## Final- Problem 1

Uniform disk of mass $M=0.5 \mathrm{~kg}$ and radius $R=0.2 \mathrm{~m}$ moves towards another disk of mass $2 M$ and radius $2 R$ on the surface of a frictionless table. The first disk has an initial velocity of $v_{0}=2.4$ $\mathrm{m} / \mathrm{s}$ and angular velocity $\omega_{0}=15 \mathrm{rad} / \mathrm{s}$ as indicated in the figure, and the second disc is initially stationary. Disks instantly stick to each other when they contact and move as a single object.
$\mathrm{I}_{\mathrm{cm}}=1 / 2 \mathrm{MR}^{2}$.


## Part A

Calculate the center of mass velocity of the combined disk after the collision.
Express your answer with the appropriate units.
ANSWER:

$$
v_{0}=
$$

## Part B

Calculate the angular velocity of the combined disk after the collision.
ANSWER:
$\square$

Part C - For what value of $\omega_{0}$ would the combined disk not rotate?
ANSWER:

```
\omega
rad/s
```

Part $\mathbf{D}$ - How much total mechanical energy is lost in this collision at part C assuming that the combined disk system is not rotating?

ANSWER:

## J

## Phys101F21_FNL_Q2

A uniform rigid rod of mass $m=7.00 \mathrm{~kg}$, length $l=1.00 \mathrm{~m}$ is free to rotate about an axis through its left end. The right (free) end of the rod is supported from below to keep the rod horizontally at rest. Note the gravitational acceleration $g=9.81 \mathrm{~m} / \mathrm{s}^{2}$. A small ball of mass $m=7.00 \mathrm{~kg}$ hits the road at a distance $d=0.73 \mathrm{~m}$ from its rotation axis with velocity $v_{0}=5.41 \mathrm{~m} / \mathrm{s}$, that makes an angle $\theta=21$ ${ }^{\circ}$ with the vertical direction as indicated on the figure. The ball immediately sticks to the rod.


## Part A

Calculate the magnitude of the angular momentum of the ball with respect to the rod's rotation axis just before the collision.
ANSWER:

$$
L_{\text {ball }}=\square \mathrm{kgm}^{2} / \mathrm{s}
$$

## Final Problem3

A thin rod of mass $m=1 \mathrm{~kg}$, length $L=0.4 \mathrm{~m}$ is hinged at a distance $d=L / 5$ from one end, and also attached to a spring horizontally from the same end as shown in the figure. The spring constant is $k=10 \mathrm{~N} / \mathrm{m}$ . There is no friction.


## Part A

Calculate the moment of inertia of the rod about the hinge pivot.
ANSWER:
$I_{\text {rod }}=\square \mathrm{kgm}^{2}$

## Part B

Calculate the frequency of oscillations when the rod is displaced by a small angle $\theta$ from the vertical position. Assume that the spring remains always horizontal during motion. Use the approximations $\sin \theta \approx \theta$ and $\cos \theta \approx 1$

ANSWER:

$$
\omega=\square s^{-1}
$$

## Part C

The rod is initially displaced from its vertical position by $\theta_{0}=\pi / 12$, and it has no angular velocity. Claculate the maximum linear speed of the rod's free (bottom) end.

ANSWER:

$$
v=\square \mathrm{m} / \mathrm{s}
$$

## Phys101F21_FNL_Q4

This problem is formulated in terms of the planetary data of Earth: mass $M_{\mathrm{E}}=5.97 \times 10^{24} \mathrm{~kg}$, radius $R_{\mathrm{E}}=6370 \mathrm{~km}$, rotation period $T_{\mathrm{E}}=24 \mathrm{hrs}$.

A planet has mass $M_{1}=0.82 * M_{\mathrm{E}}$, radius $R_{1}=0.72^{*} R_{\mathrm{E}}$, and axial rotation period $T=0.95^{*} T_{\mathrm{E}}$. The planet has a moon of mass $M_{\text {moon }}=1.70 \times 10^{-2}{ }^{*} M_{\mathrm{E}}$ with a circular orbit of radius $r=62^{*} R_{\mathrm{E}}$ from the center of the planet. A communication satellite of mass $m=105 \mathrm{~kg}$ is to be launched from the surface of the planet. The gravitational constant is $G=6.67 \times 10^{-11} \mathrm{~m}^{3} \mathrm{~kg}^{-1} \mathrm{~s}^{-2}$.
(Drawing is not to scale)


## Part A

(For this part, ignore the presence of the moon completely). Calculate the radius of the geo-stationary orbit of the planet for the satellite (a geo-stationary orbit has the same axis and the period of rotation as the planet itself).

THE ANSWER MUST BE GIVEN IN KILOMETERS.
ANSWER:

```
rgeo }=\square\textrm{km
```


## Part B

Calculate the kinetic energy of the satellite at the geostationary orbit.
THE ANSWER MUST BE GIVEN IN KILOJOULES (kJ)
ANSWER:

$$
K_{\text {satellite }}=\square \mathrm{kJ}
$$

## Part C

Suppose that we would like to launch the satellite into deep space from a circular orbit around the planet. The launch will happen when the planet, the moon and the satellite are aligned on a radial axis as shown. Let $x$ be the distance of this orbit from the center of the planet. Calculate $x$ for which the escape speed will be minimum. (Hint: The moon must be taken into account in solving this problem.)

THE ANSWER MUST BE GIVEN IN KILOMETERS.
ANSWER:

$$
x=\square \mathrm{km}
$$

