PHYS 101:General Physics1 KOÇ UNIVERSITY Fall Semester 2016

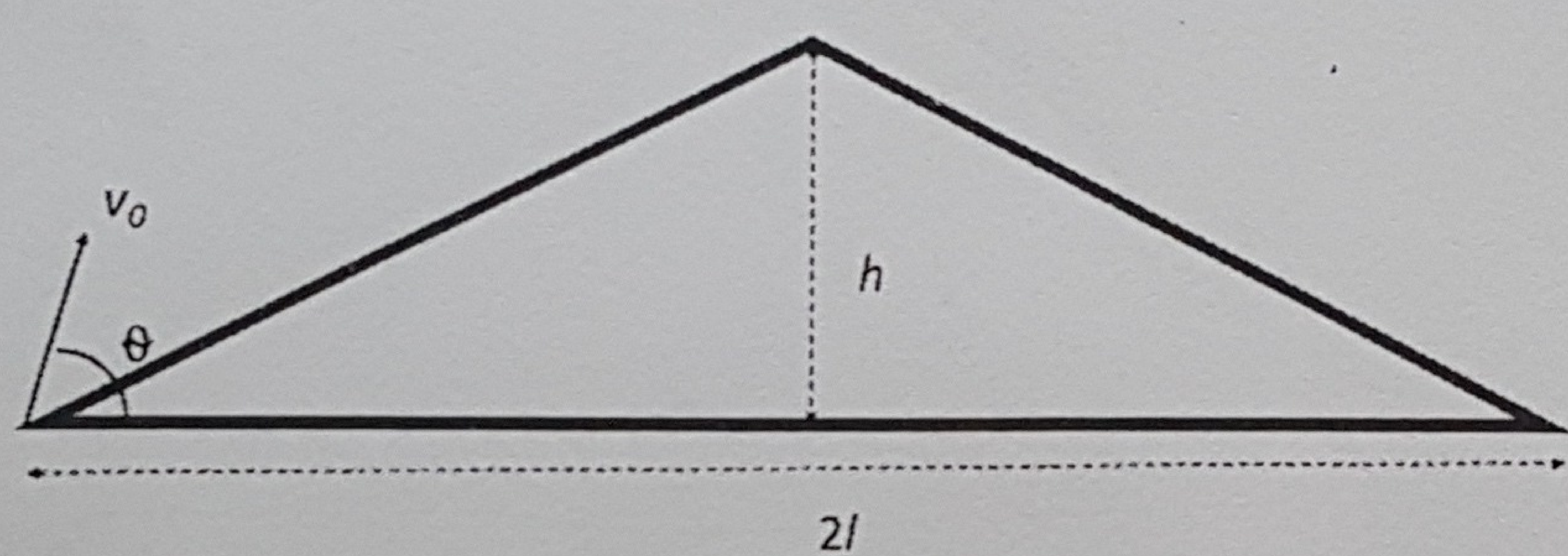
College of Sciences

Section 1 Quiz 3 October 21, 2016

Closed book. Duration: 10 minutes

Name: **Student ID:** **Signature:**

A cannon is placed at the bottom of a hill which has a conical shape of height h and base diameter $2l$. What is the minimum muzzle speed of the cannonball if we want it to go over the entire hill and reach the bottom on the opposite side? At what angle should it be fired in this case?



at max height h: $v_y = 0$

$v_y = v_{0y} - gt$

$0 = v_0 \sin\theta - gt \rightarrow t = \frac{v_0 \sin\theta}{g}$

$y = v_{0y} t - \frac{1}{2} g t^2$

$h = v_0 \sin\theta\, t - \frac{1}{2} g t^2 \rightarrow h = v_0 \sin\theta \left(\frac{v_0 \sin\theta}{g}\right) - \frac{1}{2} g \left(\frac{v_0 \sin\theta}{g}\right)^2$

$\boxed{h = \frac{v_0^2 \sin^2\theta}{2g}}$

$x = v_0 \cos\theta\, t$

$l = v_0 \cos\theta \left(\frac{v_0 \sin\theta}{g}\right) \rightarrow \boxed{l = \frac{v_0^2 \sin\theta \cos\theta}{g}}$

$\Rightarrow \boxed{\tan\theta = \frac{2h}{l}}$ $\boxed{\theta = \tan^{-1}\left(\frac{2h}{l}\right)}$

$v_0^2 = \frac{2gh}{\sin^2\theta} = \frac{2gh}{\left(\frac{2h}{\sqrt{4h^2 + l^2}}\right)^2} = \frac{2gh(4h^2 + l^2)}{4h^2}$

$\boxed{v_0 = \sqrt{\frac{g(4h^2 + l^2)}{2h}}}$

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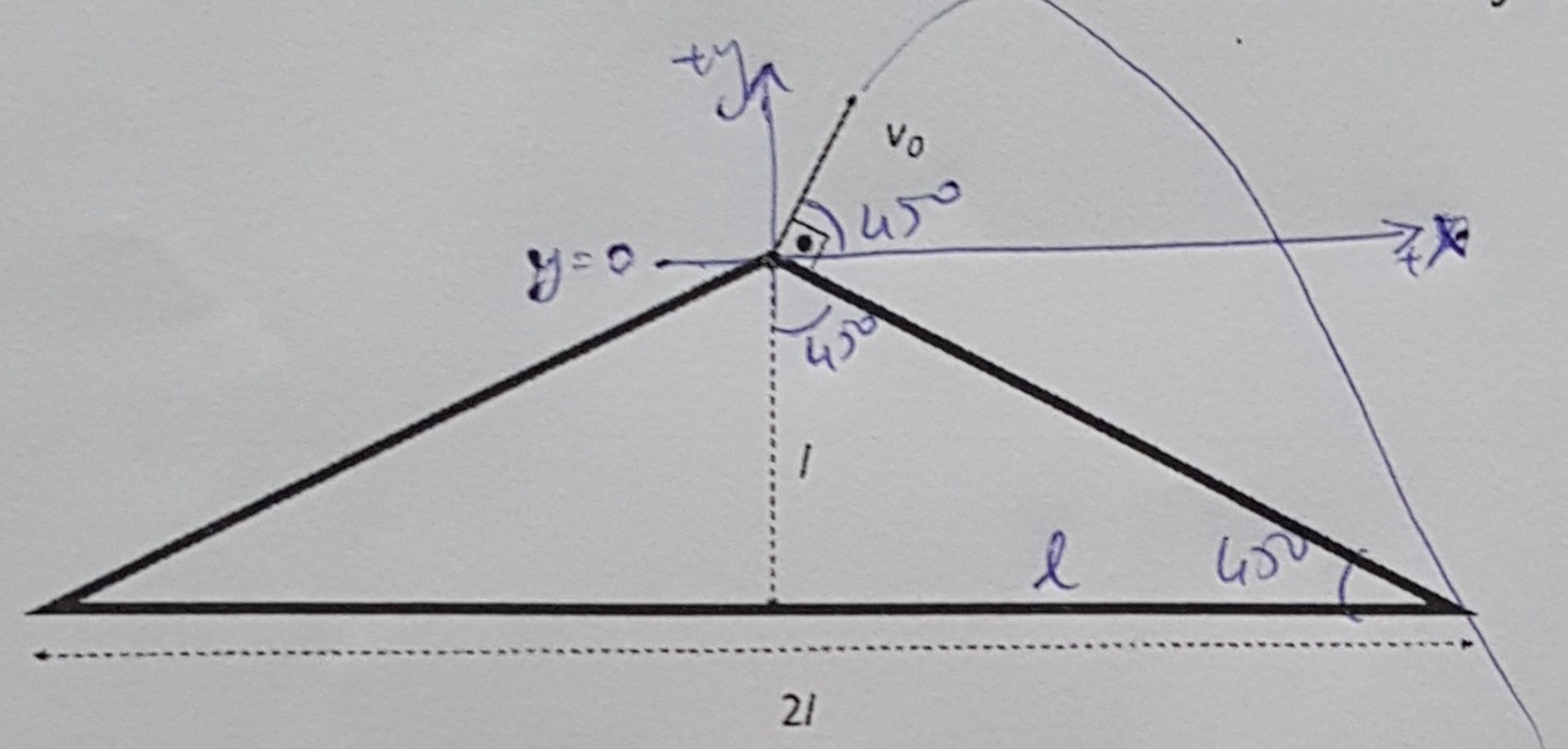
Section 2 Quiz 3 October 21, 2016

Closed book. Duration: 10 minutes

Name: **Student ID:** **Signature:**

A cannon is placed near the top of a hill. The hill has a conical shape of height *l* and base diameter 2*l*, and the cannon is buried such that it fires in a direction perpendicular to the inclined side of the hill. If we want to hit a point exactly at the bottom of the hill, what should be the muzzle velocity of the cannonball?



$$\begin{cases} y = v_{0y} t - \frac{1}{2} g t^2 \\ x = v_{0x} t \end{cases} \qquad v_{0x} = v_{0y} = \frac{\sqrt{2}}{2} v_0$$

$$\begin{cases} -l = \frac{\sqrt{2}}{2} v_0 t - \frac{1}{2} g t^2 \\ l = \frac{\sqrt{2}}{2} v_0 t \end{cases} \rightarrow t = \sqrt{2}\, l / v_0$$

insert t in upper equation

$$-l = \frac{\sqrt{2}}{2} v_0 \left(\frac{\sqrt{2}\, l}{v_0}\right) - \frac{1}{2} g \left(\frac{\sqrt{2}\, l}{v_0}\right)^2$$

$$\Rightarrow 2l = \frac{1}{2} g \frac{2 l^2}{v_0^2}$$

$$v_0^2 = \frac{g l}{2}$$

$$\boxed{v_0 = \sqrt{\frac{g l}{2}}}$$

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Section 3 Quiz 3 October 21, 2016

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Name: **Student ID:** **Signature:**

A particle follows a two dimensional trajectory given by $y=x-Ax^2$ starting from the origin at $t=0$, where A is a positive constant. The horizontal component of the velocity is constant, $v_x=v$. First find $x(t)$, and then using the trajectory, find $y(t)$. Using these, find the velocity and acceleration vectors as functions of time.

$v_x(t) = \frac{dx(t)}{dt} \rightarrow x(t) = v_x t + x_0$

at $t=0$ $x=0$, $y=0$ $\Rightarrow x_0 = 0$

$x(t) = vt$

$y(t) = x - Ax^2$

$y(t) = vt - Av^2t^2$

$\vec{r} = x\hat{i} + y\hat{j}$

$\vec{r} = (vt)\hat{i} + (vt - Av^2t^2)\hat{j}$

$\vec{v} = \frac{d\vec{r}}{dt}$

$\vec{v}(t) = v\hat{i} + (v - 2Av^2t)\hat{j}$

$\vec{a} = \frac{d\vec{v}}{dt}$

$\vec{a}(t) = (-2Av^2)\hat{j}$

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Section 4 **Quiz 3** **October 21, 2016**

Closed book. Duration: 10 minutes

Name: **Student ID:** **Signature:**

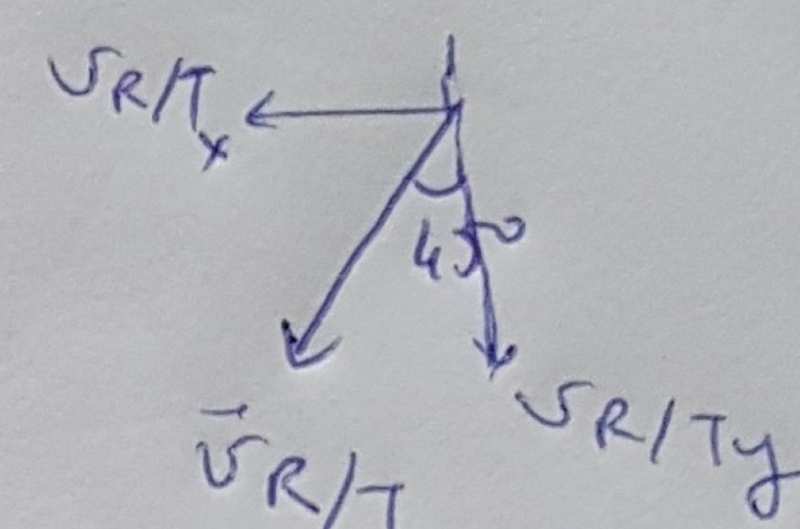
A passenger on a train travelling to the east observes that the raindrops are falling at an angle of *45º* with the vertical towards west. On his journey back, he observes this angle to be *30º*, again towards west. If the speed of the train is *20m/s* in both directions, what is the velocity of the rain for a person on the ground?

$\vec{v}_{R/G}$: raindrop relative to ground

$\vec{v}_{R/T}$: raindrop relative to train

$\vec{v}_{T/G}$: train relative to ground.

$\longrightarrow \vec{v}_{T/G} = 20\,m/s$



$$\vec{v}_{R/G} = \vec{v}_{R/T} + \vec{v}_{T/G}$$

$$v_x = v_{R/T_x} + v_{T/G_x}$$

$$v_y = v_{R/T_y} + \underset{0}{v_{T/G_y}}$$

travelling to the east :

$$v_x = v_{R/T}\sin 45° - 20$$
$$v_y = v_{R/T}\cos 45°$$
$$> \frac{v_x + 20}{v_y} = v_{R/T}\underbrace{\tan 45°}_{1}$$

travelling to the west :

$$v_x = v_{R/T}\sin 30° + 20$$
$$v_y = v_{R/T}\cos 30°$$
$$> \frac{v_x - 20}{v_y} = v_{R/T}\underbrace{\tan 30°}_{\frac{1}{\sqrt{3}}}$$

$$\Rightarrow \begin{cases} v_x + 20 = v_y \\ v_x - 20 = v_y/\sqrt{3} \end{cases}$$

$$\Rightarrow v_x(1-\sqrt{3}) = -20(1+\sqrt{3})$$

$$v_x = -\frac{20(1+\sqrt{3})}{(1-\sqrt{3})}\frac{(1+\sqrt{3})}{(1+\sqrt{3})} = -\frac{20(4+2\sqrt{3})}{-2}$$

$$\boxed{v_x = 20(2+\sqrt{3})} \quad \boxed{v_y = 20(3+\sqrt{3})}$$