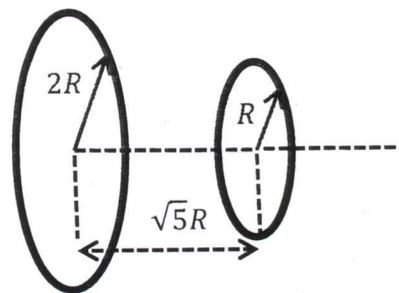
Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes

Name:

Student ID:

Signature:

Two concentric rings of equal mass M and different radius of $2R$ and R are placed with their centers on the same axis. The distance between their centers along the axis as shown in the figure. A particle of mass m will be placed to the center of one of the rings and then launched with the escape speed of its location. At which ring's center would it require larger escape speed? What is the total mechanical energy of the particle during the launch? (Gravitational constant: G . Some numerical value $\sqrt{6} \approx 2.45$)



Solution:

If m is placed in the center of $2R$ -ring:

$$u_1 = -\frac{GMm}{2R} + \frac{-GMm}{(\sqrt{5})R} = \frac{-4.45}{4.90} \frac{GMm}{R}$$

If m is placed in the center of R -ring:

$$u_2 = -\frac{GMm}{R} + \frac{-GMm}{3R} = \frac{-4.6GMm}{3R}$$

$u_2 < u_1$, hence $K_1 < K_2$, i.e., escape speed in the center of R -ring is larger.

Total mechanical energy is zero.