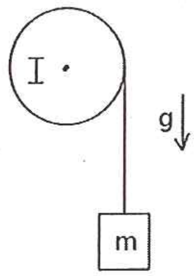
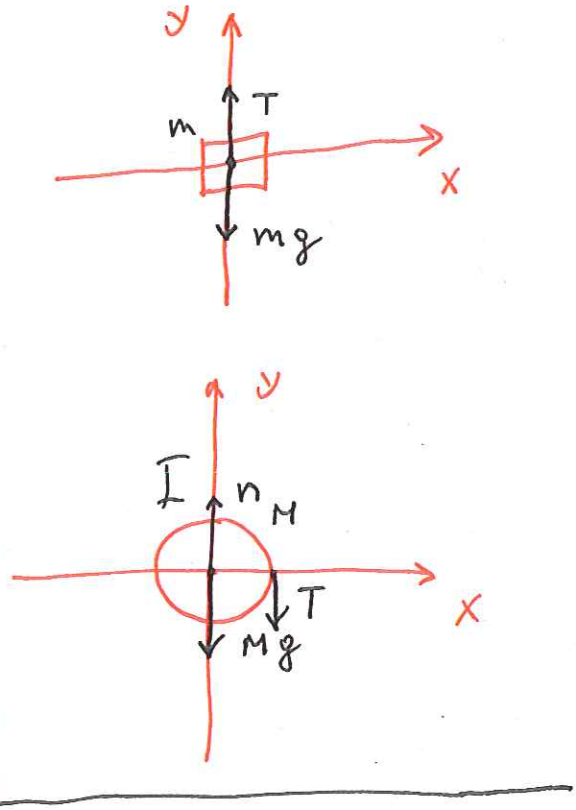
PHYS 101: General Physics KOÇ UNIVERSITY Fall Semester 2015
College of Arts and Sciences
Quiz - Ch.10

Closed book. No calculators are to be used for this quiz.
Name: Student ID: Signature:

A string with negligible mass is wrapped around a pulley with a moment of inertia I. The upper end of the string is firmly attached to the pulley which rotates freely around its fixed axis. A mass m hangs vertically from the free end of the string, as shown. Find the tension on the string.





From Newton's second law we have:

$$\sum F_y = m a_y = mg - T$$

Also we have the torque equation:

$$\sum \tau_z = I\alpha_z = RT \Rightarrow \frac{I a_y}{R} = RT \Rightarrow a_y = \frac{R^2 T}{I}$$

Substituting a_y to our first equation we have:

$$T = mg - m a_y = mg - \frac{m R^2 T}{I} \Rightarrow$$

$$\Rightarrow T + \frac{mR^2}{I} T = mg \Rightarrow T = \frac{mg}{\left(1 + \frac{mR^2}{I}\right)} \Rightarrow$$

$$\Rightarrow \boxed{T = \frac{mgI}{(I + mR^2)}}$$

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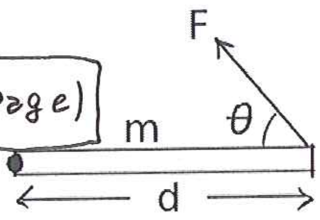
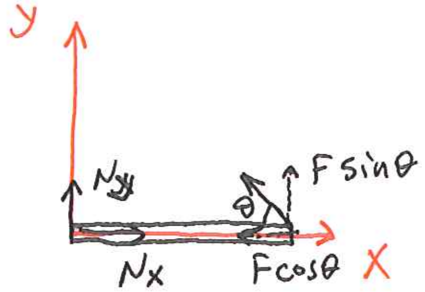
A uniform bar of length d is fixed at one end and is free to rotate about it. A force F is applied at the other end as shown in the figure. Express the magnitude and the direction of the torque due to this force in terms of F, θ and d. Then calculate the magnitude and the direction of the force acting on the bar at the other end.





$\vec{\tau} = \vec{r} \times \vec{F} \Rightarrow$

$\Rightarrow \boxed{\tau = dF\sin\theta \text{ (out of page)}}$

From Newton's second law we have:

$$\sum F_x = 0 = N_x - F\cos\theta \Rightarrow \boxed{N_x = F\cos\theta}$$

$$\sum F_y = m a_y = F\sin\theta + N_y \Rightarrow N_y = -F\sin\theta + m a_y$$

If we also write the total torque:

$$\sum \tau_z = I\alpha = dF\sin\theta \Rightarrow \frac{md^2}{3}\alpha = dF\sin\theta \Rightarrow$$

$$\Rightarrow \alpha = \frac{3dF\sin\theta}{md^2} \Rightarrow a_y = \frac{\alpha d}{2} = \frac{3F\sin\theta}{2m}$$

If we substitude a_y to above equation we have

$$N_y = -F\sin\theta + m a_y = -F\sin\theta + \frac{3F\sin\theta}{2} = \frac{F\sin\theta}{2}$$

$$\boxed{\vec{N} = F\cos\theta\,\hat{i} + \frac{F\sin\theta}{2}\,\hat{j}}$$

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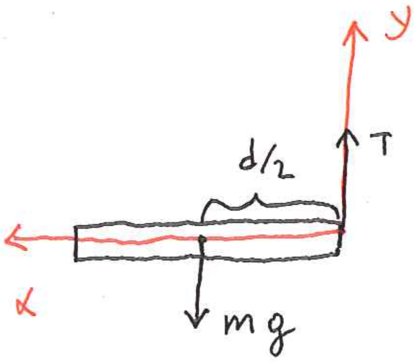
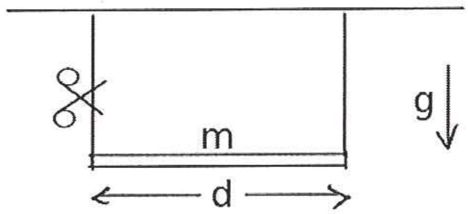
A uniform bar with mass m and length d hangs from the ceiling by two ideal strings attached to its ends. Find the tension on the right string immediately after the left string is cut.

Hint: You may assume that the bar rotates about its right end.





The torque of the system is given as:

$$\sum \tau_z = I\alpha = mg\frac{d}{2} \Rightarrow \frac{\cancel{m}d^2}{3}\alpha = \cancel{m}g\frac{d}{2} \Rightarrow$$

$$\Rightarrow \alpha = \frac{3gd}{2d^2} \Rightarrow \alpha = \frac{3g}{2d}$$

From the Newton's second law we have:

$$\sum F_x = m a_{cm} = mg - T$$

$$\Rightarrow T = m(g - a_{cm}) = m\left(g - \frac{d}{2}\cdot\alpha\right) \Rightarrow$$

$$\Rightarrow T = m\left(g - \frac{\cancel{d}}{2}\cdot\frac{3g}{2\cancel{d}}\right) = m\left(g - \frac{3g}{4}\right) \Rightarrow$$

$$\Rightarrow \boxed{T = \frac{mg}{4}}$$

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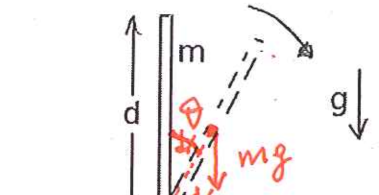
A uniform bar of length d and mass m is hinged at its base on a horizontal surface and is free to rotate about it. Initially standing upright, it looses its balance due to a light breeze and falls under the influence of gravity alone. Find the angular speed of the bar when it touches the ground, by calculating the rotational work ($W = \int \tau d\theta$) done by the bar's weight.



$$\tau = mg\frac{d}{2}\sin\theta$$

$$W = \frac{mgd}{2}\int_0^{2\pi}\sin\theta\, d\theta = \frac{mgd}{2}(-\cos\theta)\Big|_0^{2\pi} = \frac{mgd}{2}$$

From conservation of energy we have:

$$\frac{mdg}{2} = \frac{1}{2}I\omega^2 \Rightarrow mdg = \frac{md^2}{3}\omega^2$$

$$\Rightarrow \omega^2 = \frac{3.mdg}{md^2} = \frac{3g}{d} \Rightarrow$$

$$\Rightarrow \omega = \sqrt{\frac{3g}{d}}$$

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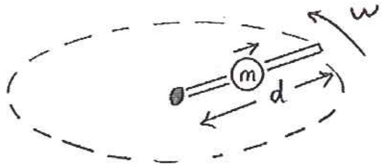
A straight rod with a length d and mass m is rotating horizontally with an angular speed ω about a fixed pivot at its end. A ball with mass m and negligible size is attached to the rod initially at a distance $d/2$ from the pivot, but it is free to slide along the rod without friction. Find the angular speed of the rod and the radial speed of the ball when the ball reaches the end of the rod.

Hint: Moment of inertia for a uniform rod with length d and mass m, rotating about one of its ends is $I = md^2/3$.



From conservation of angular momentum: $L_i = L_f \Rightarrow$

$$m\left(\frac{d}{2}\right)^2 \omega + \frac{md^2}{3}\omega = md^2\omega_f + \frac{md^2}{3}\omega_f \Rightarrow \frac{7md^2}{12}\omega = \frac{4md^2}{3}\omega_f \Rightarrow$$

$$\Rightarrow \boxed{\omega_f = \frac{7}{16}\omega}$$

From conservation of energy: $K_i + \cancelto{0}{U_i} + \cancelto{0}{W_{ot.}} = K_f + \cancelto{0}{U_f}$

$$\cancel{\frac{1}{2}}\cancel{m}\left(\frac{d}{2}\right)^2 \cdot \omega^2 + \cancel{\frac{1}{2}}\frac{\cancel{m}d^2}{3}\omega^2 = \cancel{\frac{1}{2}}\cancel{m}d^2\omega_f^2 + \cancel{\frac{1}{2}} \cdot \frac{\cancel{m}d^2}{3}\omega_f^2 + \cancel{\frac{1}{2}}\cancel{m}V^2$$

$$\frac{7}{12}d^2\omega^2 = \frac{4}{3}d^2\omega_f^2 + V^2 \Rightarrow$$

$$\Rightarrow V^2 = \frac{7}{12}d^2\omega^2 - \frac{4}{3}d^2\left(\frac{7}{16}\right)^2\omega^2 = \frac{7}{12}d^2\omega^2\left(1 - \frac{7}{16}\right) \Rightarrow$$

$$V^2 = \frac{21 d^2\omega^2}{64} \Rightarrow \boxed{V = \frac{\sqrt{21}}{8} d\omega}$$