PHYS 101 General Physics 1
Fall Semester Midterm I Exam
October 24, 2019 Thursday, 19:00-20:40

Q1-(25 pts) A car starts from rest and moves with constant acceleration a=4m/s^2 until it reaches v = 24 m/s speed. It then continues to move with this constant speed without changing direction.

a) For how long does the car accelerate? (8pts)

Sol. Initial velocity of the car $v_i = 0\text{m/s}$

Final velocity $v_f = 24\text{m/s}$

Acceleration of the car a=4m/s^2 time t?

Using first equation of kinematics;

$$v_f = v_i + at$$

By rearranging the terms $\mathbf{t}=\frac{v_f - v_i}{a}$

$$\mathbf{t}=\frac{24m/s-0m/s}{4m/s^2} = \mathbf{6s}$$

b) How much distance is covered by the car during the time it accelerates? (8 pts)

Sol. Distance covered by the car in 6s?

Recalling 2nd equation of kinematics; S=v_it+1/2 at^2

S= 0m/s 6s+ 1/2 × $4m/s^2 \times 6^2$

S=72m

c) After reaching the speed of v = 24 m/s, how much longer should the car travel until its average speed becomes $\bar{v}$ = 20 m/s? (9pts)

Ans; $v_{ave}=\frac{final\ distance - initial\ distance}{final\ time - initial\ time}$ (1)

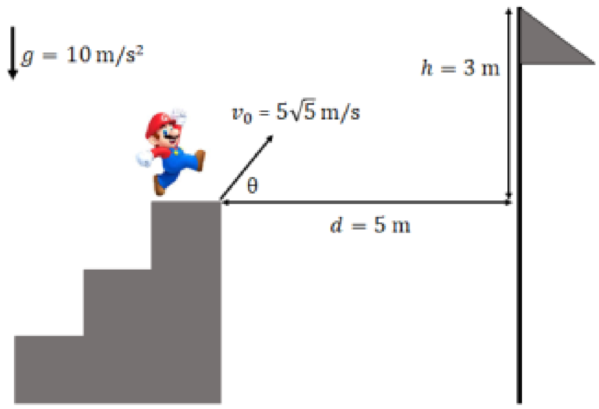
$$v_{ave} = 20m/s$$

Initial distance travelled by the car at initial time ($t_i = 0m/s$) is zero i.e., s_i=0m

Final distance travelled by the car after t_f=(6+t) is s_f = (72+24×t)

Substituting above values in (1) 20m/s=$\frac{(72+24\times t)m-0}{(6+t)s-0}$ =>4t=48s => t=12s (required time)

Q2-(25 pts) Super Mario wants to jump over the pole to reach the princess in the castle. He is to be thrown with speed $v_0 = 5\sqrt{5}$ m/s towards the pole with an angle θ(see the figure). The pole has a height of h = 3 m from the level Mario jumps, and it is away at a distance of d = 5 m. Find the tangent of minimum and maximum angles of throw (find tanθ for the angle θ shown in the figure) so that Mario can pass over the pole. (The gravitational acceleration, g = 10 m/s2, if needed: sec2θ = 1 + tan2θ)



Sol. The kinematic equations of motion for distance under the effect of gravity can be represented as

x(t)= v_{ox}t with x(t)=5 m (a) g=0 along x-axis

Vertical distance y(t)= v_{oy}t-1/2 gt^2 with y(t)=3m (b) a= -g

$v_{ox} = 5\sqrt{5}\cos\theta$; $v_{oy} = 5\sqrt{5}\sin\theta$

Solving a for t

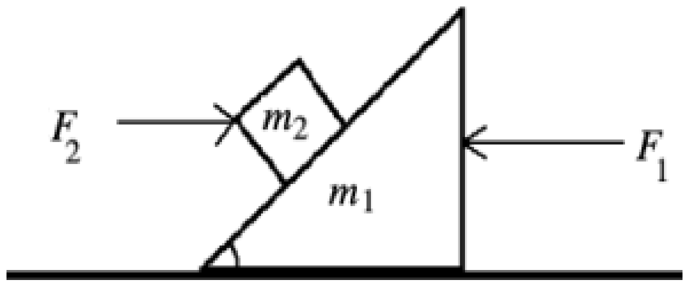
5= $5\sqrt{5}\cos\theta$ t => t=$\frac{\mathbf{1}}{\sqrt{5}\,\mathbf{cos}\,\boldsymbol{\theta}}$ Solving b 3=$\frac{5\sqrt{5}\sin\theta}{\sqrt{5}\cos\theta}$ – ½ ×10 $(\frac{\mathbf{1}}{\sqrt{5}\,\boldsymbol{cos}\,\boldsymbol{\theta}})^{\mathbf{2}}$

3=5tan θ-5(1/5$sec^2\theta$) => 3-5 tan θ-$sec^2\theta$ => $tan^2\theta - 5tan\theta$+4=0 (we used 1+ $tan^2\theta$=$sec^2\theta$) $tan\theta\,(tan\theta - 1) - 4(tan\theta - 1) = 0$ Factorized

$tan\theta = 1, and\ tan\theta = 4$ Therefore, 1≤ $tan\theta \leq 4$

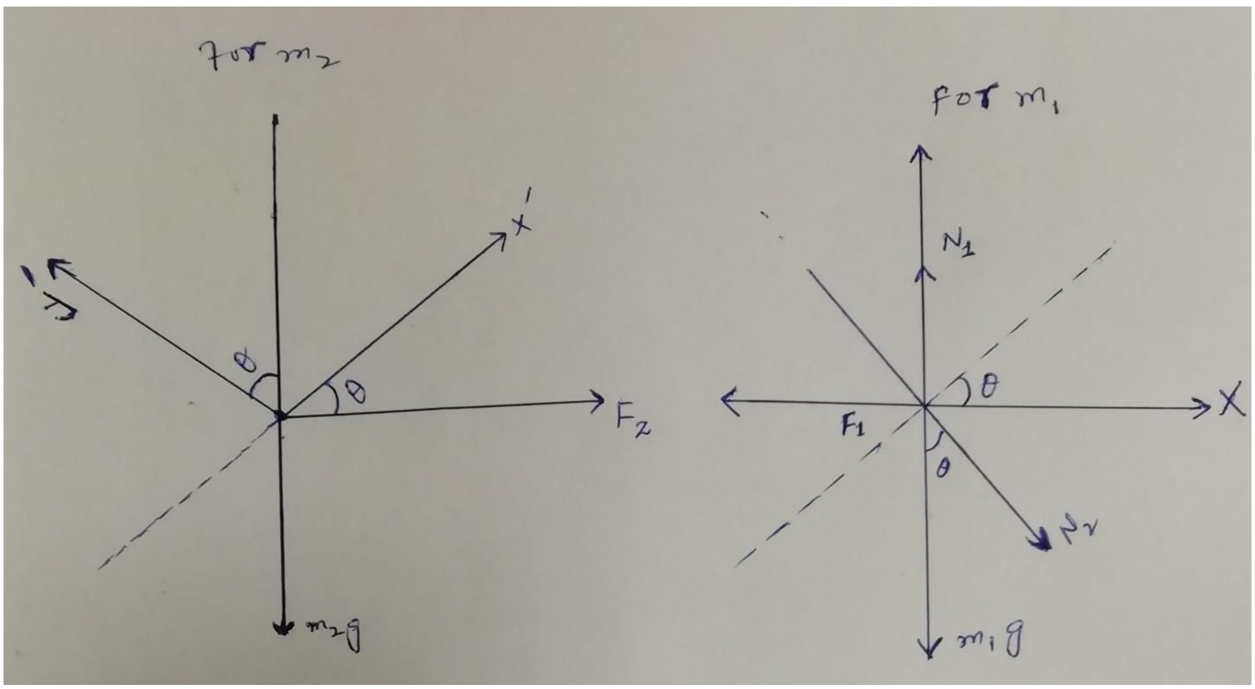
Q3-(25 pts) Consider the incline with mass $m1$ and the box with mass m_2 that are shown in the figure. A horizontal forceF_1 is applied to m_1 from right and an another horizontal force F_2 is applied to m_2 from left. Assuming there is no friction between m_1 and m_2 nor m_1 and the ground, we want to find the accelerations $\vec{a}_1$ and $\vec{a}_2$ of these masses with respect to the ground. Take the gravitational acceleration as g and the angle of the incline as θ.

Hint: The acceleration of m_2 with respect to the ground can be written as $\vec{a}_2 = \vec{a}_{21} + \vec{a}_1$ where $\vec{a}_{21}$ is the acceleration of m_2 with respect to m_1.



a) Draw the free body diagrams for masses. (10pts)

Sol. The free diagram for $masses\ m_1$ and m_2 are sketched as follows;



b) Specify a coordinate system and apply Newton's laws to your diagrams. (9pts)

Sol. $\vec{a}_2 = \vec{a}_{21}\uparrow + \vec{a}_1$ $\quad\quad$ $\vec{a}_1 = \vec{a}_1\uparrow$ $\quad$ with respect to ground

$\vec{a}_2 = (\vec{a}_{21}\cos\theta + \vec{a}_1)\uparrow + \vec{a}_{21}\sin\theta$ g

For m_2; $\quad \Sigma\vec{F} = m_2\vec{a}_2 \quad \vec{F}_2 - \vec{N}_2\sin\theta = m_2(\vec{a}_{21}\cos\theta + \vec{a}_1)$ $\quad$ (1)

For m_1; $\quad \Sigma\vec{F} = m_1\vec{a}_1 \quad \vec{N}_1 = m_1 g + \vec{N}_2\cos\theta$ $\quad$ (2)

$\vec{N}_2\cos\theta - m_2 g = m_2\vec{a}_{21}\sin\theta$ $\quad$ (3) $\quad\quad$ $\vec{N}_2\sin\theta - \vec{F}_1 = m_1\vec{a}_1$ $\quad$ (4)

c) Sol. $\vec{F}_2\cos\theta - m_2 g\sin\theta = m_2(\vec{a}_{21}\cos\theta + \vec{a}_1)$

$\Rightarrow \cos\theta + m_2\vec{a}_{21}\sin^2\theta = m_2(\vec{a}_{21} + \vec{a}_1\cos\theta)$

$\vec{F}_2 - \vec{F}_1 = m_2(\vec{a}_1 + \vec{a}_{21}\cos\theta)\ + m_1\vec{a}_1 \Rightarrow\ \vec{a}_{21}\cos\theta + (m_1 + m_2)\vec{a}_1$

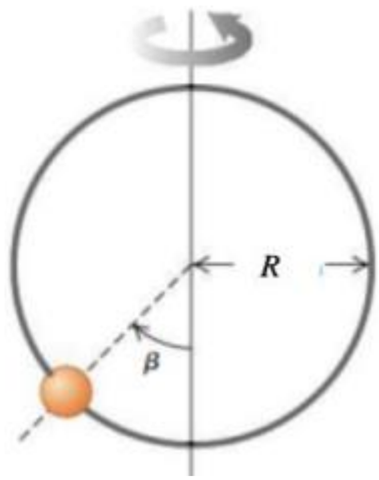
$\Rightarrow \vec{F}_2\cos^2\theta - m_2 g\sin\theta\cos\theta - \vec{F}_2 + \vec{F}_1\ \Rightarrow m_2\vec{a}_1\cos^2\theta - +(m_1 + m_2)\vec{a}_1$

Solving for $\vec{a}_1$, and $\vec{a}_{21}$, we get

$$\vec{a}_1 = \frac{\vec{F}_1 - \vec{F}_2\sin^2\theta - m_2 g\sin\theta\cos\theta}{-(m_2\sin^2\theta + m_1)},$$

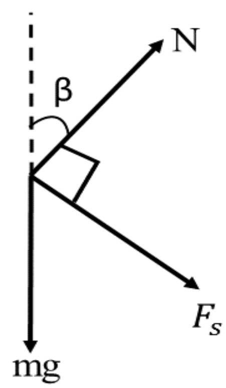
$$\vec{a}_{21} = \frac{\vec{F}_2 - \vec{F}_1}{m_2\cos\theta} - \frac{m_1 + m_2}{m_2\cos\theta}\vec{a}_1.$$

Q4-(25 pts) A small bead with mass m can slide with friction on a circular hoop that is in vertical plane and has a radius R. The hoop rotates at a constant angular frequency ω, i.e., it is a uniform circular motion. There is a range of $\omega_{min} < \omega < \omega_{max}$ for which the bead can stay in vertical equilibrium with angle β. Take the gravitational acceleration as g and the coefficient of static friction to be μ_s. We want to find ω_{max}.



a) Draw the free body diagram for the bead. (10pts)

Sol. The free body diagram for bead of mass m is given below



b) Specify a coordinate system and apply Newton's laws to your diagrams. (8pts)

Sol. For x-axis; $N\sin\beta + F_s\cos\beta = m\omega^2 R\sin\beta$

For y-aixs; $N\cos\beta - F_s\sin\beta - mg = 0$ Newton 3rd law

c) Extract ω_{max}. (7pts)

Sol. $N - mg\cos\beta = m\omega^2 R\sin\beta \Rightarrow N = mg\cos\beta + m\omega^2 R\sin\beta$

$F_s + mg\sin\beta = m\omega^2 R\sin\beta\cos\beta$

$\Rightarrow F_s = -mg\sin\beta + m\omega^2 R\sin\beta\cos\beta$

$since, F_s \leq \mu_s N$

$\Rightarrow m\omega^2{}_{max} R\sin\beta\cos\beta - mg\sin\beta \leq mg\mu_s\cos\beta + m\omega^2{}_{max} R\mu_s sin^2\beta$

$\Rightarrow m\omega^2{}_{max} R\sin\beta(\cos\beta - \mu_s sin\beta) \leq mg(\mu_s\cos\beta + \sin\beta)$

$$\Rightarrow \omega^2{}_{max} \leq \sqrt{\frac{g(\mu_s\cos\beta + \sin\beta)}{R\sin\beta(\cos\beta - \mu_s sin\beta)}}$$