Fall 2017

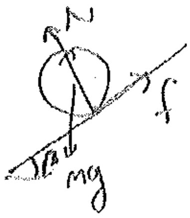
Phys 101 Final

Exam Room:	**P101_Index:**
Student ID Number:	**Signature:**

Q1-(25 pts) A spherical bowling ball with mass M and radius R rolls down a ramp that is inclined at an angle β to the horizontal. (For the bowling ball $I_{cm} = \frac{2}{5}MR^2$).

a) Assume β is small and the ball rolls without slipping. What are the ball's accelaration and the magnitude of the friction force on the ball?

$a = \alpha R$



$Mg\sin\beta - f = Ma$

$fR = \frac{2}{5}MR^2\frac{a}{R}$

$\Rightarrow Mg\sin\beta - \frac{2}{5}Ma = Ma$

$\Rightarrow \boxed{a = \frac{5}{7}g\sin\beta}$

$f = \frac{2}{5}Ma = \boxed{\frac{2}{7}Mg\sin\beta}$

b) Assume μ_s is the coefficient of static friction between the ramp and the ball. Find out an expression for the maximum angle β for which the ball will roll without slipping.

$f \leq N\mu_s \qquad N = mg\cos\beta$

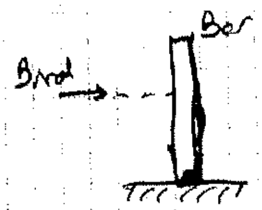
$\Rightarrow \frac{2}{7}Mg\sin\beta \leq mg\cos\beta\,\mu$

$\Rightarrow \tan\beta \leq \frac{7}{2}\mu$

$\boxed{\beta \leq \tan^{-1}\left(\frac{7}{2}\mu\right)}$

Yes ☐ No ☐

Fall 2017, Final Exam, Question 2:



a) Conservation of angular momentum during the collision,

$$L_{before} = m_{bird}\, v_{bird}\,(0.53\,m)$$

$$L_{after} = I_{rod}\,\omega \quad , \quad I_{rod} = m_{rod}\frac{L^2}{3}$$

$$\Rightarrow \omega = \frac{0.46 \times 2.3 \times 0.53}{2.5 \times \frac{(0.78)^2}{3}} = \boxed{1.106\ \text{rad/s}}$$

b) $K_1 + U_1 = K_2 + U_2$

$$\frac{1}{2} I_{rod}\,\omega^2 + m_{rod}\, g \times \frac{L}{2} = \frac{1}{2} I_{rod}\,\omega_f^2 + 0$$

$$\Rightarrow \omega_f^2 = \omega^2 + \frac{\cancel{m_{rod}}\, g\, L/2}{\frac{1}{2}\frac{\cancel{m_{rod}}\, L^2}{3}} = \omega^2 + \frac{6}{2}\frac{g}{L} = \omega^2 + 3\frac{g}{L}$$

$$\Rightarrow \omega_f = \sqrt{(1.106)^2 + 3 \times \frac{9.8}{0.78}} = \boxed{6.238\ \text{rad/s}}$$

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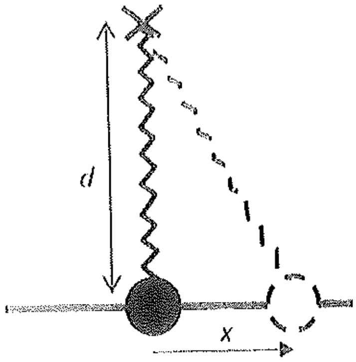
Q3-(25 pts) A small bead of mass m can move on a fixed horizontal wire without friction as in the figure. The bead is connected to an ideal spring of spring constant k, and the other end of the spring is connected to a fixed point at a perpendicular distance d from the wire. Unstretched length of the spring is very small, and can be taken to be zero.



a) What is the period of oscillations of the bead around its equilibrium position?

$F_x = -k\sqrt{d^2+x^2}\,\sin\theta$

$= -k\sqrt{d^2+x^2}\,\frac{x}{\sqrt{d^2+x^2}}$

$\Rightarrow m\frac{d^2x}{dt^2} = -kx$

$\Rightarrow \omega^2 = \frac{k}{m} \Rightarrow \boxed{T = 2\pi\sqrt{\frac{m}{k}}}$

b) What is the amplitude of the simple harmonic motion if we give the bead an initial velocity of v_0 at its equilibrium position?

$\frac{1}{2}kd^2 + \frac{1}{2}mv_0^2 = \frac{1}{2}k\left(\sqrt{d^2+A^2}\right)^2$

$\Rightarrow \frac{1}{2}mv_0^2 = \frac{1}{2}kA^2 \Rightarrow \boxed{A = v_0\sqrt{\frac{m}{k}}}$

$\boxed{= \frac{v_0}{\omega}}$

c) Find the position of the bead as a function of time, $x(t)$, for the case in part b.

$x(t) = A\cos(\omega t + \varphi)$

$v(t) = -\omega A\sin(\omega t + \varphi)$

At $t = 0 \Rightarrow \left.\begin{array}{l} A\cos\varphi = 0 \\ -\omega A\sin\varphi = v_0 \end{array}\right\} \Rightarrow \varphi = -\pi/2$

$\Rightarrow x(t) = A\cos(\omega t - \frac{\pi}{2}) \Rightarrow \boxed{x(t) = \sqrt{\frac{m}{k}}\,v_0\,\sin\left(\sqrt{\frac{k}{m}}\,t\right)}$

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Q4-(25 pts) In this problem, you will explore what would happen if the attractive gravitational force was $F = \frac{Hm_1m_2}{r^3}$, H being a given constant, r the distance between objects, m_1 and m_2 masses of the objects (i.e. we have the third power of the distance instead of the second power). Consider a planet of mass M and radius R, and assume that the force is the same for spherical bodies and point particles.

a) What is the orbital period of a small satellite on a circular orbit of radius $2R$?

$$a_r = \frac{v^2}{2R}$$

$$\Rightarrow m\frac{v^2}{2R} = \frac{HMm}{(2R)^3}$$

$$\Rightarrow v^2 = \frac{HM}{4R^2} \Rightarrow v = \sqrt{\frac{HM}{4R^2}}$$

b) What is the gravitational potential energy of the satellite in part a?

$$U(r_2)-U(r_1) = \int_{r_1}^{r_2} F\,dr = \int_{r_1}^{r_2} \frac{HMm}{r^3}\,dr = \frac{HMm}{2r_1^2} - \frac{HMm}{2r_2^2}$$

$$\Rightarrow U(r) = -\frac{HMm}{2r^2}$$

$$U(2R) = -\frac{HMm}{2(2R)^2} = \boxed{-\frac{HMm}{8R^2}}$$

c) What is the minimum initial velocity needed to escape the surface of this planet and never return? This is called the escape velocity.

To escape infinity, $E_{tot} \geq 0$

$$\frac{1}{2}mv_{esc}^2 - \frac{HMm}{2R^2} = 0$$

$$\Rightarrow \boxed{v_{esc} = \sqrt{\frac{HM}{R^2}}}$$

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