

Closed book. No calculators are to be used for this quiz.

Quiz duration: 15 minutes

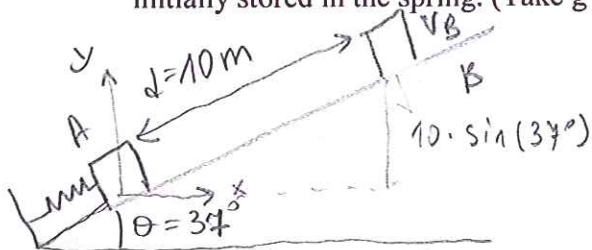
First Name:

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Student ID:

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A wooden block with mass 2 kg is placed against a compressed spring at the bottom of incline of slope 37 degrees (Point A). When the spring is released, it projects the block up the inclined plane. When the spring is released, it projects the block up the incline. At point B, a distance of 10 m up the incline from A, the block is moving up the incline at 10 m/s and is no longer in contact with the spring. The coefficient of kinetic friction between the block and the incline is $\mu_k = 0.5$. The mass of the spring is negligible. Calculate the amount of potential energy that was initially stored in the spring. (Take $g = 10 \text{ m/s}^2$)



$$m = 2 \text{ kg}$$

$$v_B = 10 \text{ m/s}$$

$$\mu_k = 0.5$$

From conservation of energy we have:

$$K_A + U_A + W_{\text{ol.}} = K_B + U_B$$

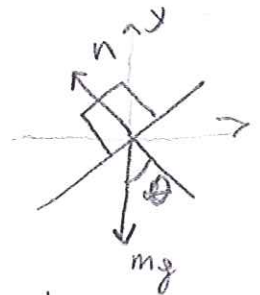
$$U_{\text{sp.}} + W_f = \frac{1}{2} m v_B^2 + m g d \sin(\theta)$$

$$W_f = -f_k d = -\mu_k n d = -\mu_k m g \cos \theta d$$

$$U_{\text{sp.}} = \frac{1}{2} m v_B^2 + m g d \sin \theta + \mu_k m g \cos \theta d$$

$$U_{\text{sp.}} = \frac{1}{2} \cdot 2 \cdot (10)^2 + 2 \cdot 10 \cdot 10 \cdot 0.6 + 0.5 \cdot 2 \cdot 10 \cdot 0.8 \cdot 10$$

$$U_{\text{sp.}} = 100 + 120 + 80 = 300 \text{ J}$$



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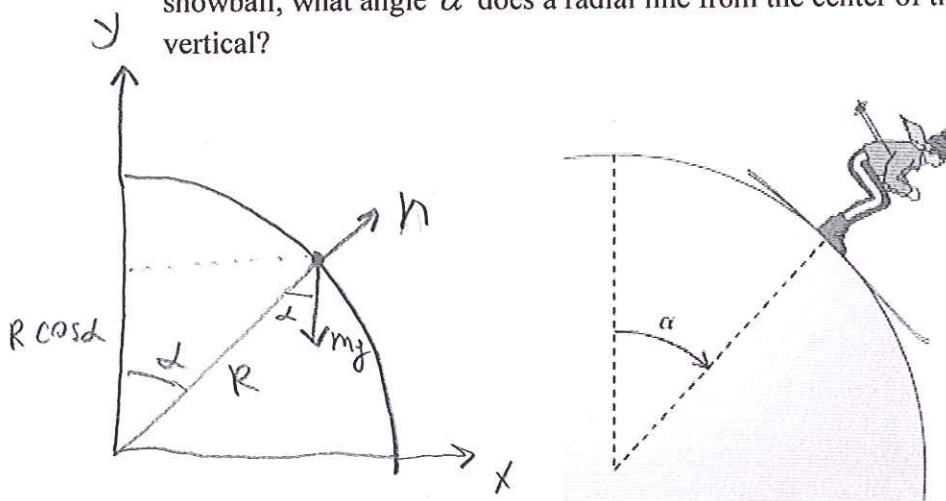
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A skier starts at the top of a very large, frictionless snowball, with a very small initial speed, and skis straight down the side as shown in the figure below. At what point does she lose contact with the snowball and fly off at a tangent? That is, at the instant she loses contact with the snowball, what angle α does a radial line from the center of the snowball to the skier make with vertical?



There will be centripetal acceleration. Skier will lose contact when $N=0$.

$$mg \cos \alpha - \cancel{N} = \frac{mv^2}{R} \Rightarrow \cancel{mg} \cos \alpha = \frac{\cancel{m} v^2}{R} \Rightarrow \boxed{v^2 = R g \cos \alpha}$$

From conservation of energy we have:

$$K_0 + U_0 + \cancel{W_{nc}} = K_\alpha + U_\alpha$$

$$mgR = \frac{mv_\alpha^2}{2} + mgR \cos \alpha \Rightarrow$$

$$\Rightarrow \cancel{mgR} = \frac{\cancel{m} R g \cos \alpha}{2} + \cancel{mgR} \cos \alpha$$

$$1 = \frac{\cos \alpha}{2} + \cos \alpha \Rightarrow 1 = \frac{3 \cos \alpha}{2} \Rightarrow$$

$$\Rightarrow \cos \alpha = \frac{2}{3} \Rightarrow \alpha = \arccos\left(\frac{2}{3}\right)$$

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A 2 Kg fish is attached to the lower end of vertical spring that has negligible mass and force constant 200 N/m. The spring initially is neither stretched nor compressed. The fish is released from rest. (a) What is its speed after it has descended 0.1 m from its initial position? (b) What is the maximum speed of the fish as it descends? (Take $g=10 \text{ m/s}^2$)

$$m = 2 \text{ kg}$$

$$k = 200 \text{ N/m}$$

$$y = 0,1 \text{ m}$$

$$a) V_2 = ?$$

$$b) V_{2\text{max}} = ?$$

a) From conservation of energy we have:

$$K_1 + U_1 + W_{\text{ext}} = K_2 + U_2$$

$$mgy = \frac{mv_2^2}{2} + \frac{1}{2}ky^2 \Rightarrow \frac{mv_2^2}{2} = mgy - \frac{1}{2}ky^2$$

$$\Rightarrow v_2 = \sqrt{\frac{2}{m} \left(mgy - \frac{1}{2}ky^2 \right)}$$

$$v_2 = \sqrt{\frac{2}{2} \left(2 \cdot 10 \cdot 0,1 - \frac{1}{2} \cdot 200 \cdot (0,1)^2 \right)} = \sqrt{2 - 1} = 1 \text{ m/s}$$

b) The maximum speed is when K_2 is maximum. That happens when $\frac{dK_2}{dy} = 0$.

Above we found that:

$$mgy = K_2 + \frac{1}{2}ky^2 \Rightarrow K_2 = mgy - \frac{1}{2}ky^2$$

$$\frac{dK_2}{dy} = mg - ky = 0 \Rightarrow y = \frac{mg}{k} = \frac{2 \cdot 10}{200} = 0,1 \text{ m}$$

$y = 0,1$ is displacement given in part a. Thus velocity calculated at part a) is maximum velocity:

$$v_{2\text{max}} = 1 \text{ m/s}$$

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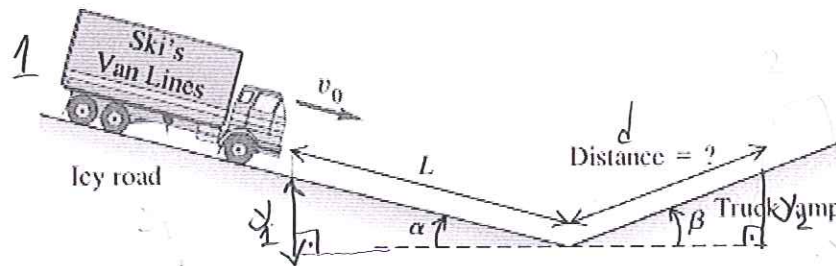
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A truck with mass m has a brake failure while going down an icy mountain road of constant downward slope angle α as shown in the figure below. Initially the truck is moving downhill at speed v_0 . After careening downhill a distance L with negligible friction, the truck driver steers the runaway vehicle onto a runaway truck ramp of constant upward slope angle β . The truck ramp has a soft sand surface for which the coefficient of rolling friction is μ_r . What is the distance that the truck moves up the ramp before coming to a halt? Solve using energy methods.



From conservation of energy we have:

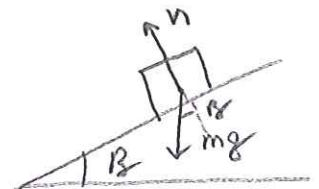
$$K_1 + U_1 + W_{\text{ext.}} = K_2 + U_2$$

$$\frac{1}{2} m v_0^2 + m g y_1 + W_{\text{ext.}} = m g y_2$$

From trigonometric relations we have:

$$y_1 = L \sin \alpha ; y_2 = d \sin \beta$$

$$W_{\text{ext.}} = -\mu_r m g \cos \beta \cdot d$$



$$n = m g \cos \beta$$

Thus:

$$\frac{1}{2} m v_0^2 + m g L \sin \alpha - \mu_r m g d \cos \beta = m g d \sin \beta$$

$$d (m g \sin \beta + \mu_r m g \cos \beta) = \frac{1}{2} m v_0^2 + m g L \sin \alpha$$

$$d = \frac{(v_0^2 / 2g) + L \sin \alpha}{\sin \beta + \mu_r \cos \beta}$$

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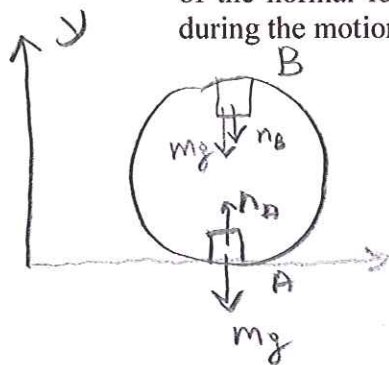
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A small block with mass 0.5 kg slides in vertical circle of Radius $R=1$ m on the inside of a circular track. During one of the revolutions of the block, when the block is at the bottom of its path, point A, the magnitude of the normal force exerted on the block by the track has magnitude 37 N. In this same revolution, when the block reaches the top of its path, point B, the magnitude of the normal force on the block is 3 N. How much work was done on the block by friction during the motion of the block from point A to point B? (Take $g=10$ m/s²)



$$m = 0,5 \text{ kg}$$

$$n_A = 37 \text{ N}$$

$$n_B = 3 \text{ N}$$

From Newton's second law
We have:

$$A: n_A - mg = \frac{m V_A^2}{R} \Rightarrow$$

$$B: n_B + mg = \frac{m V_B^2}{R}$$

$$\Rightarrow \begin{cases} V_A = \sqrt{\frac{R}{m} (n_A - mg)} \\ V_B = \sqrt{\frac{R}{m} (n_B + mg)} \end{cases} \Rightarrow \begin{cases} V_A = \sqrt{\frac{1}{0,5} (37 - 0,5 \cdot 10)} = \sqrt{64} = 8 \text{ m/s} \\ V_B = \sqrt{\frac{1}{0,5} (3 + 0,5 \cdot 10)} = \sqrt{16} = 4 \text{ m/s} \end{cases}$$

From conservation of energy we have:

$$K_A + U_A + W_{\text{tot}} = K_B + U_B$$

$$\frac{m V_A^2}{2} + W_f = \frac{m V_B^2}{2} + mg(2R)$$

$$W_f = \frac{m V_B^2}{2} - \frac{m V_A^2}{2} + 2mgR$$

$$W_f = \frac{0,5 \cdot 4^2}{2} - \frac{0,5 \cdot 8^2}{2} + 2 \cdot 0,5 \cdot 10 \cdot 1$$

$$W_f = 4 - 16 + 10 = -2 \text{ J}$$