

Closed book. Duration: 10 minutes

Name:

Student ID:

Signature:

The x -position of a particle (don't worry about the units) with respect to time t is given as $x(t) = 6t^2 - t^3$

Find

- a) the total distance traveled by the particle, and
 b) the displacement of the particle
 from $t = 0$ to $t = 4$?

a) $x(t) = 6t^2 - t^3$ the particle will eventually stop, when $v(t) = 0$.

$$\frac{dx(t)}{dt} = v(t) = 12t - 3t^2 = 0$$

$$12 - 3t = 0$$

$$t = 4 \text{ s.}$$

Let's investigate its position for every second.

$$\text{at } t=0 \Rightarrow x(0) = 0$$

$$\text{at } t=1 \Rightarrow x(1) = 5 \text{ m.}$$

$$\text{at } t=2 \Rightarrow x(2) = 16 \text{ m.}$$

$$\text{at } t=3 \Rightarrow x(3) = 27 \text{ m}$$

$$\text{at } t=4 \Rightarrow x(4) = 32 \text{ m}$$

- b) Since the particle moves only in one direction until $t = 4 \text{ s.}$, total distance traveled by the particle and the displacement of the particle are equal.

$$\Delta x = x(4) - x(0) = 32 \text{ m.}$$

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The y -position of a particle (don't worry about the units) with respect to time t is given as $y(t) = 3t^2 - t^3$. Find the position of the particle when it reaches maximum velocity?

$$y(t) = 3t^2 - t^3$$

$$\frac{dy(t)}{dt} = v(t) = 6t - 3t^2$$

$$\frac{dv(t)}{dt} = a(t) = 6 - 6t = 0 \quad (\text{max. velocity condition})$$

$$t = 1s.$$

$$y(t=1) = 2m$$

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The z-position of a particle (don't worry about the units) with respect to time t is given as $z(t) = t^2 - 2t^3$.

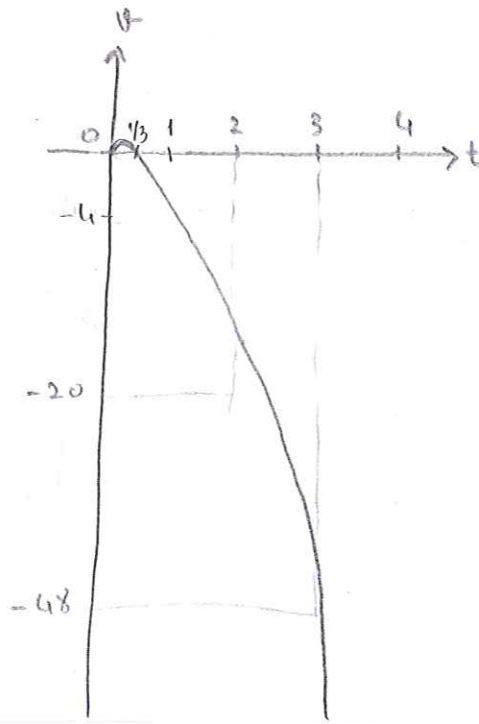
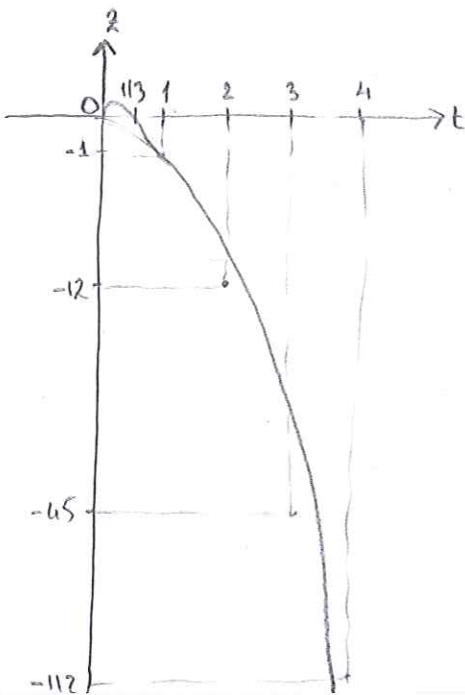
Carefully sketch

- a) the position z , and
- b) the velocity v_z

of the particle as a function of t for $0 < t < 4$. Indicate special points where necessary.

$$v_z(t) = \frac{dz(t)}{dt} = 2t - 6t^2 = 0 \Rightarrow 2t = 6t^2 \Rightarrow \boxed{t = 1/3 \text{ s.}}$$

at $t=0 \Rightarrow z(0) = 0$, $v(0) = 0$
at $t=1 \Rightarrow z(1) = -1$, $v(1) = -4$
at $t=2 \Rightarrow z(2) = -12$, $v(2) = -20$
at $t=3 \Rightarrow z(3) = -45$, $v(3) = -48$
at $t=4 \Rightarrow z(4) = -112$, $v(4) = -88$



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Consider two cannon balls that are both in free fall under the influence of Earth's gravitational acceleration g . At $t = 0$ the speed and height of the first ball are measured to be $v_1 = 0$ and $y_1 = h$, and at time $t = 1s$ the speed and height of the second ball are measured to be $v_2 = 0$ and $y_2 = H$.

Find the relation between h and H if the balls hit the ground at the same instant.



first ball: $y - y_0 = v_0 \cdot t_1 + \frac{1}{2} a t_1^2$

\downarrow \downarrow \downarrow \downarrow
 0 h 0 $-9,8$

$$-h = -4,9 t_1^2$$

$$t_1 = \sqrt{\frac{h}{4,9}}$$

second ball: $y - y_0 = v_0 \cdot t_2 + \frac{1}{2} a t_2^2$

\downarrow \downarrow \downarrow \downarrow
 0 H 0 $-9,8$

$$-H = -4,9 t_2^2$$

$$t_2 = \sqrt{\frac{H}{4,9}}$$

$$t_1 = 1 + t_2$$

$$\sqrt{\frac{h}{4,9}} = 1 + \sqrt{\frac{H}{4,9}}$$

$$\frac{\sqrt{h}}{\sqrt{4,9}} - \frac{\sqrt{H}}{\sqrt{4,9}} = 1$$

$$\boxed{\sqrt{h} - \sqrt{H} = \sqrt{4,9}}$$