B2

PHYS 101: General Physics **KOÇ UNIVERSITY** **Fall Semester 2015**

College of Sciences

Quiz 1 -1 **1 October 2015**

Closed book. No calculators are to be used for this quiz.

Quiz duration: 10 minutes

First Name: **Last name:** **Student ID:**

Signature: Solution

Find the angle between ($\vec{A}-\vec{B}$) and $\vec{A}$, for $\vec{A}=4\hat{i}+\hat{j}+2\hat{k}$ and $\vec{B}=2\hat{i}+3\hat{j}+\hat{k}$.

$$\vec{A}-\vec{B} = 4\hat{i}+\hat{j}+2\hat{k}-(2\hat{i}+3\hat{j}+\hat{k})$$

$$\vec{A}-\vec{B} = (4-2)\hat{i}+(1-3)\hat{j}+(2-1)\hat{k}$$

$$\vec{A}-\vec{B} = 2\hat{i}-2\hat{j}+\hat{k}$$

use scalar product ~~between~~ $(\vec{A}-\vec{B})\cdot\vec{A}$

$$(\vec{A}-\vec{B})\cdot\vec{A} = |\vec{A}-\vec{B}||\vec{A}|\cos\theta$$

$$(2\hat{i}-2\hat{j}+\hat{k})\cdot(4\hat{i}+1\hat{j}+2\hat{k}) = \sqrt{2^2+2^2+1}\sqrt{4^2+1^2+2^2}\cos\theta$$

$$8-2+2 = (3)(\sqrt{21})\cos\theta$$

$$\cos\theta = \frac{8}{3\sqrt{21}} \Rightarrow \theta = a\cos\left[\frac{8}{3\sqrt{21}}\right] \approx 54.4^\circ$$

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PHYS 101: General Physics **KOÇ UNIVERSITY** **Fall Semester2015**

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Quiz 1 -2 **1 October 2015**

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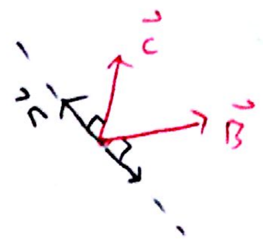
Quiz duration: 10 minutes
First Name: **Last name:** **Student ID:**
Signature:

Solution

Find the vector $\vec{A}$ that has magnitude $A = 4$ and that is perpendicular to both $\vec{B} = +5\hat{\imath} + 3\hat{\jmath} - 7\hat{k}$ and $\vec{C} = -2\hat{\imath} - 4\hat{\jmath} + 7\hat{k}$.



The unit vector perpendicular to both $\vec{C}$ and $\vec{B}$ can be written as:

$$\hat{n} = \pm \frac{\vec{B} \times \vec{C}}{|\vec{B} \times \vec{C}|}$$

$$\vec{B} \times \vec{C} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 5 & 3 & -7 \\ -2 & -4 & 7 \end{vmatrix} = (21-28)\hat{\imath} - (35-14)\hat{\jmath} + (-20+6)\hat{k}$$

$$\vec{B} \times \vec{C} = -7\hat{\imath} - 21\hat{\jmath} - 14\hat{k}$$

$$\hat{n} = \pm \frac{(-7\hat{\imath} - 21\hat{\jmath} - 14\hat{k})}{\sqrt{7^2 + 21^2 + 14^2}}$$ and has magnitude of 1.

vector $\vec{A}$ must have direction along $\hat{n}$ and magnitude 4.

So,

$$\boxed{\vec{A} = \frac{\pm 4}{\sqrt{7^2 + 21^2 + 14^2}}(-7\hat{\imath} - 21\hat{\jmath} - 14\hat{k})}$$

Note that vector product of two vectors u and v is found as follows:

$u = u_x i + u_y j + u_z k;\quad v = v_x i + v_y j + v_z k;$

$$u \times v = \begin{bmatrix} i & j & k \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{bmatrix} = (u_y v_z - u_z v_y) i + (u_z v_x - u_x v_z) j + (u_x v_z - u_z v_x) k$$

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Quiz 1 -3 **1 October 2015**

Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes
First Name: **Last name:** **Student ID:**
Signature: Solution

Calculate the vector product of ($\vec{A}+\vec{B}$) and ($\vec{A}-\vec{B}$) where $\vec{A}=5\hat{i}-2\hat{j}+4\hat{k}$ and $\vec{B}=3\hat{i}+4\hat{j}-2\hat{k}$.

$$\vec{A}+\vec{B} = 8\hat{i} + 2\hat{j} + 2\hat{k}$$

$$\vec{A}-\vec{B} = 2\hat{i} - 6\hat{j} + 6\hat{k}$$

$$(\vec{A}+\vec{B}) \times (\vec{A}-\vec{B}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & 2 & 2 \\ 2 & -6 & 6 \end{vmatrix} = \boxed{24\hat{i} - 44\hat{j} - 52\hat{k}}$$

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Quiz 1 -4 **1 October 2015**

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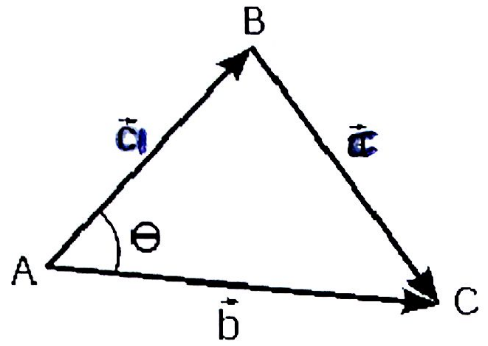
Quiz duration: 10 minutes
First Name: **Last name:** **Student ID:**
Signature:

Solution



Using vector techniques and considering the triangle formed by three vectors as shown above, show that $c^2 = a^2 + b^2 - 2ab\cos\theta$. (Hint: Consider the vector relation $\vec{\mathbf{c}} = \vec{\mathbf{b}} - \vec{\mathbf{a}}$)

$\vec{c} = \vec{b} - \vec{a}$, so

$\vec{c}\cdot\vec{c} = (\vec{b}-\vec{a})\cdot(\vec{b}-\vec{a})$

$|\vec{c}|^2 = \vec{b}\cdot\vec{b} - \vec{a}\cdot\vec{b} - \vec{b}\cdot\vec{a} + \vec{a}\cdot\vec{a}$

Take $|\vec{c}| = c$, $|\vec{b}| = b$, $|\vec{a}| = a$

Then, $c^2 = b^2 + a^2 - 2\,\vec{a}\cdot\vec{b}$

It is known that $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Hence $\boxed{c^2 = a^2 + b^2 - 2ab\cos\theta}$

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Quiz 1 -5 **1 October 2015**

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Quiz duration: 10 minutes
First Name: **Last name:** **Student ID:**
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Solution

By direct substitution show that $\vec{\mathbf{A}}\times(\vec{\mathbf{B}}\times\vec{\mathbf{C}}) = \vec{\mathbf{B}}(\vec{\mathbf{A}}\cdot\vec{\mathbf{C}}) - \vec{\mathbf{C}}(\vec{\mathbf{A}}\cdot\vec{\mathbf{B}})$ for the vectors $\vec{\mathbf{A}} = \hat{\imath} - 2\hat{\jmath}$, $\vec{\mathbf{B}} = \hat{\jmath} + \hat{k}$, and $\vec{\mathbf{C}} = \hat{k}$.

$$\vec{B}\times\vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \hat{i} + 0\hat{j} + 0\hat{k} = \hat{i}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \end{vmatrix} = 0\hat{i} + 0\hat{j} + 2\hat{k} = \boxed{2\hat{k}}$$

$$\vec{A}\cdot\vec{C} = (\hat{i} - 2\hat{j})\cdot\hat{k} = 0$$

$$\vec{A}\cdot\vec{B} = (\hat{i} - 2\hat{j})\cdot(\hat{j} + \hat{k}) = -2$$

So, $\vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B}) = \vec{B}(0) - \vec{C}(-2) = 2\vec{C}$

$$= \boxed{2\hat{k}}$$

Which confirms that:

$$\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$$