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PHYS 101: General Physics 1 **KOÇ UNIVERSITY** **Fall Semester 2015**

College of Sciences

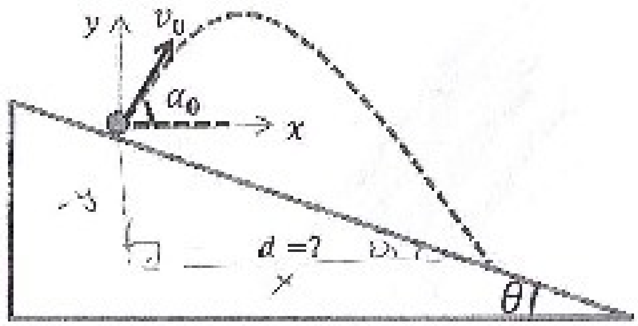
Section 1 **Quiz 2** **08 October 2015**

Closed book. No calculators are to be used for this quiz.
Quiz duration: 15 minutes

Name: **Student ID:** **Signature:**

A projectile is launched from the origin with initial speed v_0 on an inclined surface. The launch angle with the horizontal is $\alpha_0 = \frac{\pi}{3}$. The inclination angle of the surface is $\theta = \frac{\pi}{4}$. Determine the distance of the landing point from the launch point in terms of the given parameters. The gravitational acceleration is g.

Note: $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$, $\cos\frac{\pi}{3} = \frac{1}{2}$, $\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$



$\tan\theta = \frac{-y}{x} = \tan\left(\frac{\pi}{4}\right) = 1$

$\Rightarrow y = -x$

$x = (v_0 \cos\alpha_0)t$

$y = (v_0 \sin\alpha_0)t - \frac{1}{2}gt^2$

$\sin\alpha_0 = \frac{\sqrt{3}}{2}, \ \cos\alpha_0 = \frac{1}{2}$

$(v_0 \cos\alpha_0)t = \frac{1}{2}gt^2 - (v_0 \sin\alpha_0)t$

$t = \frac{2v_0(\cos\alpha_0 + \sin\alpha_0)}{g} = \frac{v_0}{g}(1+\sqrt{3})$

$d = \sqrt{x^2+y^2} = \sqrt{y^2+y^2} = \sqrt{2}\,y$

$y = (v_0 \sin\alpha_0)\cdot\frac{v_0(1+\sqrt{3})}{g} - \frac{1}{2}g\left(\frac{v_0}{g}(1+\sqrt{3})\right)^2$

$y = \frac{v_0^2}{2g}(1+\sqrt{3})$

$d = \sqrt{2}\,\frac{v_0^2}{2g}(1+\sqrt{3}) = \frac{v_0^2}{\sqrt{2}g}(1+\sqrt{3})$

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Section 2 | **Quiz 2** | **08 October 2015**

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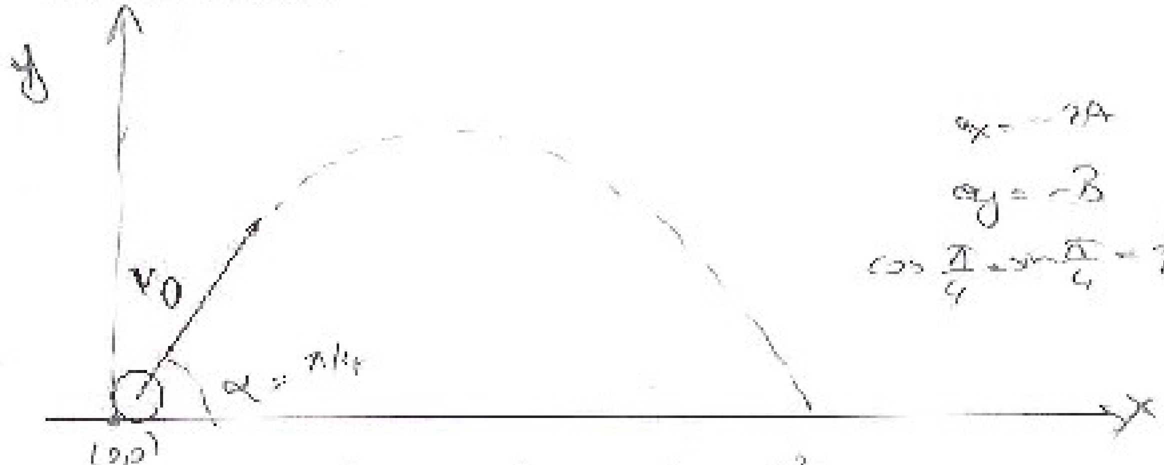
Quiz duration: 15 minutes

First Name: **Last name:** **Student ID:** **Signature:**

Consider the motion shown in the figure, where a ball starts moving at time $t = 0$ with speed v_0 and making an angle of $\pi/4$ radians with the horizontal axis. If the horizontal and vertical accelerations of the ball are given by $a_x = -2A$ and $a_y = -B$, respectively, find its vertical and horizontal displacements with respect to the initial position as a function of time. Here A and B are positive constants. You may ignore the effects of gravity.



$a_x = -2A$

$a_y = -B$

$\cos\frac{\pi}{4} = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$x(t) = x(0) + (v_0 \cos\alpha)t + \frac{1}{2} a_x t^2$

$y(t) = y(0) + (v_0 \sin\alpha)t + \frac{1}{2} a_y t^2$

$x(t) - \cancel{x(0)}^{0} = \frac{v_0}{\sqrt{2}} t + \frac{1}{2}(-2A)t^2$

$y(t) - \cancel{y(0)}^{0} = \frac{v_0}{\sqrt{2}} t + \frac{1}{2}(-B)t^2$

$\frac{dx(t)}{dt} = 2At^* - \frac{1}{\sqrt{2}} v_0 = 0 \Rightarrow t^* = \frac{v_0}{2\sqrt{2}A}$ max. point

$\frac{dy(t)}{dt} = -Bt^* + \frac{v_0}{\sqrt{2}} = 0 \Rightarrow t^* = \frac{v_0}{\sqrt{2}B}$ max

$\frac{d^2x}{dt^2} < 0, \quad \frac{d^2y}{dt^2} < 0$

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Section 3 | Quiz 2 | 08 October 2015

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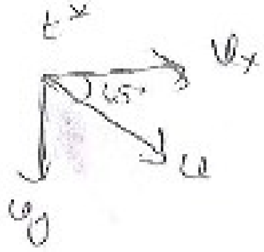
Quiz duration: 15 minutes

First Name: **Last name:** **Student ID:** **Signature:**

A missile is thrown at an angle 53° into air from the ground with initial speed $v_0 = 720$ km/h (there is no air resistance). Find the altitude of the missile from the ground at the instant when its velocity vector makes an angle -45° with the x-axis. (Hint: $\tan(53°) = 4/3$, $g = 10 \text{m/s}^2$).

$V_0 \sin 53° =$



$v_x = v_y$

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ $\quad a_x = 0$

$\Rightarrow v_x = v_{0x}$ constant speed in x-direction

$v_{0x} = v_0 \cos 53 = v_x = v_y$

$v_y^2 = v_{0y}^2 + 2a_y(y - \cancel{y_0})$ $\quad a_y = -10 \text{ m/s}^2$

$(v_0 \cos 53)^2 = (v_0 \sin 53)^2 + 2 \cdot (-10 \frac{m}{s^2}) y$

$\left(v_0 = 720 \frac{km}{h} \cdot \frac{1000 m}{km} \cdot \frac{1}{3600} \frac{h}{s} = 200 \text{ m/s} \right)$

Take y

$y = \left(200 \frac{m}{s}\right)^2 \frac{1}{2(10 \frac{m}{s^2})} (\cos^2 53 - \sin^2 53)$

$\sin 53 = 4/5$

$\cos 53° = 3/5$

$y = 560$ m ✓

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Section 4 — Quiz 2 — 08 October 2015

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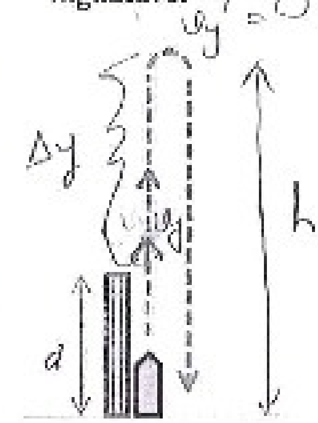
Quiz duration: 15 minutes
First Name: **Last name:** **Student ID:** **Signature:**

A vertical launch ramp is $d = 10\ m$ long from the ground. During the launch, the ramp applies a constant vertical acceleration $a_0 = 5\ m/s^2$ to an object until it leaves the ramp. ($g = 10\ m/s^2$).

1. The launch starts at $t = 0s$ with the object at rest on the base of the ramp at the ground. When does the object return to the ground?
2. Plot the velocity vs. time graph of the object during this motion qualitatively. Indicate the important data points on the graph.



1. Object's velocity just after launch

$v_y^2 = \cancel{v_0}^{2} + 2a_y(\underbrace{y - y_0}_{d = 10 m})$ $\quad a_y = 5 m/s^2$

$v_y = 10 m/s$

$y - y_0 = d = v_{0y} t + \frac{1}{2} a_y t^2$

$10 m = 0 + \frac{1}{2}(5 m/s^2) t^2 \Rightarrow t = 2s$ object stays launcher

$\cancel{v_y'} = v_y - gt \Rightarrow (g = 10 m/s^2) \Rightarrow t = 1s$ object reaches max. height

$h = \Delta y + d$

$\Delta y = v_y t - \frac{1}{2} g t^2 \Rightarrow \Delta y = (10 \frac{m}{s})(1s) - \frac{1}{2}(10 \frac{m}{s^2})(1s)^2$

$\Delta y = 5 m$

$h = 5m + 10m = 15m$

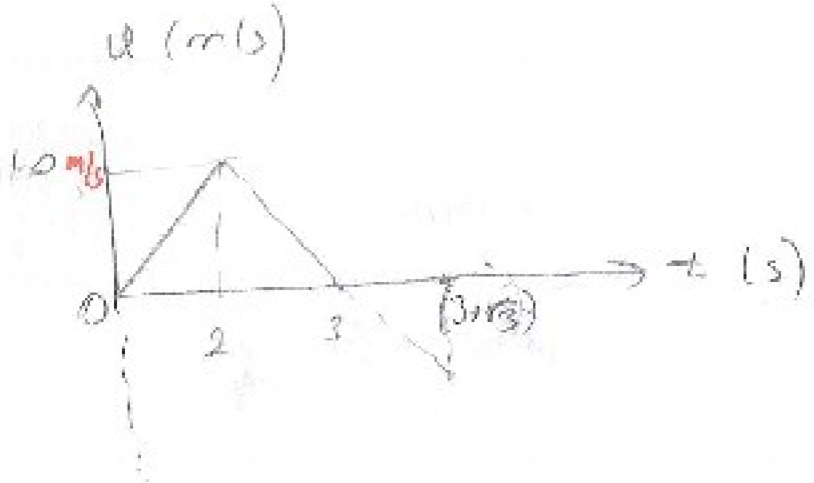
Object reaches ground from $h = 15 m$ in

$h = \cancel{v_y} t + \frac{1}{2} g t^2 = \frac{1}{2}(10 \frac{m}{s^2}) t^2 = 15 m$

$\underline{t = \sqrt{3}\ s}$

In total, object reaches ground in $\Sigma t = \underline{3+\sqrt{3}}$ s

2.



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Section 5 | Quiz 2 | 08 October 2015

Closed book. No calculators are to be used for this quiz.
Quiz duration: 15 minutes

First Name: **Last name:** **Student ID:** **Signature:**

The position of a particle as a function of time is given as
$\vec{r}(t) = R[1+\cos(\omega t)]\hat{i} + R\sin(\omega t)\hat{j}$; where R and ω are some constants.

a) Determine the instantaneous velocity vector, $\vec{v}(t)$ (using unit vectors $\hat{i}$ and $\hat{j}$).

b) Determine the instantaneous acceleration vector, $\vec{a}(t)$ (using unit vectors $\hat{i}$ and $\hat{j}$).

c) What is the angle between $\vec{v}(t)$ and $\vec{a}(t)$?

d) Based on the information obtained in previous parts, what is the name for this particular motion?

A useful identity: $\sin^2\theta + \cos^2\theta = 1$

a) $\vec{v}(t) = \dfrac{d\vec{r}(t)}{dt} = -R\omega\sin(\omega t)\hat{i} + R\omega\cos(\omega t)\hat{j}$

b) $\vec{a}(t) = \dfrac{d\vec{v}(t)}{dt} = -R\omega^2\cos(\omega t)\hat{i} - R\omega^2\sin(\omega t)\hat{j}$

c) $\vec{v}\cdot\vec{a} = |\vec{v}||\vec{a}|\cos\alpha$

$\cos\alpha = \dfrac{\vec{v}\cdot\vec{a}}{|\vec{v}||\vec{a}|} = \dfrac{R^2\omega^3\sin(\omega t)\cos(\omega t) - R^2\omega^3\sin(\omega t)\cos(\omega t)}{(\omega^2 R^2)(\omega^4 R^2)}$

$\cos\alpha = 0 \Rightarrow \alpha = \pi/2 \Rightarrow \vec{v} \perp \vec{a}$

d) $|\vec{r}(t)| = R\sqrt{(1+\cos\omega t)^2 + \sin^2(\omega t)} = 2R\cos\left(\dfrac{\omega t}{2}\right)$

$|\vec{v}| = \omega R$, $|\vec{a}| = \omega^2 R$

Uniform circular motion