PHYS 101: General Physics KOÇ UNIVERSITY Fall Semester 2015
College of Arts and Sciences
Quiz - Ch.14

Closed book. No calculators are to be used for this quiz.
Name: Student ID: Signature:

A guitar string stretched horizontally vibrates at a frequency of $(1000/2\pi)$ Hz. The point at its center moves in SHM with an ant resting on it. What is the maximum amplitude of oscillation at the center point if the ant does not fall off? ($g \simeq 10$ m/s^2)

Hint: The ant will remain on the string as long as the string applies an upward normal force on it. What does this say about the acceleration of the string?

The ant will remain on the string as long as the acceleration of the string will be smaller than g:

$$a_{max} \leq g$$

$$A\omega^2 \leq g \quad ; \quad \omega = 2\pi f$$

$$A(2\pi f)^2 \leq g$$

$$A \leq \frac{g}{(2\pi f)^2} \Rightarrow A_{max} = \frac{g}{(2\pi f)^2}$$

$$A_{max} = \frac{10\,m/s^2}{\left(2\pi \frac{1000}{2\pi} s^{-1}\right)^2} = \frac{10}{10^6} m \Rightarrow$$

$$\Rightarrow \boxed{A_{max} = 10^{-5}\,m}$$

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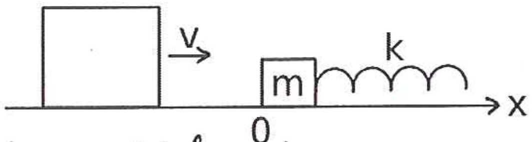
A block with mass m is at rest, attached to an ideal spring with spring constant k on a horizontal, frictionless table. A much larger block with infinite mass and speed v collides elastically with it from the opposite side of the spring. All motion is along the x-axis and you may assume that the collision takes place at $x = 0$ (ignore the sizes of the boxes). Draw x vs t graphs for the two blocks on the same (x,t) axes and write down an equation for the position of their next collision.

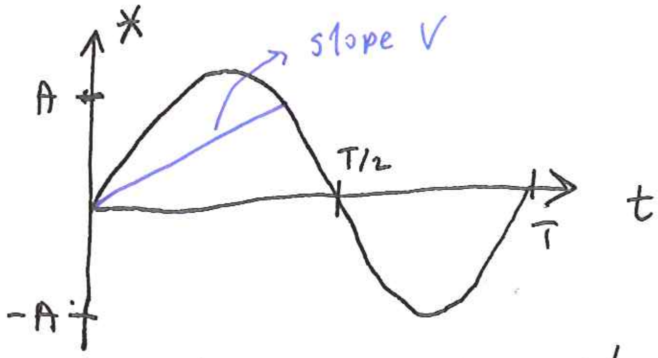
Hint: Immediately after the collision, the speed of the small block is $\lim_{M\to\infty} \frac{2M}{M+m} v = 2v$.



Since big block has infinite momentum it will not be effected by the collision. Small block will make harmonic motion with maximum velocity 2V. From conservation of energy we have:

$$\cancel{\tfrac{1}{2}} m (2V)^2 = \cancel{\tfrac{1}{2}} k A^2 \Rightarrow \boxed{A = 2V\sqrt{\frac{m}{k}} = \frac{2V}{\omega}}$$

- Period of m is: $T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}}$
- Position in general: $X = A\cos(\omega t)$



- The collision will happen at: $Vt' = A\cos\omega t'$ and
- Position will be: $x' = Vt'$, where t' is solution of above equation.

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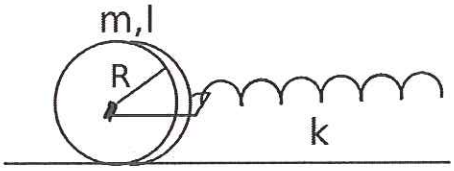
A wheel with mass m, radius R, and moment of inertia I is connected from its center to an ideal spring with spring constant k. The wheel oscillates while rolling without slipping at all times. Derive the SHM equation of motion ($d^2x/dt^2 = -\omega^2 x$) starting from the mechanical energy E of the system and setting $dE/dt = 0$ by energy conservation. Find the frequency of the SHM. (Caution: ω above is not the angular velocity of the wheel.)



The mechanical energy of the system can be written as: (where Ω denotes the angular velocity)

$$E = \frac{1}{2} m V_{cm}^2 + \frac{1}{2} I \Omega^2 + \frac{1}{2} k x^2 = \frac{1}{2} m V^2 + \frac{1}{2} I \left(\frac{V}{R}\right)^2 + \frac{1}{2} k x^2$$

From energy conservation we have:

$$\frac{dE}{dt} = 0 = \cancel{\tfrac{1}{2}} m \cdot \cancel{2} V \frac{dV}{dt} + \cancel{\tfrac{1}{2}} \frac{I}{R^2} \cdot \cancel{2} \cdot V \frac{dV}{dt} + \cancel{\tfrac{1}{2}} k \cdot \cancel{2} x \frac{dx}{dt}$$

Using $V = \frac{dx}{dt}$:

$$m \cancel{V} \frac{dV}{dt} + \frac{I}{R^2} \cancel{V} \frac{dV}{dt} + k x \cancel{V} = 0 \Rightarrow$$

$$\Rightarrow \left(m + \frac{I}{R^2}\right) \frac{dV}{dt} + kx = 0 \Rightarrow \left(m + \frac{I}{R^2}\right) \frac{d^2x}{dt^2} = -kx \Rightarrow$$

$$\Rightarrow \boxed{\frac{d^2x}{dt^2} = -\omega^2 x} \quad \text{where} \quad \boxed{\omega^2 = \frac{k}{\left(m + \frac{I}{R^2}\right)}} \quad \boxed{f = \frac{\omega}{2\pi}}$$

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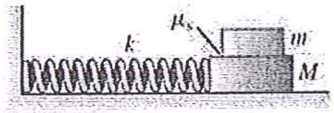
A block with mass M rests on a frictionless surface and is connected to a horizontal spring of force constant k. A second block with mass m rests on top of the first block and they oscillate together with an amplitude A. What is the minimum value of the coefficient of static friction between the blocks?

Hint: Static friction is the only horizontal force acting on the block with mass m.



The maximum acceleration mass m feels is given as:

$$a_{max} = \frac{K}{M_{Tot.}} \cdot A \quad ; \quad M_{Tot.} = M + m$$

$$a_{max} = \frac{KA}{M+m}$$

If we write the Newton's second law for the mass m we have:

$$F_x = m a_x \Rightarrow f_s = m\, a_{max} \Rightarrow$$

$$\Rightarrow \mu_s \cancel{m} g = \cancel{m} \frac{KA}{M+m} \Rightarrow$$

$$\Rightarrow \boxed{\mu_s = \frac{KA}{g(M+m)}}$$

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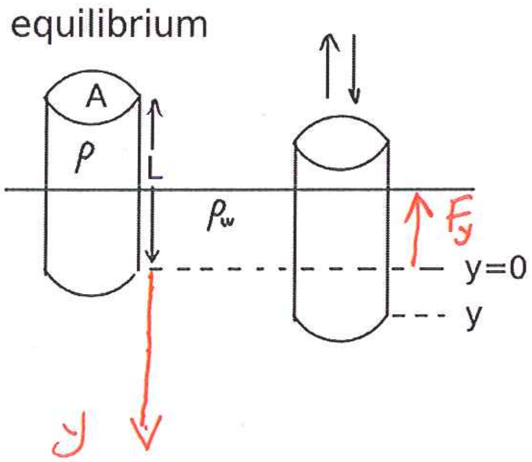
Archimedes observed that an object in water is subject to an upward buoyant force which is equal in magnitude to the weight of the water displaced by the object:

$$F_y = V_s \times \rho_w \times g = (\text{submerged volume}) \times (\text{density of water}) \times (\text{gravitational acceleration}) .$$

Find the frequency of oscillations for a cylindrical object floating vertically in the water. The object's height is L, cross sectional area is A, density is ρ.

Hint: Express the upward restoring force on the object when it is pushed down by a distance y from its equilibrium and show that $F_y = -ky$, where k is a function of given parameters.



$$F_y = -V_s \rho_w g = -yA\rho_w g \Rightarrow F_y = -A\rho_w g y$$

$$ma_y = F_y \Rightarrow \rho \cdot L \cdot A\, a_y = -A\rho_w g y \Rightarrow$$

$$\Rightarrow a_y + \frac{\rho_w g}{L\rho} y = 0 \Rightarrow$$

$$\Rightarrow \omega^2 = \frac{\rho_w g}{L\rho} \Rightarrow \omega = \sqrt{\frac{\rho_w g}{L\rho}}$$

$$\boxed{f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{\rho_w g}{L\rho}}}$$