# PHYS 101 General Physics 1
# Fall Semester 2019
# Midterm 2 Exam
# November 20, 2019 Wednesday, 19:00-20:40

**Q1-(25 pts)** (The questions are not related to each other.)

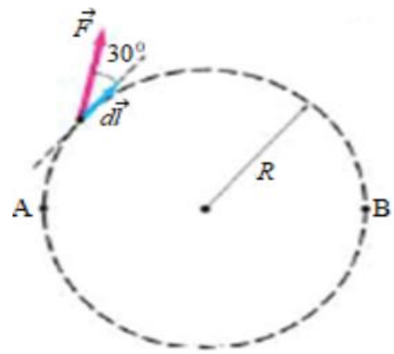
**i)** The force needed to hold a particular spring compressed an amount $x$ from its normal length is given by $F = kx + ax^3 + bx^4$. How much work must be done to stretch it by $d$, starting from $x = 0$? (10 pts)

**Sol.** By using the definition of work

W=$\int_0^d F.dx$=> W= $\int_0^d (\text{kx} + \text{a}x^3 + \text{b}x^4)dx$ integrating by sum rule

W= $kd^2/2 + ad^4/4 + bd^5/5$

**ii)** An object moving along the circumference of a circle with radius $R$, is acted upon by a force of constant magnitude $F$. The force is directed at all times at a 30 degree angle with respect to the tangent to the circle as shown in the figure. Determine the work done by this force when the object moves along the half circle from A to B? (8 pts)




**Sol**. W=$\int_A^B F.cos30.dx$

W= $\int_0^\pi \frac{\sqrt{3}}{2} \vec{F}.Rdl$ where dx= $Rdl$ and cos30=$\sqrt{3}$/2

W= $\sqrt{3}$/2 π $\vec{F}$ R

**iii)** How much work must be done to stop a 1000 kg car travelling at 72 km/hr? (7 pts)

By definition of work energy relation

Given data, mass of the car m=1000kg

Speed of the car final velocity of the car $V_f$= 72km/hr => $\frac{72\times1000m}{3600s}$=20m/s (we convert speed into standard unit)

Note that initially the car is in rest; therefore, initial speed of the car is zero; $V_i = 0$

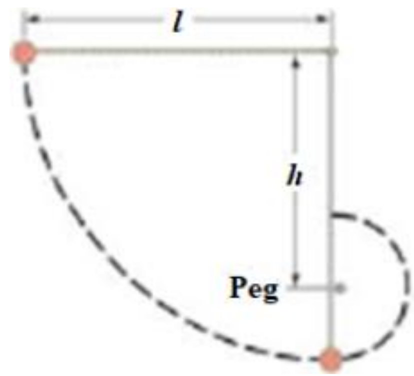
Using work energy theorem;

W=ΔK.E=> W=$1/2\text{m}V_f^{\,2}$- $1/2\text{m}V_i^{\,2}$

W= ½ ×1000kg $(20m/s)^2$-1/2× 1000kg×0

W=500kg.400$(m/s)^2$= 2,00000J= 2×$10^5$J

**Q2-(25 pts)** A ball is attached to a horizontal cord of length $l$ whose other end is fixed (see the figure).




**a)** If the ball is released, what will be its speed at the lowest point of its path? (8 pts)

**Sol.** Recalling the conservation law of energy

K.E= P.E

$1/2\text{m}v^2$=$mgl$ => v=$\sqrt{2gl}$

**b)** A peg (or a nail) is located at a distance $h$ directly below the point of attachment of the cord. If $h$ = 0.80 $l$, what will be the speed of the ball when it reaches the top of its circular path? (8 pts)

Sol. ½ m$v^2 = mgl - 0.4mgl$=> ½ m$v^2 = 0.6mgl$ note= mgΔh=mg($h_2$-$h_1$)

v=$\sqrt{1.2gl}$

**c)** What should be the minimum value of $h$ if the ball is to make a complete circle about the peg? (9 pts)

Sol. Change in P. E= Change in K. E

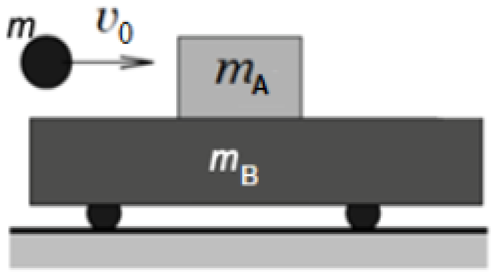
$mgl - 2mg(l-h)$=½ m$v^2 - 0$

$3mgh - 2mgl = \frac{1}{2}mg(l-h)$ $\qquad v = \sqrt{g(l-h)}$

$5/2mgh = 5/2mgl - mgl => 5/2mgh = 3/2mgl$

$5h = 3l => h = 3l/5$

**Q3-(25 pts)** A block of mass $m_A$ and a platform of mass $m_B$ are initially at rest. A bullet of mass $m$ hits the block with velocity $v_o\ \hat{\imath}$, and gets stuck in it at t = 0. After sliding a short distance of $d$ on the platform, the block comes to a rest at $t = T$. Assume that the coefficient of the kinetic friction between the block and the platform is $\mu_k$ and that there is no friction between the platform and the ground.




**a)** With respect to the ground, find the velocities of the block and the platform immediately after the bullet is stuck, that is for t = 0+ (6 pts).

**Sol.** By momentum conservation law

Momentum before collision=Momentum after collision

$$m v_0 = (m+m_A) v_A(0^+) + m_B V_B(0^+) \qquad (1)$$

When the bullet strikes with the block, it sticks with the block and comes to rest so the final velocity of the bullet becomes zero i.e. $V_B(0^+)=0$

From equation (1), the velocity of the block can be found as

$$v_A(0^+) = \frac{m v_0}{(m+m_A)}$$

**b)** With respect to the ground, find the velocity of the block and the platform long after the bullet is stuck, that is for $t > T$ (6 pts).

Sol. For t >T, the block and platform has the same velocity, i.e. $v_A$(t)= $v_B$(t)

Equation (1) becomes

$m v_0 = (m+m_A) v_A(t) + m_B V_A(t)$

$$V_A(t) = \frac{m v_0}{(m+m_A+m_B)} \qquad (1a)$$

**c) Sol.** For block , consider the Fig. 1

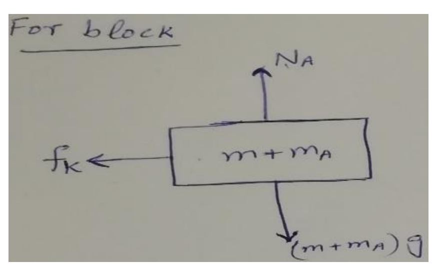



**Fig. a**

$$a_A(t) = -\frac{f_k}{(m+m_A)} = -\mu g \qquad \text{where } f_k = (m+m_A)\,\mu g$$

$V_A$(t) = $\frac{mv_0}{(m+m_A)} - \mu g t$ (1a) First kinematics equation in friction and gravity form

$x_A$(t) = $\frac{mv_0}{(m+m_A)} t - \frac{1}{2}\mu g t^2$ with $0^+ \leq t \leq T$

**Now for platform, consider the figure b**

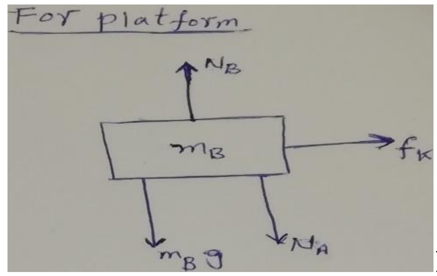



**Fig. b**

$a_B$(t)= - $\frac{f_k}{m_B}$= $\mu g(\frac{m+m_A}{m_B}$ ) => $v_B$(t)=$\mu g(\frac{m+m_A}{m_B}$ )t (1b)

$x_B$(t) = $\frac{1}{2}\mu g(\frac{m+m_A}{m_B})t^2$ Second kinematics equation

Since $v_A$(t)= $v_B$(t) from 1a and 1b

$$\frac{mv_0}{(m+m_A)} - \mu g t = \mu g(\frac{m+m_A}{m_B})t \quad t \rightarrow T$$

Solving for T,

$$T = \frac{m m_B v_0}{(m+m_A)(m+m_A+m_B)}$$

**d)** Sol. We have $x_A$(T)= $x_B$(T) + d (1)

$x_A$(T) = $\frac{mv_0}{\mu g(m+m_A)} \frac{m m_B v_0}{(m+m_A)(m+m_A+m_B)}$ - $\frac{1}{2}$ $\mu g/(\mu g)^2(\frac{m m_B v_0}{(m+m_A)(m+m_A+m_B)})^2$

$x_A$(T) = $\frac{m m_B v_0}{\mu g(m+m_A)(m+m_A+m_B)}[\frac{mv_0}{m+m_A} - \frac{1}{2\mu g} \frac{m m_B v_0}{(m+m_A)(m+m_A+m_B)}]$

$x_A$(T) = $\frac{{v_0}^2 m^2\, m_B(2m+2m_A+m_B)}{2\mu g[(m+m_A)(m+m_A+m_B)]^2}$ (i)

$Now\ x_B$(T)

$x_B$(T)= $\frac{1}{2}\mu g \frac{{v_0}^2 m^2\, {m_B}^2}{(\mu g)^2[(m+m_A)(m+m_A+m_B)]^2}(\frac{m+m_A}{m_B})$

$x_B$(T)= $\frac{{v_0}^2 m^2\, m_B}{2\mu g(m+m_A)(m+m_A+m_B)^2}$ (ii)

Now for d, from 1

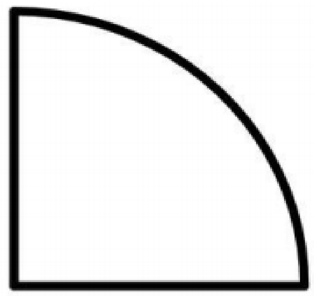
$d = x_A$(T)$-x_B$(T) substituting i and ii

$$d = \frac{{v_0}^2 m^2\, m_B}{2\mu g(m+m_A)(m+m_A+m_B)^2}\left[\frac{(2m+2m_A+m_B)}{(m+m_A)} - 1\right]$$

$$d = \frac{{v_0}^2 m^2\, m_B}{2\mu g(m+m_A)^2(m+m_A+m_B)} \qquad \text{(iii)}$$

**Q4-(25 pts)** A long wire with total mass $M$ is bent into the shape of a quarter circle with radius $R$ that is shown in the figure. We want to calculate the kinetic energy of this wire when it is rotating in the *x-y* plane through an axis that is passing (out of the page) through its center of mass. For this purpose, we will use the parallel axis theorem.

**Hint:** feel free to use $I_{CM} = \frac{mL^2}{12}$ for a rod of mass $m$ and length $L$.



**a)** What is the parallel axis theorem (5 pts)?

Ans. As we know the parallel axis theorem

$I_p = I_{CM} + Md^2$ where $I_p$ is the moment of inertia at point of the rigid body.

**b)** Find the position of the center of mass of the wire (8 pts)

**Sol.** $r_{CM}$=1/m$\int_0^r dm.r$

=>2R/π$\int_0^{\pi/2}(cos\theta i + sin\theta j)d\theta$ where dm=2m/πdθ and =R$(cos\theta i + sin\theta j)$

$r_{CM}$ =2R/π $(i + j)$

**c) Sol.** Mass density $= \frac{M}{2+\frac{\pi R}{2}}$

$\frac{M}{2+\frac{\pi}{2}}$ = m (1) $\frac{M}{2+\frac{\pi}{2}}$ = m (2) $\frac{M}{2+\frac{\pi}{2}}\frac{\pi}{2}$= m (3)

From 1, 2 and 3 the mass density become, $(\frac{2}{3}+\frac{\pi}{2})(\frac{M}{2+\frac{\pi}{2}})$R

Now, the moment of inertia of the entire wire can be found as

$$I_0 = \frac{4+3\pi}{3(4+\pi)} MR^2$$

**d)** Find the moment of inertia of the entire wire with respect to a rotation axis that is passing through its center of mass (4 pts)

**Sol.** $I_{CM} = I_0 + M{r_{CM}}^2$

$$r_{CM} = [\frac{1}{(2+\frac{\pi}{2})^2} + \frac{1}{2+\frac{\pi}{2}}]\mathrm{i}+[\frac{1}{(2+\frac{\pi}{2})^2} + \frac{1}{2+\frac{\pi}{2}}]\mathrm{j}$$

$$r_{CM} = \frac{3R}{2(2+\frac{\pi}{2})}(\mathrm{i}+\mathrm{j})=\frac{3R}{4+\pi}(\mathrm{i}+\mathrm{j})$$