A body in simple harmonic motion with angular frequency $\omega = 0.2$ rad/s is initially 100 cm away from its equilibrium position and moving back toward the equilibrium position with a velocity-20 cm/s. (i) Find the period of the motion. (ii) Find the coordinate and velocity of the body as a function of time. (iii) How long does it take the body to return to its equilibrium position?

i) $T = \frac{2\pi}{\omega} = \frac{2\pi}{(0.2 rad/s)} = 10\pi$

ii) $x(t) = A\cos(\omega t + \phi)$, $v(t) = \frac{dx(t)}{dt} = -A\omega \sin(\omega t + \phi)$

$x_0 = 1m = A\cos\phi$ $\quad v_0 = -0.2 m/s = -A\omega \sin\phi$

$\frac{v_0}{x_0} = -0.2 = -\omega \tan\phi$

$\tan\phi = 1$

$\phi = \pi/4$

, $1m = A\cos \pi/4 \Rightarrow$

$A = \sqrt{2} m$

$x(t) = \sqrt{2} m \cos\left(\frac{t}{5} + \pi/4\right)$

$v(t) = -(0.2\sqrt{2}) \cos\left(\frac{t}{5} + \frac{\pi}{4}\right)$

iii) $x(t_e) = 0$, t_e: time at equilibrium.

$x_0 = 1m$

$0 = \sqrt{2} m \cos\left(\frac{t_e}{5} + \frac{\pi}{4}\right)$

$\frac{t_e}{5} = \frac{\pi}{2} - \frac{\pi}{4}$, $\boxed{t_e = 5\pi/4}$

A body of mass $m = 3$ kg is attached to a wall by a spring of constant $k = 27$ N/m. It is initially pulled a distance $x = 80$ cm from its equilibrium position and released from rest. (i) What are the amplitude, frequency and period of the motion? (ii) Find the displacement and the velocity for time $t = 10$ s after release.

i) $x(t) = A\cos(\omega t + \phi)$

$t = 0;\quad x_0 = A\cos\phi\ ,\ 0.8m = A\cos\phi\ ,\ v_0 = 0.$

$A = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}}\ ;\ A = x_0 \rightarrow \boxed{A = 0.8m}\quad \phi = 0°.$

$f = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{27 N/m}{3kg}} = \frac{3}{2\pi}\ ,\ \boxed{T = \frac{2\pi}{3}}$

ii) $x(t) = A\cos\omega t$

$t = 10s;\quad x(t = 10s) = 0.8m\cos(3 rad/s\,(10s))$

$= (0.8m)\frac{\sqrt{3}}{2}$

$x = \frac{2\sqrt{3}}{5} m$

A body of mass $m = 2$ kg is attached to a wall by a spring of constant $k = 8$ N/m. It is given an initial velocity, at $x = 0$ cm (its equilibrium position), of 20 m/s. Plot the time evolution of the displacement $x(t)$, velocity $v(t)$ and acceleration $a(t)$ for one period.

$\omega = \sqrt{\frac{k}{m}} = 2 \text{rad/s}$, $T = \frac{2\pi}{\omega} = \pi$

$x(t) = A\cos(\omega t + \phi)$

$t = 0,\ x_0 = 0$; $0 = A\cos\phi$

$A \neq 0$; $\phi = \pi/2$

$v(t) = -A\omega\sin(\omega t + \phi)$, $t = 0,\ v_0 = 20 \text{m/s}$

$v(t) = -A\omega\sin(\omega t + \phi)$

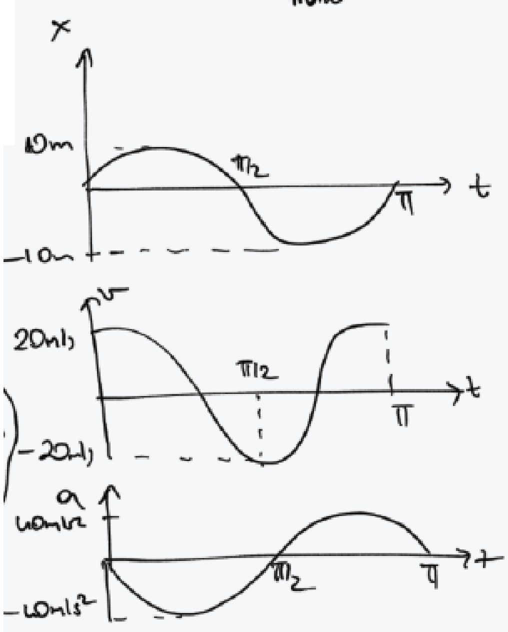
$t = 0$; $20 \text{m/s} = -A\,(2 \text{rad/s})\sin(\pi/2)$

$A = -10 \text{m}$

$a(t) = -A\omega^2\cos(\omega t + \phi)$

$t = 0,\ a = 0,\ a_{max} = \omega^2 A = (2 \text{rad/s})^2(+10 \text{m}) = +40 \text{m/s}^2$

$-a_{max} = -40 \text{m/s}^2$



A 4 kg mass vibrates in simple harmonic motion with amplitude 200 cm and angular frequency 4 rad/s. Find (i) the energy of motion (ii) the maximum values of the velocity and the acceleration (iii) the time it takes to move from 100 cm below to 100 cm above the equilibrium position.

i) $E = \frac{1}{2}kA^2$, k; $\omega = \sqrt{\frac{k}{m}}$

$16\,rad/s = \sqrt{\frac{k}{4kg}}$

$k = 1024\,N/m$

$E = \frac{1}{2}kA^2 = \frac{1}{2}(1024\,N/m)(2m)^2 = 2048\,J$

ii) $v_{max} = \omega A = (4\,rad/s)(2m) = 8\,m/s$

$a_{max} = \omega^2 A = 16(rad/s)^2(2m) = 32\,m/s^2$

iii) $x(t_1) = 1m$ $\quad x(t_2) = -1m$ $\quad |t_2 - t_1| = ?$

$x(t) = A\cos(\omega t + \phi) = A\sin\omega t$

$1m = 2m\sin((4\,rad/s)t_1)$, $-1m = 2m\sin(4\,rad/s\, t_2)$

$4t_1 = 30$ $\qquad t_2 = -\frac{30}{4}$

$t_1 = \frac{30}{4}$

$t_2 - t_1 = \frac{60}{4} = 15\,s.$

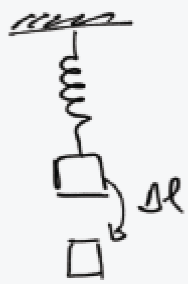
A spring stretches 40 cm when its tension is 80 N. A body of mass 4 kg is hung from the spring. When at rest the body is given an initial upward velocity of 2 m/s. (Take $g = 10 m/s^2$) (i) Find the amplitude and the frequency of the motion. (ii) Find the acceleration of the mass when it is 20 cm above its equilibrium position. (iii) Find the force of tension in the spring at this point.

$x(t) = A\sin\omega t$

$t = 0;\quad v_0 = A\omega$

$(2 m/s) = A(5 rad/s)$

$\boxed{A = 0.4 m}$



$k\,\Delta l = mg$

$k(0.4 m) = (4 kg)(10 m/s^2)$

$k = 100 N/m$

i) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{100 N/m}{4 kg}} = 5 rad/s$, $f = \frac{2\pi}{5 rad/s} = \frac{2\pi}{5}$

$\boxed{f = 2\pi/5}$

ii) $a(t) = -\omega^2 A\sin\omega t$

$x(t_1) = 0.2 m = (0.4 m)\sin(5 rad/s\, t_1)$

$5t_1 = 30$

$t_1 = 6 s$

$a(t = 6 s) = -\omega^2 A\sin(5 rad/s\, 6 s)$

$a = 5 m/s^2$

iii) $F = kx = (100 N/m)(0.2 m) = 20 N.$