

Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes

Name:

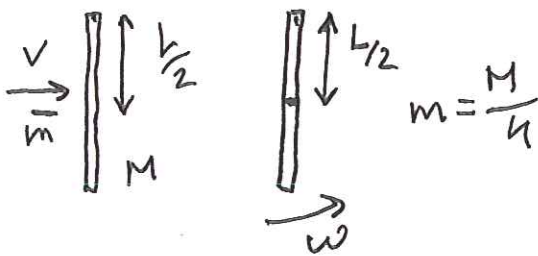
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A uniform rod of length L rests on a frictionless horizontal surface. The rod pivots about a fixed frictionless axis at one end. The rod is initially at rest. A bullet traveling parallel to the horizontal surface and perpendicular to the rod with speed v strikes the rod at its center and becomes embedded in it. The mass of the bullet is one-fourth the mass of the rod.

a) What is the final angular speed of the rod?

What is the ratio of the kinetic energy of the system after the collision to the kinetic energy of the bullet before the collision?



a) From conservation of angular momentum we have:

$$L_1 = L_2$$

$$mv \frac{L}{2} = (I_{\text{rod}} + I_B) \omega$$

$$\frac{M}{4} v \frac{L}{2} = \left(m \left(\frac{L}{2} \right)^2 + \frac{1}{3} ML^2 \right) \omega$$

$$\frac{M}{8} v L = \left(\frac{ML^2}{16} + \frac{1}{3} ML^2 \right) \omega$$

$$\frac{v}{8} = \frac{19}{48} L \omega$$

$$\omega = \frac{48}{8} \frac{v}{19L} = \frac{6}{19} \frac{v}{L}$$

b)

$$K_1 = \frac{1}{2} mv^2 = \frac{1}{8} MV^2$$

$$K_2 = \frac{1}{2} I \omega^2 = \frac{1}{2} (I_{\text{rod}} + I_B) \omega^2$$

$$K_2 = \frac{1}{2} \left(\frac{1}{3} ML^2 + \frac{1}{16} ML^2 \right) \omega^2$$

$$K_2 = \frac{1}{2} \frac{19}{48} ML^2 \cdot \left(\frac{6}{19} \right)^2 \cdot \frac{v^2}{L^2}$$

$$K_2 = \frac{3}{152} MV^2$$

$$\frac{K_2}{K_1} = \frac{\frac{3}{152} MV^2}{\frac{1}{8} MV^2} = \frac{3}{19}$$

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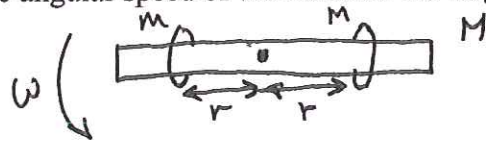
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A uniform, 0.03 kg rod of length 0.4 m rotates in a horizontal plane about a fixed axis through its center and perpendicular to the rod. Two small rings, each with mass 0.02 kg, are mounted so that they can slide along the rod. They are initially held by catches at positions 0.05 m on each side of the center of the rod, and the system is rotating at 30 rev/min. With no other changes in the system, the catches are released, and the rings slide outward along the rod and fly off at the ends.

a) What is the angular speed of the system at the instant when the rings reach the ends of the rod?

b) What is the angular speed of the rod after the rings leave it?



a) Angular momentum is conserved: $L_1 = L_2$

$$I_1 \omega_1 = I_2 \omega_2 \Rightarrow \omega_2 = \omega_1 \frac{I_1}{I_2}$$

$$I_1 = \frac{1}{12} M L^2 + 2 m r_1^2 = \frac{1}{12} \cdot 3 \cdot 10^{-2} \cdot 4^2 \cdot 10^{-2} + 2 \cdot 2 \cdot 10^{-2} \cdot 5^2 \cdot 10^{-4} = 5 \cdot 10^{-4}$$

$$I_2 = \frac{1}{12} M L^2 + 2 m r_2^2 = \frac{1}{12} \cdot 3 \cdot 10^{-2} \cdot 4^2 \cdot 10^{-2} + 2 \cdot 2 \cdot 10^{-2} \cdot 2^2 \cdot 10^{-2} = 20 \cdot 10^{-4}$$

$$\omega_2 = 30 \cdot \frac{5 \cdot 10^{-4}}{20 \cdot 10^{-4}} = 7.5 \text{ rev/min}$$

b) The forces and torques that the rings and the rod exert on each other will vanish. But the common angular velocity will be same 7.5 rev/min.

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Occasionally, a rotating neutron star undergoes a sudden and unexpected speedup called a glitch. One explanation is that a glitch occurs when the crust of the neutron star stiles slightly, decreasing the moment of inertia about the rotation axis. A neutron star with angular speed $\omega_0 = 70.4 \text{ rad/s}$ underwent such a glitch in October 1975 that increased its angular speed to $\omega = \omega_0 + \Delta\omega$, where $\Delta\omega/\omega_0 = 2 \times 10^{-6}$. If the Radius of the neutron star before the glitch was 11 km, by how much did its Radius decrease in the star-quake?

Assume that the neutron star is a uniform sphere.

Angular momentum is conserved:

$$L_1 = L_2$$

$$R_0^2 \omega_0 = R^2 \omega$$

$$R_0^2 \omega_0 = (R_0 + \Delta R)^2 (\omega_0 + \Delta\omega)$$

$$R_0^2 \omega_0 = (R_0^2 + 2\Delta R R_0 + (\Delta R)^2) (\omega_0 + \Delta\omega)$$

If we ignore very small terms $(\Delta R)^2$ and $\Delta R \Delta\omega$:

$$R_0^2 \omega_0 = R_0^2 \omega_0 + 2R_0 \omega_0 \Delta R + R_0^2 \Delta\omega$$

$$2R_0 \omega_0 \Delta R = -R_0^2 \Delta\omega$$

$$\Delta R = -\frac{R_0}{2} \frac{\Delta\omega}{\omega_0} = -\frac{11 \cdot 10^3}{2} \cdot 2 \cdot 10^{-6} \text{ m}$$

$$\Delta R = -11 \cdot 10^{-3} \text{ m} = -1.1 \text{ cm}$$

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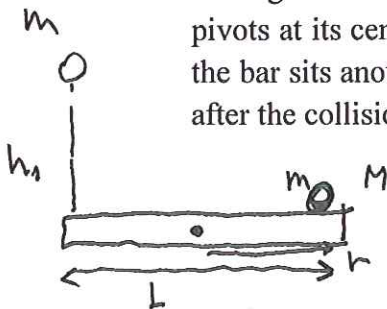
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A 5 kg ball is dropped from a height of 12.8 m above one end of a uniform bar that pivots at its center. The bar has mass 7.5 kg and is 4 m in length. At the other end of the bar sits another 5 kg ball, unattached to the bar. The dropped ball sticks to the bar after the collision. How high will the other ball go after the collision?



$$r = \frac{L}{2}$$

From conservation of energy, the speed of the ball before it hits the bar can be determined:

$$\frac{1}{2} m v_1^2 = m g h_1 \Rightarrow v_1 = \sqrt{2 g h_1} = \sqrt{2 \cdot 10 \cdot 12,8} = \sqrt{256} = 16 \text{ m/s}$$

From conservation of angular momentum we have:

$$L_1 = L_2 \Rightarrow m v_1 r = I \omega \Rightarrow m v_1 r = \left(\frac{1}{12} M L^2 + 2 m r^2 \right) \omega \Rightarrow$$

$$\Rightarrow 5 \cdot 16 \cdot 2 = \left(\frac{1}{12} \cdot 7,5 \cdot 4^2 + 2 \cdot 5 \cdot 2^2 \right) \omega \Rightarrow$$

$$160 = (10 + 40) \omega \Rightarrow \omega = \frac{160}{50} = \frac{16}{5}$$

Just after the collision the speed of the second ball is:

$$v_2 = r \omega = 2 \cdot \frac{16}{5} = \frac{32}{5}$$

Again using conservation of energy:

$$\frac{1}{2} m v_2^2 = m g h_2 \Rightarrow h = \frac{v_2^2}{2g} = 2,05 \text{ m}$$

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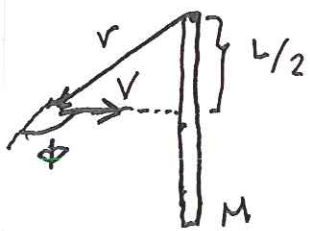
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A target in a shooting gallery consists of a vertical square wooden board with length L on a side and with mass M , that pivots on a horizontal axis along its top edge. The board is struck face-on at its center by a bullet with mass m that is traveling at speed v and that remains embedded in the board. Give your answer in terms of M , m , L and v .

- What is the angular speed of the board just after the bullet's impact?
- What maximum height above the equilibrium position does the center of the board reach before starting to swing down again?

a) Before

After



Using Conservation of Angular momentum we have:

$$L_1 = L_2$$

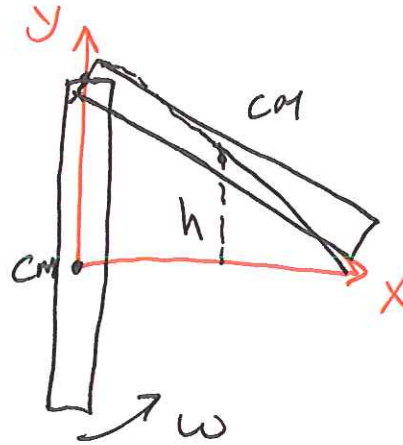
$$L_1 = mvr \sin \phi = mV \frac{L}{2}$$

$$L_2 = I\omega ; I = I_{\text{Board}} + I_{\text{Bullet}}$$

$$I = \frac{1}{3} ML^2 + m \left(\frac{L}{2} \right)^2$$

$$L_1 = mV \frac{L}{2} = \omega \left(\frac{1}{3} ML^2 + \frac{1}{4} mL^2 \right) = L_2$$

$$\omega = \frac{mV}{\left(\frac{2}{3} ML + \frac{1}{2} mL \right)}$$



Using conservation of energy we have:

$$U_1 + K_1 + W_{\text{tot}} = U_2 + K_2$$

$$K_1 = U_2$$

$$\frac{1}{2} I \omega^2 = (m+M)gh$$

$$h = \frac{I \omega^2}{2(m+M)g}$$