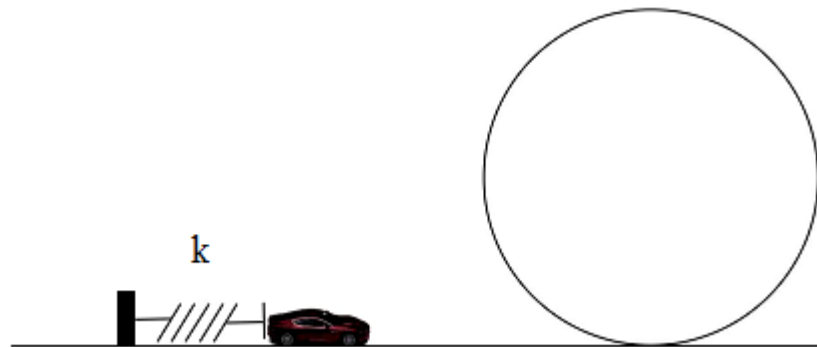


Closed book. Duration: 20 minutes**Name:****Student ID:****Signature:**

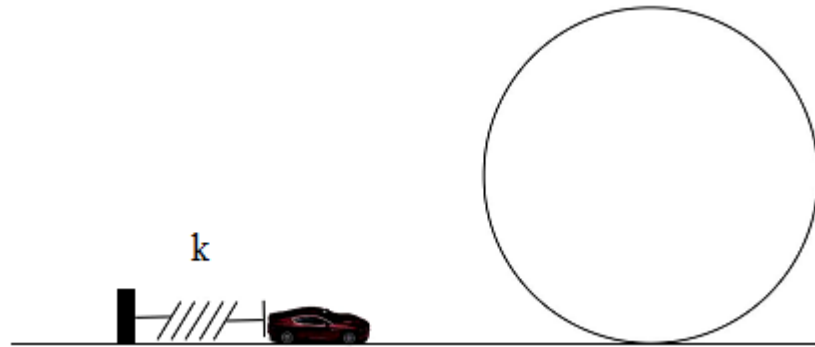
You are trying to launch a 100g slot car around the inside of a 1m diameter loop-the-loop using a spring with a natural (unstressed) length of 20 cm and a spring constant of 100 N/m. Assume the slot car is frictionless. Take the gravitational acceleration as $g = m/s^2$. What is the minimum amount you need to compress the spring in order for the car to make it around the loop without falling off?



- You will lose credit if you don't show all relevant coordinate systems.
- You will lose credit if you don't draw all relevant free-body diagrams.
- You will lose credit if you use a conservation theorem without any explanation.
- You will lose credit for missing units.

Closed book. Duration: 20 minutes**Name:****Student ID:****Signature:**

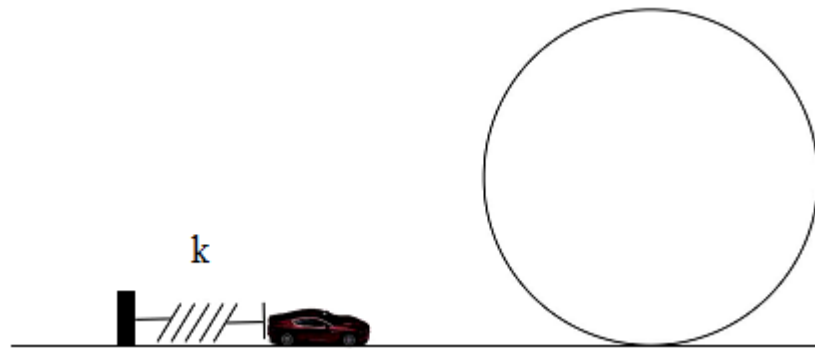
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- You will lose credit if you don't show all relevant coordinate systems.
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Closed book. Duration: 20 minutes**Name:****Student ID:****Signature:**

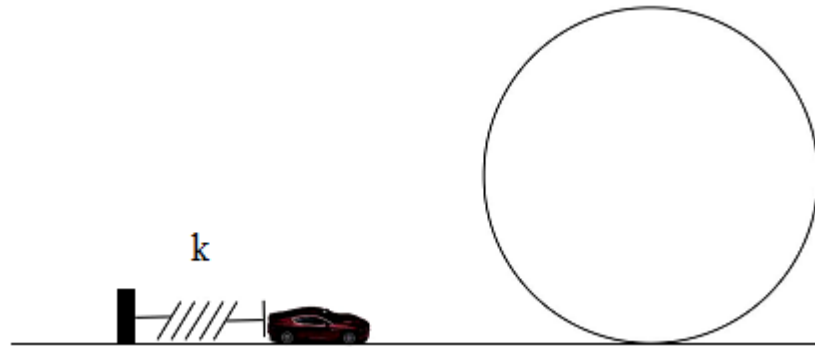
You are trying to launch a 100g slot car around the inside of a 1m diameter loop-the-loop using a spring with a natural (unstressed) length of 20 cm and a spring constant of 100 N/m. Assume the slot car is frictionless. Take the gravitational acceleration as $g = 10 \text{ m/s}^2$. If you compress the spring all the way to the wall, what normal force will the track exert on the car at the top of the loop?



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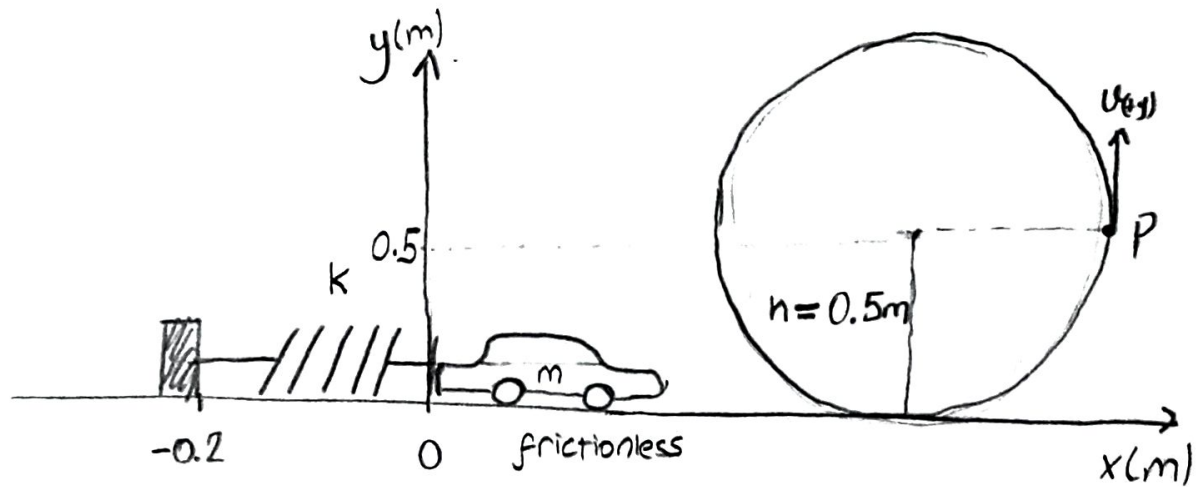
Closed book. Duration: 20 minutes**Name:****Student ID:****Signature:**

You are trying to launch a 100g slot car around the inside of a 1m diameter loop-the-loop using a spring with a natural (unstressed) length of 20 cm and a spring constant of 100 N/m. Assume the slot car is frictionless. Take the gravitational acceleration as $g = m/s^2$. If you compress the spring all the way to the wall, what is the acceleration of the car when its velocity is purely in the upward (+y) direction?



- You will lose credit if you don't show all relevant coordinate systems.
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QUIZ 8&9 SOLUTIONS



- (a) The car's velocity is purely upward when it reaches the point P . The elastic potential energy stored in spring will be converted into kinetic and potential energy with the total energy being conserved.

Therefore we write ($K_1 = 0$, $U_1 = \frac{1}{2} k x^2$, $K_2 = \frac{1}{2} m v_{(y)}^2$, $U_2 = mgh$),

$$\frac{1}{2} k x^2 = mgh + \frac{1}{2} m v_{(y)}^2$$

$$\Rightarrow v_{(y)} = \sqrt{\frac{k}{m} x^2 - 2gh}$$

$$\Rightarrow v_{(y)} = \sqrt{\frac{(100 \text{ N/m}) (0.2 \text{ m})^2}{(0.1 \text{ kg})} - 2 \left(\frac{\text{m}}{\text{s}^2} \right) (0.5 \text{ m})}$$

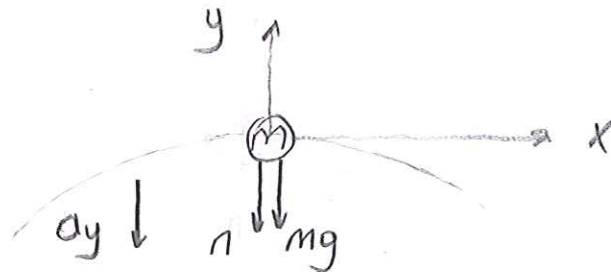
$$= 6.25 \text{ m/s} \quad \text{where } \text{N} : \text{kg} \frac{\text{m}}{\text{s}^2}.$$

The radial acceleration is thus,

$$a_{\text{rad}} = \frac{v_{(ty)}^2}{R} = \frac{(6.25 \text{ m/s})^2}{0.5 \text{ m}} = 78.1 \text{ m/s}^2$$

The tangential acceleration is the same as $g = 9.8 \text{ m/s}^2$, as there is only F_g along y direction.

(b) The minimum compression to make it around the loop corresponds to the minimum speed at top



$$\Sigma \vec{F} = m\vec{a}$$

$$\Rightarrow \underbrace{-n - mg}_{\downarrow} = -m \frac{u_m^2}{R}$$

Normal force should be zero for minimum speed

$$\Rightarrow u_m = \sqrt{gR}$$

Conservation of the mechanical energy gives

$$\frac{1}{2} k x_{\min}^2 = mg2R + \frac{1}{2} m \frac{u_m^2}{gR}$$

$$\Rightarrow x_{\min} = \sqrt{\frac{4gmR}{k} + \frac{gmR}{k}} = \sqrt{\frac{5gmR}{k}}$$

$$= \sqrt{\frac{5 \left(\frac{m}{s^2}\right) (0.1 \text{ kg}) (0.5 \text{ m})}{100 \text{ N/m}}} = 0.05 \text{ m}$$

$$\begin{aligned}
 (c) \quad \frac{1}{2} k x^2 &= \frac{1}{2} m v_b^2 \Rightarrow v_b = \sqrt{\frac{k}{m}} x \\
 &= \sqrt{\frac{100 \text{ N/m}}{0.1 \text{ kg}}} (0.2 \text{ m}) \\
 &= 6.32 \text{ m/s}
 \end{aligned}$$

at the bottom of the loop

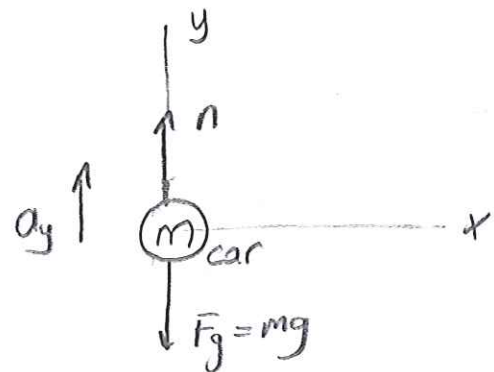
$$\sum \vec{F} = m \vec{a}$$

$$n - mg = m \frac{v_p^2}{R}$$

$$\Rightarrow n = m \left(g + \frac{v_p^2}{R} \right)$$

$$= (0.1 \text{ kg}) \left[\frac{m}{s^2} + \frac{(6.32 \text{ m/s})^2}{(0.5 \text{ m})} \right]$$

$$\cong 8.1 \text{ kg} \cdot \frac{\text{m}}{\text{s}^2} = 8.1 \text{ N}$$



$$(d) \quad \frac{1}{2} kx^2 = mg \frac{h}{2R} + \frac{1}{2} m u_t^2$$

$$\Rightarrow u_p = \sqrt{\frac{kx^2}{m} - 4gR} = \sqrt{\frac{100 \text{ N/m} (0.2 \text{ m})^2}{0.1 \text{ kg}} - (9.8 \text{ m/s}^2)(2 \text{ m})}$$

$$= 6.16 \text{ m/s}$$

at the top of the loop

$$\Sigma \vec{F} = m\vec{a}$$

$$\Rightarrow n + mg = m \frac{u_p^2}{R}$$

$$\Rightarrow n = m \left(\frac{u_p^2}{R} - g \right)$$

$$= (0.1 \text{ kg}) \left[\frac{(6.16 \text{ m/s})^2}{(0.5 \text{ m})} - \frac{\text{m}}{\text{s}^2} \right]$$

$$= 7.50 \text{ kg} \frac{\text{m}}{\text{s}^2} = 7.50 \text{ N}$$

