Name:	Signature:
Surname:	Number:

KOÇ UNIVERSITY

College of Sciences

PHYS 101 General Physics 1

Fall Semester 2018

Final Exam

December 30, 2018 Sunday, 08:30-10:10

Solution

Please read.

- Count to make sure that there are 5 pages in this question booklet
- Check your **name, number, on front page, and student ID on each page.**.
- This examination is conducted with closed books and notes.
- Put all your personal belongings underneath your seat and make sure that pages of books or notebooks are not open.
- Absolutely no talking or exchanging anything (like rulers, erasers) during the exam.
- You must show all your work to get credit; you will not be given any points unless you show the details of your work (this applies even if your final answer is correct).
- Write neatly and clearly; unreadable answers will not be given any credit.
- If you need more writing space, use the backs of the question pages and put down the appropriate pointer marks.
- Make sure that you include units in your results.
- Make sure that you label the axis and have units in your plots.
- You are not allowed to use calculators during this exam.
- Turn off your mobile phones, and put away.
- You are not allowed to leave the class during the first 15 minutes, and last 15 minutes.
- Write your final answers into the boxes. No points will be given to unjustified answers. Incomplete calculations will not be graded.

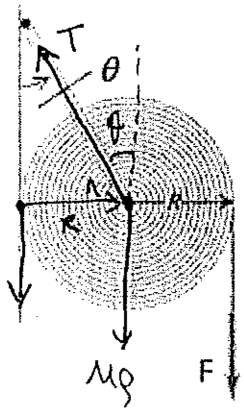
P101_Index:

1	2	3	4	Total

F18 _ Final

Exam Room:	**P101_Index:**
Student ID Number:	**Signature:**

Q1-(25 pts) A large roll of paper with mass M and radius R rests against the wall and is held in place by a bracket attached to a rod through the center of the roll. The rod turns without friction in the bracket, and the moment of inertia of the paper and rod about the axis is I. The other end of the bracket is attached by a frictionless hinge to the wall such that the bracket makes and angle of θ with the wall. The weight of the bracket is negligible. The coefficient of kinetic friction between the paper and the wall is 0.5. A constant vertical force F is applied to the paper, and the paper unrolls.



a) What is the magnitude of the force that the rod exerts on the paper as it unrolls in terms of M, R, I, F and θ?

$$\boxed{T = \frac{F + Mg}{\cos\theta - \mu_k \sin\theta}}$$

$n = T\sin\theta$

$a_{pulley} = 0 \Rightarrow T\cos\theta - \mu_k n - F - Mg = 0$

$$\Rightarrow T = \frac{F + Mg}{\cos\theta - \mu_k \sin\theta}$$

b) What is the angular acceleration of the roll in terms of M, R, I, F and θ?

$$\boxed{\alpha = \frac{(F - \mu_k n)R}{I}}$$

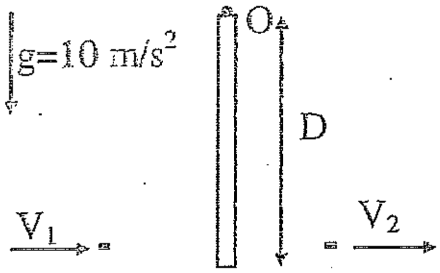
$\sum \tau_z = I\alpha$

$$\Rightarrow \frac{(F - \mu_k n)R}{I} = \alpha$$

Yes □ No □

Exam Room: [redacted]	P101_Index: [redacted]
Student ID Number: [redacted]	Signature:

Q2-(25 pts) A uniform rod of mass $M = 3\ kg$ and length D is pivoted at its edge so that it can rotate in a vertical circle. A bullet of mass $m = 100g$ and $V1 = 40\ m/s$ strikes the rod at its unpivoted edge and leaves it with a final velocity of $V2 = 10\ m/s$.



How high does the center of mass of rod rises? You may use $I = MD^2/12$ for the rod about its center of mass.

+ $I_0 = I_{cm} + Md^2$

$$= \frac{1}{12} M D^2 + M\left(\frac{D}{2}\right)^2 = \frac{1}{3} M D^2 \quad (+5)$$

+ Collision $\rightarrow$ $L_i = L_f$ (+5)

Not: $P_i \neq P_f$ only angular momentum is conserved.

$$m V_1 D = I_0 \omega + m V_2 D$$

$$\omega = \frac{m D (V_1 - V_2)}{I_0} = \frac{3}{D} \quad (+5)$$

+ After collision $E_{i,rod} = E_{f,rod}$. (+5)

$$\frac{1}{2} I_0 \omega^2 = M g h$$

- Not: Esystem is not conserved!

$$h = \frac{3}{20} \quad (+5)$$

- During collision energy is not conserved

Solution continues on the back side of the page Yes □ No □

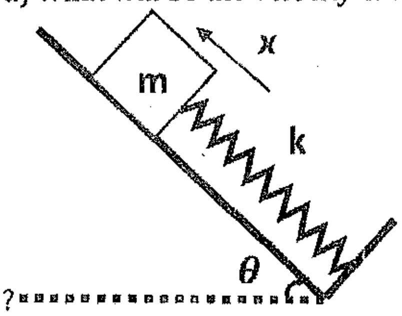
3) A block of mass m is connected to a spring of spring constant k, and can slide on an inclined plane without friction as in the figure. The position of the mass on the inclined plane is given by x, $+x$ direction is up the inclined plane, and at $x = 0$ the spring is not stretched or compressed. The mass performs simple harmonic motion around its equilibrium point.

a) What is the equilibrium point of the mass, x_{eq}? *Hint: $x = 0$ is not the equilibrium point of the mass.*

b) What is the period of oscillations of the mass around its equilibrium point x_{eq}?

c) If we leave the mass with no speed from $x = 0$ at $t = 0$, what will be its position as a function of time?

d) What will be the velocity of the mass as a function of time in part (c)?



a) $-mg\sin\theta - kx_{eq} = 0$

$\Rightarrow x_{eq} = -\dfrac{mg\sin\theta}{k}$

b) $F = -mg\sin\theta - kx = m\dfrac{d^2x}{dt^2}$

$\Rightarrow -k(x - x_{eq}) = m\dfrac{d^2x}{dt^2} = m\dfrac{d^2(x-x_{eq})}{dt^2}$

$-\dfrac{k}{m}(x - x_{eq}) = \dfrac{d^2(x-x_{eq})}{dt^2}$

So $x - x_{eq}$ does SHM with $\omega^2 = \dfrac{k}{m} \Rightarrow T = 2\pi\sqrt{\dfrac{m}{k}}$

c) $x - x_{eq}$ does SHM so

$x - x_{eq} = A\cos(\omega t + \phi) \Rightarrow x = x_{eq} + A\cos(\omega t + \varphi)$

$\dfrac{dx}{dt} = -\omega A\sin(\omega t + \varphi)$

At $t = 0$

$0 = -\dfrac{mg\sin\theta}{k} + A\cos\varphi$

$0 = -\sqrt{\dfrac{k}{m}}A\sin\varphi$

$\sin\varphi = 0 \Rightarrow \varphi = 0$

$A = \dfrac{mg\sin\theta}{k}$

$x(t) = -\dfrac{mg\sin\theta}{k} + \dfrac{mg\sin\theta}{k}\cos\left(\sqrt{\dfrac{k}{m}}t\right)$

d) $v(t) = \dfrac{dx(t)}{dt} = \dfrac{mg\sin\theta}{k}\sqrt{\dfrac{k}{m}}\sin\left(\sqrt{\dfrac{k}{m}}t\right)$

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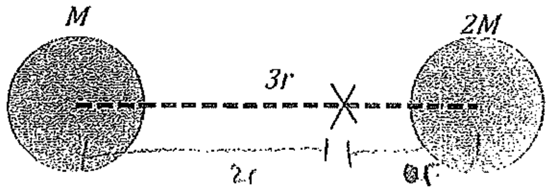
Q4-(25 pts)

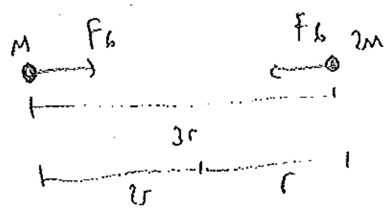
13 pts a) Two black holes with mass M and $2M$ separated by a center to center distance of $3r$ orbit around their common center of mass in circular trajectories. Find the period of their rotation.



$$\boxed{\frac{6\pi r\sqrt{r}}{\sqrt{GM}}}$$

Find Center of Mass $\frac{2M \cdot 3r + M \cdot 0}{3M} = 2r$ (3 pts)

FBD



$F_G = \sum \vec{F} = \frac{G \cdot M \cdot 2M}{(3r)^2}$ = centripetal force

For 2M $\Rightarrow$ $2M \cdot \frac{v_{2m}^2}{r} = \frac{G \cdot 2M^2}{9r^2}$ (5 pts)

For M $\Rightarrow$ $M \cdot \frac{v_m^2}{2r} = \frac{G \cdot 2M^2}{9r^2}$

$T = \frac{2\pi \cdot r}{v_{2m}} = \frac{2\pi\, 2r}{v_m} = \frac{2\pi r}{\sqrt{\frac{G \cdot M}{9r}}} = \frac{6\pi r\sqrt{r}}{\sqrt{GM}}$ (5 p

12 pts b) What is the escape speed from a solid spherical asteroid with radius R and uniform mass density d?

$$\boxed{\sqrt{\frac{8}{3}\pi R^2 G d}}$$

$\frac{1}{2} \cdot m \cdot v_{esc}^2 - \frac{G \cdot m \cdot M_{ast}}{R} = 0$ (6 pts)

$v_{esc} = \left(\frac{2 G M_{ast}}{R}\right)^{1/2}$ (3 pts) $M_{ast} = \frac{4}{3}\pi R^3 d$ (1 pts)

$v_{esc} = \sqrt{2G\frac{4}{3}\pi R^3 d \cdot \frac{1}{R}} = \sqrt{\frac{8}{3}\pi R^2 G d}$ (2 pts)

Solution continues on the back side of the page Yes ☐ No ☐