

Closed book. Duration: 10 minutes

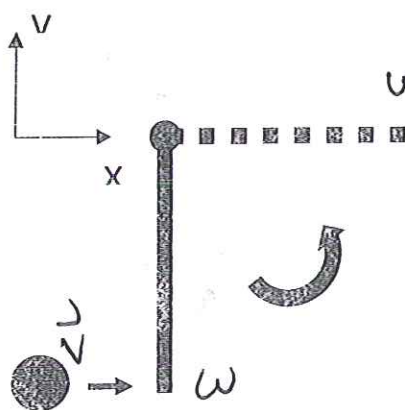
Name: A

Student ID:

Signature:

A rod of length  $l$  and mass  $m$  is initially vertical as in the figure and can freely rotate around a pivot on one end. A ball of mass  $m$  moving in the  $+x$  direction hits the rod on the other end, and comes to a stop immediately after the collision.

- Find the moment of inertia of the rod with respect to the pivot. Moment of inertia of the rod for an axis through its center is  $\frac{1}{12}ml^2$
- If the angular velocity of the rod when it becomes horizontal is half its initial angular velocity right after the ball hits it, what is the initial velocity of the ball?
- What are the  $x$  and  $y$  components of the force the pivot exerts on the rod when the rod is horizontal.



$$a) I = I_{cm} + m d^2 = \frac{1}{12}ml^2 + m\left(\frac{l}{2}\right)^2 = \frac{1}{3}ml^2$$

b) After collision  $E_i = E_f$

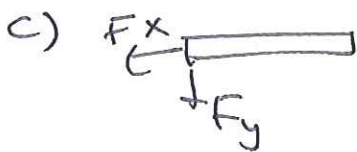
$$\frac{1}{2}I\omega^2 - mg\frac{l}{2} = \frac{1}{2}I\left(\frac{\omega}{2}\right)^2$$

$$\omega^2 = \frac{4mgl}{3I} \Rightarrow \omega = \frac{2\sqrt{8L}}{3}$$

During collision

$$L_i = L_f$$

$$mv\frac{l}{2} = I\omega \Rightarrow v = \frac{I\omega}{m \cdot l/2} \Rightarrow v = \frac{2\sqrt{8L}}{3}$$



$$F_x = ma_r = -m\omega^2 \cdot \frac{l}{2} \hat{x} = -2mg \hat{x}$$

$$F_y = ma_y = m\alpha \cdot \frac{l}{2}$$

$$\tau = I\alpha$$

$$-mg\frac{l}{2} = \frac{1}{3}ml^2 \cdot \alpha$$

$$\alpha = -\frac{3}{2}g/L$$

insert  $\rightarrow$

$$F_y = -\frac{3}{4}mg \hat{y}$$

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Section 3

Quiz 11

December 16, 2016

Closed book. Duration: 10 minutes

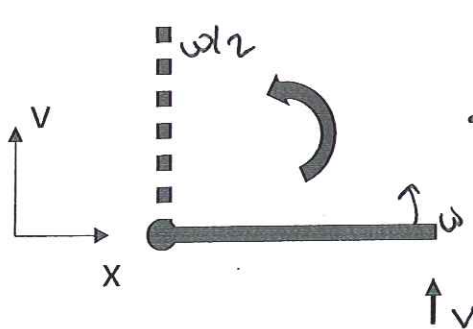
Name:

Student ID:

Signature:

A rod of length  $l$  and mass  $m$  is initially horizontal as in the figure, and can freely rotate around a pivot on one end. A ball of mass  $m$  moving in the  $+y$  direction hits the rod on the other end, and comes to a stop immediately after the collision.

- Find the moment of inertia of the rod with respect to the pivot. Moment of inertia of the rod for an axis through its center is  $\frac{1}{12}ml^2$
- If the angular velocity of the rod when it becomes vertical is half its initial angular velocity right after the ball hits it, what is the initial velocity of the ball?
- What are the  $x$  and  $y$  components of the force the pivot exerts on the rod right after the ball hits it and it starts rotating?



$$I = I_{cm} + md^2$$

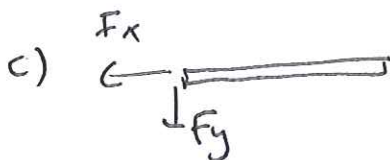
$$a) \quad = \frac{1}{12}ml^2 + m\left(\frac{l}{2}\right)^2 = \frac{1}{3}ml^2 //$$

$$b) \quad \text{After collision } \vec{r}_i = \vec{r}_f \Rightarrow \frac{1}{2}I\omega^2 = \frac{1}{2}I\left(\frac{\omega}{2}\right)^2 + mg\frac{l}{2}$$

$$\omega^2 = \frac{4mgL}{3I} \Rightarrow \omega^2 = 4g/L$$

collision  $L_i = L_f$

$$mvL = I\omega \quad v = \frac{\frac{1}{3}ml^2 \cdot 2\sqrt{g/L}}{mL} = \frac{2}{3}\sqrt{g/L} //$$



$$c) \quad F_x = ma_x = -m\omega^2 \frac{l}{2} \hat{x} = -2mg \hat{x}$$

$$F_y = ma_y = ma \frac{l}{2}$$

$$\tau = I\alpha$$

$$-mg\frac{l}{2} = \frac{1}{3}ml^2 \alpha$$

$$\alpha = -\frac{3}{2}g/L$$

$$F_y = -\frac{3}{4}mg \hat{y}$$

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Section 2

Quiz 11

December 16, 2016

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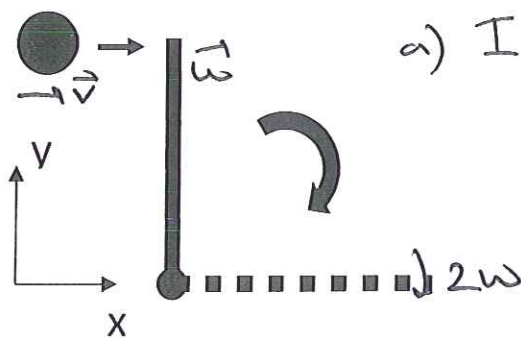
Name:

Student ID:

Signature:

A rod of length  $l$  and mass  $m$  is initially vertical as in the figure, and can freely rotate around a pivot on one end. A ball of mass  $m$  moving in the  $+x$  direction hits the rod on the other end, and comes to a stop immediately after the collision.

- Find the moment of inertia of the rod with respect to the pivot. Moment of inertia of the rod for an axis through its center is  $\frac{1}{12}ml^2$
- If the angular velocity of the rod when it becomes horizontal is twice its initial angular velocity right after the ball hits it, what is the initial velocity of the ball?
- What are the  $x$  and  $y$  components of the force the pivot exerts on the rod when the rod is horizontal.



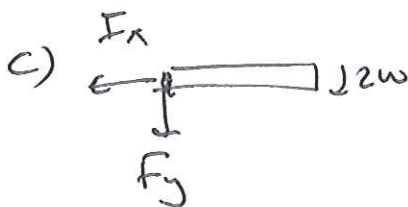
$$a) I = I_{cm} + md^2 = \frac{1}{12}ml^2 + m\left(\frac{l}{2}\right)^2 = \frac{1}{3}mL^2$$

$$b) \text{ After collision } E_i = E_f; \quad \frac{1}{2}I\omega^2 + mg\frac{l}{2} = \frac{1}{2}I(2\omega)^2$$

$$\omega^2 = \frac{mgL}{3I} = \frac{g}{L}$$

collision  $L_i = L_f$

$$mvL = I\omega \Rightarrow v = \frac{\frac{1}{3}mL^2 \sqrt{\frac{g}{L}}}{mL} = \frac{1}{3}\sqrt{gL}$$



$$F_x = ma_x = -m(2\omega)^2 \cdot \frac{l}{2} = -4m\omega^2 \cdot \frac{l}{2} = -2mg \hat{x}$$

$$F_y = ma_y = m\alpha \cdot \frac{l}{2}$$

$$\tau = I\alpha$$

$$-mg \cdot \frac{l}{2} = \frac{1}{3}mL^2 \cdot \alpha$$

$$\alpha = -\frac{3mg}{2L}$$

inset

$$F_y = -\frac{3mg}{4} \hat{y}$$



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Section 4

Quiz 11

December 16, 2016

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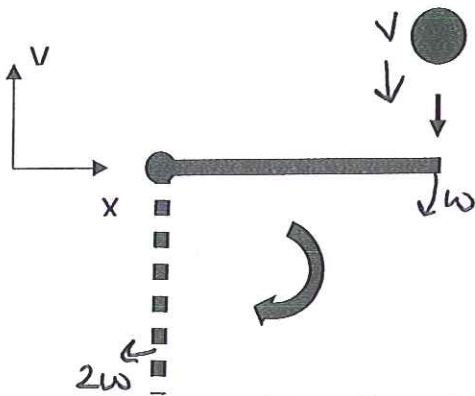
Name:

Student ID:

Signature:

A rod of length  $l$  and mass  $m$  is initially horizontal as in the figure, and can freely rotate around a pivot on one end. A ball of mass  $m$  moving in the  $-y$  direction hits the rod on the other end, and comes to a stop immediately after the collision.

- Find the moment of inertia of the rod with respect to the pivot. Moment of inertia of the rod for an axis through its center is  $\frac{1}{12}ml^2$
- If the angular velocity of the rod when it becomes vertical is twice its initial angular velocity right after the ball hits it, what is the initial velocity of the ball?
- What are the  $x$  and  $y$  components of the force the pivot exerts on the rod right after the ball hits it and it starts rotating?



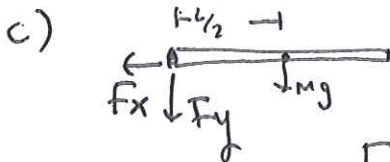
a)  $I = I_{cm} + md^2$   
 $= \frac{1}{12}ml^2 + m\left(\frac{l}{2}\right)^2 = \frac{1}{3}ml^2$

b) After collision  $E_i = E_f \Rightarrow \frac{1}{2}I\omega^2 = \frac{1}{2}I(2\omega)^2 - mgL/2$

collision  $L_i = L_f$

$\omega^2 = \frac{mgL}{3I} = \frac{g}{L}$

$m v \cdot L = I\omega \Rightarrow v = \frac{1}{3} \frac{mL^2}{mL} \sqrt{\frac{g}{L}} = \frac{1}{3} \sqrt{gL}$



$F_x = m a_r = -m\omega^2 \cdot \frac{l}{2} \hat{x} = -\frac{3}{2}mg \hat{x}$

$F_y = m \cdot a_y = m \cdot \alpha \cdot \frac{l}{2}$

$\tau = I\alpha$

$-mg \cdot \frac{l}{2} = \frac{1}{3}ml^2 \cdot \alpha$

$\alpha = -\frac{3}{2} \frac{g}{L}$

$F_y = -m \cdot \frac{3}{2} \frac{g}{L} \cdot \frac{l}{2} = -\frac{3}{4}mg \hat{y}$