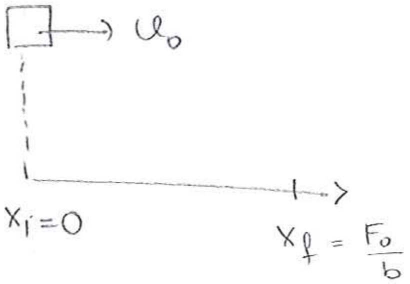
PHYS 101: General Physics KOÇ UNIVERSITY Fall Semester 2014
College of Arts and Sciences
Quiz 6-1 6 Nov 2014

Closed book. No calculators are to be used for this quiz.
Name: Ongun ARISEV **Student ID:** **Signature:**

oarisev14@ku.edu.tr

A net force in the x-direction with magnitude $F_x = F_0 + bx$ is applied to an object with mass m. The object is initially at the origin ($x_i = 0$) and moving in the $+x$ direction with a speed v_0. What is the speed of the object when it reaches the position $x_f = F_0/b$?

$$[b] = \frac{N}{m}$$



$\Delta K = W_{tot.}$ Work-energy theorem

$$W_{tot} = \int_0^{\frac{F_0}{b}} F_x \, dx = F_0 x + \frac{b}{2} x^2 \Big|_0^{F_0/b} = \frac{F_0^2}{b} + \frac{F_0^2}{2b} = \frac{3}{2}\frac{F_0^2}{b}$$

$$\frac{3}{\cancel{2}}\frac{F_0^2}{b} = \frac{1}{\cancel{2}} m v_f^2 - \frac{1}{\cancel{2}} m v_0^2$$

$$m v_f^2 = \frac{3F_0^2}{b} + m v_0^2$$

$$v_f = +\sqrt{\frac{3F_0^2}{mb} + v_0^2}$$

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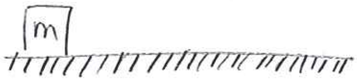
A block of ice with mass m is initially at rest on a frictionless, horizontal surface. A worker then applies a non-constant, horizontal force $\vec{F}$ to it. As a result, the block moves along the x-axis such that its position as a function of time is given by $x(t) = at^3$. Calculate the work done by the force $\vec{F}$ during the first τ seconds of the motion.

By Newton's 2^{nd} law



$\mu = 0$

$$\vec{F} = m\vec{a}$$

$$[a] = \frac{L}{T^3}$$

$$\frac{d^2x(t)}{dt^2} = 6at = \frac{F(t)}{m} \Rightarrow F(t) = 6atm$$

$$F(x) = 6\sqrt[3]{\frac{x}{a}}\, ma$$

$$x(t) = at^3$$

$$dx = 3at^2 dt$$

$$W_{tot} = \int_{x_0}^{x_f} F(x)\, dx = \int_0^{t_f} F(t)\, 3at^2 dt = 18a^2 m \int_0^{t_f} t^3 dt$$

$$W_{tot} = \frac{18}{4} a^2 m\, t_f^4 \Big|_0^{\tau} = \frac{9}{2} a^2 m \tau^4$$

$$[a^2] = \frac{L^2}{T^6} \quad [m] = M$$

$$[W_{tot}] = \frac{L^2}{T^6} M T^4 = \frac{ML^2}{T^2} \checkmark$$

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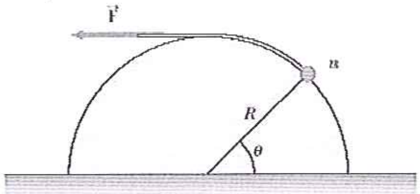
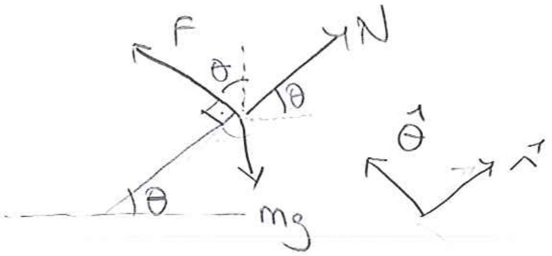
$\boxed{\omega = \frac{d\theta}{dt} \text{ No info}}$

A small particle of mass m is pulled to the top of a frictionless half-cylinder (of radius R) by a cord that passes over the top of the cylinder as in the figure. Calculate, by integration, the work done by the force $\vec{F}$ in moving the particle *at constant speed* from the bottom to the top of the half-cylinder.

* The mass of the rope is not important since there is no tangential acceleration.
Furthermore, $F = T$ in magnitude





$N < mg\sin\theta \Rightarrow$ centripetal acc. $\quad d\vec{l} = dr\,\hat{r} + r\,d\theta\,\hat{\theta}$

Constant speed $\Rightarrow$ No acceleration in $\hat{\theta}$ direction so

$F = mg\cos\theta \qquad \vec{F} = mg\cos\theta\,\hat{\theta}$

$$W = \int_{(R,0)}^{(R,\frac{\pi}{2})} \vec{F}\cdot d\vec{r} = \int_{(R,0)}^{(R,\frac{\pi}{2})} (mg\cos\theta\,\hat{\theta})\cdot(dr\,\hat{r} + r\,d\theta\,\hat{\theta})$$

$$= R\int_0^{\pi/2} mg\cos\theta\, d\theta = Rmg\sin\theta\Big|_0^{\pi/2} = mgR$$

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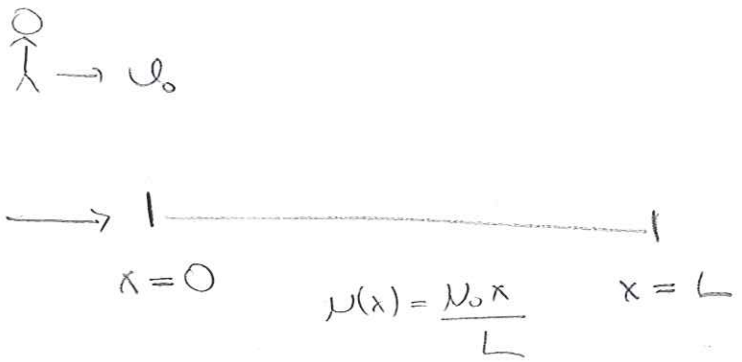
A skier with mass m is moving on a horizontal field along the x-direction with a speed v_0. She is approaching a region between $x = 0$ and $x = L$, where the snow has partially melted to yield a coefficient of kinetic friction $\mu(x) = \mu_0\, x/L$ (rest of the field can be assumed to be frictionless). How large should v_0 be so that she will not come to a full stop?



$N = mg$

$f_s = \mu(x) N (-\hat{x})$

$f_s = -\frac{\mu_0 x}{L} mg\, \hat{x}$

$d\vec{l} = dx\, \hat{x}$

$$W_{friction} = \int_{x=0}^{x=L} \vec{f_s} \cdot d\vec{l}$$

$$W_{friction} = -\int_0^L \frac{\mu_0}{L} x\, mg\, dx = -\frac{\mu_0}{L} mg \frac{x^2}{2} = -\frac{\mu_0 L mg}{2}.$$

$$W_{tot} = W_{fric} = K_2 - K_1$$

K_2: 0 (for min v_0)

K_1: $\frac{1}{2} m v_0^2$

$\mu_0 L mg = m v_0^2$

$v_0 = +\sqrt{\mu_0 L g}$

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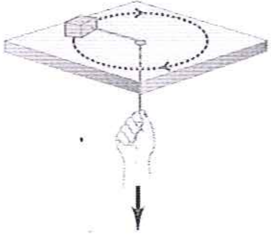
A small block with a mass m is attached to a cord passing through a hole in a frictionless, horizontal surface. The block is originally revolving at a distance r_0 from the hole with a speed v_0. The cord is then pulled from below until the distance from the hole is reduced to $r_0/2$. The speed of the block is observed to be $v(r) = r_0 v_0 / r$ when it is at a distance r from the hole. How much work is done by the person who pulled the cord?



$$F_{centripetal} = \frac{m v^2(r)}{r}$$

$$F_{centripetal} = \frac{m r_0^2 v_0^2}{r^3} \qquad \vec{F}_{cp} = -\frac{m r_0^2 v_0^2}{r^3}\hat{r}$$

$$W_{tot} = \int_{r=r_0}^{r=\frac{r_0}{2}} \vec{F}_{cp} \cdot \underbrace{d\vec{l}}_{\hat{r}dr + r d\theta \hat{\theta}} = -\int_{r_0}^{r_0/2} \frac{m r_0^2 v_0^2}{r^3} dr = +m r_0^2 v_0^2 \frac{r^{-2}}{2}\Bigg|_{r_0}^{r_0/2}$$

$$\frac{m r_0^2 v_0^2}{2}\left(\frac{4}{r_0^2} - \frac{1}{r_0^2}\right) = \frac{3}{2} m v_0^2$$

$$W = K_2 - K_1 \qquad K_1 = \frac{1}{2} m [v(r_0)]^2 = \frac{1}{2} m \frac{r_0^2 v_0^2}{r_0^2} = \frac{1}{2} m v_0^2$$

$$K_2 = \frac{1}{2} m [v(r_0/2)]^2 = \frac{1}{2} m \frac{r_0^2 v_0^2}{\frac{r_0^2}{4}} = 2 m v_0^2$$

$$W = \left(2 - \frac{1}{2}\right) m v_0^2$$